Life table or Mortality Table : (1) The life table gives the life history of a hypothetical group as it is gradually diminished by deaths. (2) It is a conventional method of expressing the most fundamental and essential fact about the age distn of mortality in tabular form and is a powerful tool for measuring the probability of life and death of various age sectors.

(3) The life table thus gives a summary of the mortality experience of any popn group during a given period and is a very effective and comprehensive method for providing concise measures of the longevity of that popn

(4) The data used for constructing a life table are the census data and death registration data.

(5) Life tables are generally constructed for various sections of the people which, as experience shows, have sharply different patterns of mortality. Life tables are as well constructed on regional basis and other factors accounting differential mortality.

Assumptions : The following are a few simplified assumptions which are used in the construction of the life tables.

(1) The Cohort is closed for emigration or immigation. In other words, there is no change in the census except the losses due to deaths.

(2). Individuals die at each age according to pre-determined schedule which is fixed and does not change.

(3). The Cohort originates form some standard number of births, say 10,000 or 1,00,000 which is called the radix of the table.

(4) The deaths are distributed uniformly over the period (x, x+1) for each x (except for first few years). In other words deaths are uniformly distributed between one birthday and the next.

Description of a life table: A typical life table has generally the following columns:

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
x	l_x	d_x	q_x	p_x	m_x	μ_x	L_x	T_x	$\mathring{e}_x$	e_x

(1) The first column of the life table is age and is represented by x. It may be exact age or an age interval.

(2) l_x is the number of persons living at any specified age x in any year out of an assumed number of birth, say, l_0 usually called the cohort or radix of the life table.

(3) d_x is the no of persons, among the l_x persons (reaching age x), who die before reaching ~~the~~ age (x+1). Obviously we have $\boxed{d_x = l_x - l_{x+1}}$

(4) q_x is the probability that a person of exact age x will die ~~within one year following the attainment of that age~~ before reaching age (x+1). Thus we have

$$\boxed{q_x = \frac{d_x}{l_x}}$$

(5) p_x is the prob that a person aged x survives upto age (x+1). Thus we have

$$\boxed{p_x = \frac{l_{x+1}}{l_x} = 1 - q_x}.$$

(6) m_x (central mortality ~~rate~~ or death rate) is the prob that a person whose exact age is not known but lies in between x and (x+1) will die within one year following the attainment

of that age. Thus

$$\boxed{m_x = \frac{\text{number of deaths within age interval } x \text{ to } (x+1)}{\text{average } l_x \text{ of the cohort in that interval}}}$$

$$= \boxed{\frac{d_x}{L_x}} = \frac{d_x}{l_x - \frac{d_x}{2}} = \frac{2\frac{d_x}{l_x}}{2 - \frac{d_x}{l_x}} = \boxed{\frac{2q_x}{2-q_x}}$$

(7) μ_x is the force of mortality at age x is defined as the ratio of instantaneous rate decrease in l_x to the value of l_x. Thus

$$\boxed{\mu_x = -\frac{1}{l_x} \cdot \frac{d}{dx} l_x = -\frac{d}{dx}(\log l_x)}$$

It gives 'nominal annual rate of mortality'.

(8) L_x is the no. of years lived in the aggregated by cohort of l_0 person between age x and $(x+1)$.

Thus, $\boxed{L_x = \int_0^1 l_{x+t}\, dt = \frac{l_x + l_{x+1}}{2}}$

(9) T_x is the total no of years lived by the cohort l_0 after attaining the age x i.e. T_x is the total future life time of the l_x persons who reach age x.

Thus $\boxed{T_x = L_x + L_{x+1} + L_{x+2} + \cdots}$

(10) e_x is average no of complete years of life lived after age x by any of the l_x persons who attain ~~of the given~~ age x. (curtate expectation of life)

$$e_x = \frac{\sum_{i=0}^{\omega} i\, d_{x+i}}{l_x} = \frac{(d_{x+1} + 2d_{x+2} + 3d_{x+3} + \cdots)}{l_x}$$

$$= \frac{1}{l_x}[l_{x+1} + l_{x+2} + \cdots] = \frac{\sum_{n=1} l_{x+n}}{l_x}$$

(11) $\mathring{e}_x$ the average no. of years lived after age x by each of the l_x persons who attain that age. It is called the (complete) expectation of life (or life expectancy) at age x and is obtained from the relation

$$\mathring{e}_x = e_x + \frac{1}{2} = \frac{T_x}{l_x}$$

The columns (5), (6), (7) do not occur in all the life tables. However, the remaining columns are a must in any life table. → $\mathring{e}_0$, the expectation of life at age 0, is the average age at death, or the average longevity of a person belonging to the given community.

