Since $g(x)$ and $G(x)$ are the solutions of D.Es. (2) and (1) respectively, therefore they will satisfies these equations and we have

$$a_0 g(x) + a_1 g(x+1) + \cdots + a_n g(x+n) = 0 \qquad (3)$$

and

$$a_0 G(x) + a_1 G(x+1) + \cdots + a_n G(x+n) = Q(x) \qquad (4)$$

Adding (3) and (4) we get

$$a_0 [g(x) + G(x)] + a_1 [g(x+1) + G(x+1)] + \cdots + a_n [g(x+n) + G(x+n)] = Q(x)$$

Hence $g(x) + G(x)$ is also a solution of (2)

<u>Exercise-1</u> Solve the following D.E.

$$u_{x+2} - 5u_{x+1} + 6u_x = 36$$

<u>Solution</u> – The given D.E. can be written as

$$(E^2 - 5E + 6)u_x = 36$$

$$\therefore K(E) = (E-2)(E-3)$$

$$\Rightarrow K(1) = (1-2)(1-3) = (-1)\times(-2) = 2$$

Hence particular solution of the given D.E is

$$u_x = \frac{36}{2} = 18$$

The general solution of the corresponding homogeneous D.E. is

$$u_x = C_1 2^x + C_2 3^x$$

Hence the general solution of the given non-homogeneous D.E. is

$$u_x = C_1 2^x + C_2 3^x + 18$$

<u>Exercise-2</u> Solve the D.E.

$$u_{x+2} - 5u_{x+1} + 6u_x = 5^x$$

<u>Solution</u> — In this equation $Q(x) = 5^x$ and $K(E) = E^2 - 5E + 6$