Construction of life table : It will be seen, as discussed below, that the complete life table can be constructed if we can compute the quantities q_x (or p_x) for all $x \geq 0$. The only other data which is needed is the radix l_0. The q_x column is then called the pivotal column of the life table.

Starting with radix l_0 and q_x ($x = 0, 1, 2, \ldots$), we have

$$d_0 = l_0 q_0 \Rightarrow l_1 = l_0 - d_0$$
$$d_1 = l_1 q_1 \Rightarrow l_2 = l_1 - d_1$$

and so on.

We are now in a position to complete l_x, d_x columns. After completing l_x, d_x, we construct L_x, T_x and e_x can also be completed.

In this way we observe that all the columns of life table can be completed, if the value of q_x are given for all ages.

Uses of Life table : Although the basic objective of life tables is to give a clear picture of the age distn of mortality in a given popn group. Today life table is widely accepted as important basic material in demographic and public health studies. In the words of W. Farr, life table is the 'Biometer' of the popn. We enumerate below some of imp. applications of life tables.

① Life tables are indispensable for the solution of all questions concerning the duration of human life.

②. Life tables are used by demographers to devise measures such "Net Reproduction Rate" to study the rate of growth of pop^n.

③ Life tables for two or more different groups of pop^n may be used for the relative comparison of various measures of mortality such as death rate, expectation of life at various ages etc.

④ A life table is useful from the points of view of business and Government as well. It is employed by life insurance companies in determining rates of premium for policies of persons of different ages, while the Government or a firm may use it for the determination of rates of retirement benefits for its employees.

Stationary population: A pop^n is said to be stationary if it is of constant size and constant age and sex composition over time. Such a pop^n may be conceived of under the following conditions.

(i) if every year, the no. of birth is exactly l_0 (say) and is equal to the no. of deaths and these are distributed uniformly throughout the year, and

(ii) if the pop^n is not affected by emigration or immigration.

Under the above conditions, in each year the no. of persons between the age x and $(x+1)$ denoted by L_x will always be the same. Thus the columns L_x and T_x of the life table may be interpreted as giving respectively the age-$dist^n$ and the no. of persons with age x or more in a stationary pop^n.

Stable Population : Concept of a stable popn was due to A.J. Lotka and is very much akin to that of Stationary popn. A popn is said to be stable.
(i) if it has a fixed age and sex distn
(ii) if constant mortality and fertility rates are experienced at each age, and
(iii) if the popn is closed to emigration or immigration.

In other words for a stable popn the overall rates of births and deaths remain constant and consequently such a popn increases at a constant rate, thus supporting the compound interest law of popn growth.

In particular, if the constant overall birth and death rates are equal, then the popn size remain fixed and in this case stable popn becomes stationary popn.

Ex- Prove $L_x = \frac{1}{2}(l_x + l_{x+1})$.

Pf. L_x is the no. of years lived in the aggregated by Cohort of l_0 persons between age x and $(x+1)$. Thus, if deaths are assumed to be uniformly distributed over the whole year or equivalently, if we assume the linearity of l_{x+t} for $t \in [0,1]$, then

$$l_{x+t} = l_x - t d_x \quad (*)$$

we get $L_x = \int_0^1 l_{x+t}\, dt$ and

$$= \int_0^1 (l_x - t d_x)\, dt = l_x |t|_0^1 - d_x \left|\frac{t^2}{2}\right|_0^1$$

$$= l_x - \frac{d_x}{2} \overset{(*,*)}{=} l_x - \frac{1}{2}(l_x - l_{x+1})$$

$$\boxed{L_x = \frac{1}{2}(l_x + l_{x+1})}.$$

From $(*)$ and $(*,*)$

$$\boxed{L_x = l_{x+\frac{1}{2}}}.$$

Th. $_nP_x = P_x \cdot P_{x+1} \cdots P_{x+n-1}$

Pf: we know that $_nP_x = \frac{l_{x+n}}{l_x}$

$$= \frac{l_{x+1}}{l_x} \cdot \frac{l_{x+2}}{l_{x+1}} \cdots \frac{l_{x+n}}{l_{x+n-1}}$$

$$= p_x \cdot p_{x+1} \cdots p_{x+n-1}$$

Th: $_nq_x = \frac{d_{x+n-1}}{l_x}$

Pf. $_nq_x$ = Prob that a person aged x may die in nth year
= Prob that a person aged x survives till age $(x+n-1)$ but dies in the age period $(x+n-1, x+n)$
= P(A person aged x survives for $(n-1)$ years)
× P(A person aged $x+n-1$ dies within one year)

$$= \frac{l_{x+n-1}}{l_x} \times \frac{d_{x+n-1}}{l_{x+n-1}} = \frac{d_{x+n-1}}{l_x}.$$

Coor $_nP_x - {}_{n+1}P_x = \frac{l_{x+n}}{l_x} - \frac{l_{x+n+1}}{l_x} = \frac{l_{x+n} - l_{x+n+1}}{l_x}$

$$= \frac{d_{x+n}}{l_x} = {}_{n+1}q_x$$

Ex: Given the following table for l_x, the no. of rabbits living at age x, complete the life table for rabbits.

Sol.

x	0	1	2	3	4	5	6
l_x	100	90	80	75	60	30	0

x, y, z are three rabbits of age 1, 2 and 3 years resp. Find the prob that
(i) at least one of them will be alive for one year more
(ii) x, y, z will be alive for two years time.
(iii) One of the three is alive in two years, and
(iv) all will be dead in two years time.

Sol: The complete life table for the above data is given below

Age x	l_x	$d_x = l_x - l_{x+1}$	$q_x = \frac{d_x}{l_x}$	$L_x = \frac{l_x + l_{x+1}}{2}$	$T_x = \sum L_x$	$e_x^0 = \frac{T_x}{l_x}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0	100	10	0·10	95	385	3·85
1	90	10	0·11	85	290	3·22
2	80	5	0·06	77·5	205	2·56
3	75	15	0·20	67·5	127·5	1·70
4	60	30	0·50	45	60	1·0
5	30	30	1·00	15	15	0·5
6	0	—	—	—	—	—

(i) Let p_1 = Prob that x will die in one year $= \frac{d_1}{l_1} = \frac{10}{90}$

p_2 = " " y " " " $= \frac{d_2}{l_2} = \frac{5}{80}$

p_3 = " " z " " " $= \frac{d_3}{l_3} = \frac{15}{75}$

Hence the prob that all will die in one year is

$$p = p_1 \cdot p_2 \cdot p_3 = \frac{10}{90} \cdot \frac{5}{80} \cdot \frac{15}{75} = \frac{1}{720}$$

$\therefore$ P[at least one will survive for one year more]

$= 1 - P$[none will survive for one year more]

$= 1 - \frac{1}{720} = \frac{719}{720}$

(ii) Let E_1, E_2, E_3 denote the event that x, y and z survive for two year more resp. then

$P(E_1) = \frac{l_3}{l_1} = \frac{75}{90} = \frac{5}{6}$, $P(E_2) = \frac{l_4}{l_2} = \frac{60}{80} = \frac{3}{4}$

$P(E_3) = \frac{l_5}{l_3} = \frac{30}{75} = \frac{2}{5}$

Hence the required prob that all the three will survive for two years is given by

$= P(E_1 \cap E_2 \cap E_3) = P(E_1) \cdot P(E_2) \cdot P(E_3) = \frac{5}{6} \cdot \frac{3}{4} \cdot \frac{2}{5} = 0.25$

(5)

iii) Let E denote the event one of three rabbits is alive for two years more. Then

$$P(E) = P(E_1 \cap \overline{E_2} \cap \overline{E_3}) + P(\overline{E_1} \cap E_2 \cap \overline{E_3}) + P(\overline{E_1} \cap \overline{E_2} \cap E_3)$$

$$= P(E_1) \cdot P(\overline{E_2}) \cdot P(\overline{E_3}) + \text{---} \quad \text{---} + \text{---} \quad \text{---}$$

$$= \frac{5}{6} \cdot \frac{1}{4} \cdot \frac{3}{5} + \frac{1}{6} \cdot \frac{3}{5} \cdot \frac{3}{5} + \frac{1}{6} \cdot \frac{1}{4} \cdot \frac{2}{5} = \frac{26}{120} = 0.2167$$

(iv) P[all will die in two years]

$$= P(\overline{E_1} \cap \overline{E_2} \cap \overline{E_3}) = P(\overline{E_1}) \cdot P(\overline{E_2}) \, P(\overline{E_3})$$

$$= \frac{1}{6} \cdot \frac{1}{4} \cdot \frac{3}{5} = \frac{1}{40} = 0.025.$$

Ex — Complete the life table of the pop^n of a certain type of insects, x being the age in days and $l_x = 1000$ for $x = 0$:

x --	0	1	2	3	4	5	6	7	8
q_x --	0.120	0.005	0.010	0.050	0.100	0.500	0.800	0.900	0.952

Sol — Since we are given the values of q_x, in order to complete the life table, first of all we shall find the value of l_x, $(x = 0, 1, \text{--} 8)$ by using the relations $q_x = \dfrac{d_x}{l_x}$ and $l_{x+1} = l_x - d_x$ —(*)

we are given $l_0 = 1000$

$\therefore$

$$d_0 = l_0 q_0 = 1000 \times 0.12 = 120 \quad ; \; l_1 = l_0 - d_0 = 1000 - 120 = 880$$

$$d_1 = l_1 q_1 = 880 \times 0.05 = 4 \quad ; \; l_2 = l_1 - d_1 = 880 - 4 = 876$$

$$d_2 = l_2 q_2 , \; l_3 = l_2 - d_2 \quad \text{and so on.}$$

The values of l_x and d_x $(x = 0, \text{--} 8)$ obtained on using (*) repeatedly are given in the following table.

x	q_x	l_x	$d_x = l_x q_x$	
0	0.120	1000	$1000 \times 0.120 = 120$	The remaining columns of the life table, viz L_x, T_x and e_x^0 can be computed as above examples.
1	0.005	$1000 - 120 = 880$	$880 \times 0.005 = 4$	
2	0.010	$880 - 4 = 876$	$876 \times 0.10 = 8.76 \simeq 9$	
⋮	⋮	$876 - 9 = 867$	⋮	
⋮	⋮	⋮		

Ex: The no. of persons dying at age 75 is 476 and the complete expectation of life at 75 and 76 years are 3.92 and 3.66 years. Find the nos living at ages 75 and 76.

Sol. We have. $d_{75} = 476$, $\mathring{e}_{75} = 3.92$, $\mathring{e}_{76} = 3.66$

We want l_{75} and l_{76}.

We have $e_{75} = \mathring{e}_{75} - \frac{1}{2} = 3.42$

$e_{76} = \mathring{e}_{76} - \frac{1}{2} = 3.16$

We know. $$\boxed{p_x = \frac{l_{x+1}}{l_x} = \frac{e_x}{1+e_{x+1}}}$$

$$\therefore \quad l_{76} = \frac{l_{75}\, e_{75}}{1+e_{76}} = \frac{3.42}{4.16}\, l_{75}$$

Also. $d_x = l_x - l_{x+1}$

$d_{75} = l_{75} - l_{76} = 476$

$$\Rightarrow \quad l_{75} = \frac{3.42}{4.10}\, l_{75} + 476 \quad \Rightarrow l_{75} = 2675$$

and $l_{76} = l_{75} - d_{75} = 2675 - 476 = 2199$.

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Abridged Life table : (6)

In complete life table, the age interval is a year throughout the table and the life table functions such as l_x, d_x, q_x, m_x etc, are given for all integral values of x. On the other hand in abridged life table, as the name suggests, the values of these fun^s are given

(i) either for some integral values of x which are at some distance apart, usually 5 years or 10 years.

(ii) or they are given for age groups of values of x, usually of width 5 years or 10 years.

The principal methods used for the construction of abridged life tables are

(a) Reed-Merrell method (b) Greville's method

(c) King's method

Of these, (a) and (b) are meant for the type of abridgement in (ii) and (c) is used for type of abridgement in (i)

A typical abridged life table consists of the following columns.

1. x to $(x+n)$ exact age interval
2. l_x, the no. of persons out of a cohort of l_0 persons, living at the begning of the interval x to $(x+n)$.
3. ${}_nq_x$, the prob of persons dying in the age interval x to $x+n$ and is given by
$${}_nq_x = 1 - {}_np_x = 1 - \frac{l_{x+n}}{l_x}.$$
4. ${}_nd_x$ the no. of deaths in the interval x to $(x+n)$ and is given by
$${}_nq_x = \frac{{}_nd_x}{l_x} \Rightarrow {}_nd_x = l_x \times {}_nq_x.$$
5. ${}_nL_x$, the no. of members of the life table stationary pop^n in the age group $(x, x+n)$ and is given by
$${}_nL_x = \int_0^n l_{x+t}\,dt.$$

(6) $T_x = \int_0^\infty l_{x+t}\, dt$, is the no of persons lived after age x.

(7). $\mathring{e}_x = \frac{T_x}{l_x}$. Complete expectation of life at age x.

King's Method : (A method of constructing abridged life table)

Suppose the life table fn's $q_x, l_x, \mathring{e}_x$ are to be given at 5 years intervals in the abridged table. Then the first step would be to compute probabilities of death q_x at the pivotal ages by the usual procedure. Then we are also able to get the column $p_x = 1 - q_x$

Now for calculating other life table fn's, we first consider the computation of l_x at the pivotal ages.

Here we note that

$$l_{x+5} = l_x \times {}_5p_x$$

or $$\log l_{x+5} = \log l_x + \log {}_5p_x$$

Therefore it is necessary to estimate ${}_5p_x$ from the given p_x values for the first pivotal ages. ${}_5p_x$ can be evaluated from Newton's forward formula ignoring the differences higher than third. Then we get

$$\log p_{x+1} = E^{\cdot 2} \log p_x \qquad \left[\because \log({}_np_x) = \sum_{i=0}^{n-1} \log p_{x+i}\right.$$

$$= (1+\Delta)^{\cdot 2} \log p_x$$

$$= \log p_x + \cdot 2\, \Delta \log p_x - \cdot 08\, \Delta^2 \log p_x + 0\cdot 48\, \Delta^3 \log p_x$$

$$\log p_{x+2} = E^{\cdot 4} \log p_x = (1+\Delta)^{\cdot 4} \log p_x$$

$$= \log p_x + \cdot 4\, \Delta \log p_x - \cdot 12\, \Delta^2 \log p_x + 0\cdot 64\, \Delta^3 \log p_x$$

$$\log p_{x+3} = E^{\cdot 6} \log p_x = (1+\Delta)^{\cdot 6} \log p_x$$

$$= \log p_x + \cdot 6\, \Delta \log p_x - \cdot 12\, \Delta^2 \log p_x + 0\cdot 56\, \Delta^3 \log p_x$$

$$\log p_{x+4} = E^{\cdot 8} \log p_x = (1+\Delta)^{\cdot 8} \log p_x$$

$$= \log p_x + \cdot 8\, \Delta \log p_x - \cdot 08\, \Delta^2 \log p_x + 0\cdot 32\, \Delta^3 \log p_x$$

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Hence we get

$${}_5p_x = p_x \cdot p_{x+1} \cdot p_{x+2} \cdot p_{x+3} \cdot p_{x+4}$$

$$\log {}_5p_x = \sum_{i=0}^{4} \log p_{x+i}$$

$$= 5\log p_x + 2\Delta \log p_x + .4\Delta^2 \log p_x + .2\Delta^3 \log p_x$$

$$= 2.4 \log p_x + 3.4 \log p_{x+5} - \log p_{x+10} + .2 \log p_{x+15} \quad \text{——} ①$$

This last eq^n is obtained by the relation

$$\Delta^r \log p_x = (E^5 - 1)^r \log p_x$$

For the remaining pivotal ages, one can also use Everett's formula, which is as follows

$$\log p_x = E^5 \log p_{x-5} = (1+\Delta)^5 \log p_{x-5}$$

$$= \log p_{x-5} + \Delta \log p_{x-5}$$

Using this formula, we get

$$\log p_{x+1} = \log p_{x-5} + 1.2\Delta \log p_{x-5} + .12\Delta^2 \log p_{x-5} - 0.032\Delta^3 \log p_{x-5}$$

$$\log p_{x+2} = \log p_{x-5} + 1.4\Delta \log p_{x-5} + .280\Delta^2 \log p_{x-5} - 0.056\Delta^3 \log p_{x-5}$$

$$\log p_{x+3} = \log p_{x-5} + 1.6\Delta \log p_{x-5} + .480\Delta^2 \log p_{x-5} - 0.064\Delta^3 \log p_{x-5}$$

$$\log p_{x+4} = \log p_{x-5} + 1.8\Delta \log p_{x-5} + .72\Delta^2 \log p_{x-5} - 0.048\Delta^3 \log p_{x-5}$$

$$\therefore \log {}_5p_x = \sum_{i=0}^{4} \log p_{x+i}$$

$$= 5\log p_{x-5} + 7\Delta \log p_{x-5} + 1.6\Delta^2 \log p_{x-5} - 0.2\Delta^3 \log p_{x-5} \quad \text{——} ②$$

Similarly we obtain the sum

$$N'_{x\,\overline{5|}} = \sum_{i=1}^{5} l_{x+i}$$

for each pivotal age. These sums are similar to eq^n ① and ②.

The formula corresponding to ① is

$${}_5L_x = N'_{x\,\overline{5|}}$$

$$= 5l_x + 2\Delta l_x - .4\Delta^2 l_x + .2\Delta^3 l_x$$

$$= 2.4\, l_x + 3.4\, l_{x+5} - l_{x+10} + .2\, l_{x+15} \quad \text{—} ③$$

and corresponding to ②

$$_5L_x = N'_{x\,\overline{5}|}$$

$$= 5l_{x-5} + 7\Delta l_{x-5} + 1.6\Delta^2 l_{x-5} - .2\Delta^3 l_{x-5} \quad \text{④}$$

using the expression ③ and ④ we complete the column $N'_{x\,\overline{5}|}$.

In case its value comes out to be negative, we pressume it to be zero. Now, ~~we~~ finally we calculate

$$N_x' = \sum_{t=1}^{\infty} l_{x+t} = N'_{x\,\overline{5}|} + N'_{x+5}$$

Lastly, one evaluates $\mathring{e}_x$ for the pivotal ages by using the fact that

$$\mathring{e}_x = \frac{\int_0^{\infty} l_{x+t}\,dt}{l_x} = \frac{T_x}{l_x} \simeq \frac{\frac{1}{2} l_x + N_x'}{l_x}$$

$$= 0.5 + \frac{N_x'}{l_x}.$$

Since. $T_x = \int_0^{\infty} l_{x+t}\,dt = \sum_{t=0}^{\infty} L_{x+t}$

$$= \frac{1}{2}\sum_{t=0}^{\infty} l_{x+t} + \frac{1}{2}\sum_{t=1}^{\infty} l_{x+t}$$

$$= \frac{1}{2} l_x + \sum_{t=1}^{\infty} l_{x+t}$$

$$= \frac{1}{2} l_x + N_x'.$$

$$L_x = \frac{l_x + l_{x+1}}{2}$$

$$L_{x+1} \simeq \frac{l_{x+1} + l_{x+2}}{2}$$

Vital Statistics : (Demography) Vital Statistics is defined as that branch of Biometry which deals with data and the laws of human mortality, morbidity and demography. By vital events, again we mean such events of human life as birth, death, sickness, migration marriage, divorce, adoption etc.

Uses : Vital statistics are being extensively used in almost all the spheres of human activity. We outline below some of the imp application of vital statistics.

1. Study of popn trend. The vital statistics, as already pointed out, reflect the changing pattern of popn of any region, community or country in terms of the no. of births, deaths and marriages.
2. Use in Public Administration. The study of popn estimation, popn projection, popn movement, i.e., popn estimation together with birth and other allied studies together with birth and death statistics according to age and sex distn provides any administration with fundamental tools which are indispensable for the overall planning and evaluation of economic and social development programmes.
3. Mortality and natality statistics also provide guide posts for use by the researchers in medical and pharmaceutical profession
4. The facts and figures relating to births, deaths and marriages are of extreme importance to various official agencies for a variety of administrative purposes.
5. The whole of actuarial science, including life insurance is based on the mortality or life table.

Sources of Vital Statistics: The raw data of vital statistics are generally obtained from the following sources.

(a) Census: Popⁿ censuses are now undertaken in almost all countries, generally at ten-year intervals. A census may be defined as an enumeration at specified time of the individuals living in specified area; during which particulars are also collected regarding age, sex, some socio-economic or familial characteristic of the individuals.

(b) Vital Statistics registers: In many countries, there is a system of registering the occurrence of every imp vital event under legal requirement. For instance, when a child is born, the matter has to be reported to the proper authorities, together with such information as the age of mother, religion of parents etc.

(c) Hospital records: every hospital (as well as health centre or nursing home), maintains a record, for each patient, of such particulars as ~~the~~ the age, sex, etc., of the patients, the nature of illness, the type of treatment administered, and the outcome.

(d) Adhoc survey: In countries with defective registration systems, occasionally surveys are conducted to collect data on vital events. Some round of NSS in India, for instance, have been used to collect such data.

Measurement of Mortality: The following are the principal rates used in measuring mortality.

Crude Death Rate (C.D.R.): This is the simplest of all the indices of mortality and is defined as the number of deaths per k persons in the popn of any given region or community during a given period. Thus, in particular, the annual crude death rate (CDR) denoted by m for any given region or community is given by

$$m = \frac{\text{Annual deaths}}{\text{Annual mean pop}^n} \times k$$

where $k = 1000$, usually.

Merit: 1. It is simplest to understand and calculate.
2. CDR is perhaps the most widely used of any vital statistics.

Demerits: 1. Most serious drawback of CDR is that it completely ignores the age and sex distn of popn. Experience shows that mortality is different in different segments of popn. Children in the early ages of their life and the older generation are exposed to higher risk of mortality as compared to younger people.

Specific Death Rates (S.D.R.): Mortality pattern is different in different segments of the popn. Various segments may be age, sex, occupation, social status, etc. For example, the people engaged in infant or child welfare work would be interested to know that mortality condition in the age group below 1 year, 1-4 year, 5-9 year etc, those engaged in maternal health programmes would like to know the number of deaths occurring among women in the reproductive period (usually 15 to 49 year); insurance authorities would be interested in the mortality pattern at different ages of the popn.

Death rate computed for a particular

specified section of the pop^n is termed as S.D.R. given geographical region during a given period is defined as

$$S.D.R = \left[\frac{\text{Total no. of deaths in a specified section of pop}^n \text{ in the given period}}{\text{Total pop}^n \text{ of the specified section in the same period}}\right] \times k$$

where $k = 1000$ usually. Usually S.D.R. is computed specific to (i) age and (ii) sex.

Age-Specific Death Rate (Age-S.D.R.) : To formulate ideas mathematically, let

${}_nD_x$ = Number of deaths in the age group $(x, x+n)$ i.e. no. of deaths among the persons with age x or more but less than $x+n$, in a given region during a given period, t (say)

${}_nP_x$ = Total pop^n of the age-group x to $(x+n)$.

Then the age-specific death rate for the age group x to $x+n$, usually denoted by ${}_nm_x$ is given by

$${}_nm_x = \frac{{}_nD_x}{{}_nP_x} \times 1000$$

Taking $n=1$, we get the annual age-S.D.R. given by

$$m_x = \frac{D_x}{P_x} \times 1000$$

To be more specific, the age-SDR for males is given by

$${}_n^m m_x = \frac{{}_n^m D_x}{{}_n^m P_x} \times 1000 \qquad \text{—— } (*)$$

where ${}_n^m P_x$ is the no. of males in the pop^n in the age group x to $x+n$ and ${}_n^m D_x$ is the no. of deaths amongst this pop^n.
Similarly, the age-S.D.R for female is given by the formula

$${}_n^f m_x = \frac{{}_n^f D_x}{{}_n^f P_x} \times 1000 \qquad \text{—— } (*,*)$$

Formula (*) and (*,*) gives the death rates specific to both age and sex.

Merits: 1. By eliminating the variation in the death rates due to age-sex distn of popn S.D.R's provide more appropriate measures of the relative mortality situation in the region.

2. For general analytical purposes, the death rate specific for age and sex is one of the most imp and widely applicable types of death rates. It also supplies one of the essential components required for the computation of net reproduction rate and construction of life tables.

Demerits: However, S.D.R's are not of much utility for overall comparison of mortality conditions prevailing in ~~the~~ ~~into~~ two different regions. In order to draw some valid conclusions, the different age or/and sex specific death rates must be combined to give a single figure, reflecting the true picture of mortality in the region.

2. Moreover, in addition to age and sex distn of the popn social, occupational and topographical factor comes into operation causing what is called differential mortality. S.D.R's completely ignore these factor. In order to eliminate such ~~the~~ spurious effects, standardised death rates are computed.

Standardised Death Rates: The crude death rates in terms of age-specific death rates for two regions A and B are given respectively by

$$\cancel{} m^a = \frac{D^a}{P^a} \times 1000 = \frac{\sum_x m_x^a P_x^a}{\sum_x P_x^a} \quad \text{—— (x)}$$

$$\text{and } m^b = \frac{D^b}{P^b} \times 1000 = \frac{\sum_x m_x^b P_x^b}{\sum_x P_x^b} \quad \text{—— (x, x)}$$

The expressions in (x) and (x,x) are nothing but the weighted arithmetic means of the age-S.D.R, the weights being the corresponding popns in the age-groups. We observe that even if age-S.D.R's are same i.e. $m_x^a = m_x^b \ \forall x$, $m^a \neq m^b$

since in general $\frac{P_x^a}{\sum_x P_x^a} \neq \frac{P_x^b}{\sum_x P_x^b}$

i.e., since the age-distn of the popns in two regions A and B are not identical. This drawback is removed if the same set of weights is used in (x) and (x,x) for computing the weighted average of the age-SDR's. This is what is done in standardised death rates (STDR) or adjusted death rates, used with a prefix to identify the basis of adjustment as, for example, age-adjusted death rates and so on. We discuss below the two method of age-adjustments, which are in common use.

Direct Method of Standardisation: This method consists in weighting the age specific death rates not by the corresponding popn of the area to which they refer but by the popn distn of another region chosen as a standard. Thus if P_x^s is the no of persons in the age-group x to $x+1$ in ~~the~~ the standard popn, then the standard death rates for the regions A and B are given respectively by

$$(STDR)_A = \frac{\sum_x m_x^a P_x^s}{\sum_x P_x^s} \quad \text{—— (A)}$$

$$(STDR)_B = \frac{\sum_x m_x^b P_x^s}{\sum_x P_x^s} \quad \text{—— (A.A)}$$

These age adjusted death rates for regions A and B respectively are nothing but the crude death rates that would be observed in the standard popn if it were subject to the age-SDR of the regions A and B.

The death rate may ~~be~~ similarly be adjusted for other factors also such as sex, race, etc., and may be interpreted accordingly.

the age-sex adjusted (standardised) death rate for the region A is given by the formula

$$\frac{\sum_x \left[{}^m m_x^a \times {}^m P_x^a + {}^f m_x^a + {}^f P_x^a \right]}{\sum_x \left[{}^m P_x^\beta + {}^f P_x^\beta \right]}$$

Similarly we can obtain STDRs adjusted for any other two factors, e.g. age-race, sex-race and so on.

Merits: 1. Standardised death rates are readily comprehensible and easy to calculate

2. Age-adjusted death rates are comparable since they eliminate the difference caused by the different distns of the age-specific popn for region A and B.

Demerits: The main drawback of the method is the choice of a 'standard popn': The choice of this 'standard' is bound to affect the magnitude of the resulting adjusted rates and may change their relative positions with respect to each other. This bias may, however, be eliminated by taking the standard popn as actual popn of a bigger region of which A and B are subsets.

Indirect standardisation: In direct standardised rate, it is necessary to know the no. of persons and the age-specific death rates for different age-groups. Quite often we have a popn classified by age but the age-SDR's may not be known. However, the total no of deaths and hence CDR may be known. In such case, we use the indirect method of standardisation which consists in multiplying the crude death rate of region A, say, by an adjustment factor 'c' measuring the relative 'mortality' proneness of the popn of the region s.t. the result is equal to the standardised death rate. Thus the popn is to find c s.t.

~~CDR~~ CDR × C = STDR

$$\Rightarrow \frac{\sum_x m_x^a P_x^a}{\sum_x P_x^a} \times c = \frac{\sum_x m_x^a P_x^\beta}{\sum_x P_x^\beta} \Rightarrow c = \frac{-}{-}$$

Since m_x^a are not usually known, an approximate value of c is obtained on replacing m_x^a by m_x^β, the age-SDRs for the standard pop^n thus giving

$$\hat{C} = \left[\frac{\sum_x m_x^s P_x^s}{\sum_x P_x^s} \right] \Big/ \left[\frac{\sum_x m_x^s P_x^a}{\sum_x P_x^a} \right]$$

Finally, the (indirect) standardized death rate for region A is given by

STDR for A = (CDR for A) × $\hat{C}$

Remark: Actually there is no point in comparing the two methods of standardization. We use indirect standardization only when the as an approximation to direct standardization only when the necessary data for the latter is not available. The two methods would be exactly equivalent if the age-SDR for the given pop^n happen to be proportional to the SDR's of the standard pop^n.

Both the methods of direct and indirect standardization are subject to the obvious criticism that the mortality indices so obtained depend ~~on~~ on the age-sex composition of the standard pop^n used and as such the greater gains (losses) in mortality reduction obtained at ~~young~~ younger (older) age are not adequately accounted for.

Fertility: In demography, the word fertility is used in relation to the actual production of children or occurence of births, specially live births. Fertility must be distinguished from fecundity which refers to the capacity to bear children. In fact, fecundity provides an upper bound for fertility. As a measure of the rate of growth of popⁿ various fertility rates are computed.

Crude Birth Rate (CBR.): This is the simplest of all the measures of fertility and consists in relating the no of live births to the total ~~births~~ popⁿ. This provides an index of the relative speed at which additions are being made through child birth. The fertility pattern of the above mentioned measure is given by CBR defined as follows:

$$CBR = \frac{B^t}{P^t} \times R$$

where B^t = Total no. of live births in the given region or locality during a given period, say t.

P^t = Total popⁿ of the given region during the period t.

R = A constant, usually 1000

Merits: It is simple, easy to calculate and readily comprehensible.

Demerits: ① CBR, though simple, is only a crude measure of fertility and is unreliable since it completely ignores the age and sex distⁿ of the popⁿ.

② CBR is not a probability ratio, since the whole popⁿ P^t cannot regarded as exposed to the risk of producing children.

③. As a consequence of variation of climate conditions in various countries, the child bearing age-groups are not identical in all the countries.

(7) - CBR assumes that women in all the ages have same fertility, an assumption which is not true since younger ~~wom~~ women have, in general higher fertility than elderly women

General Fertility Rate (G.F.R.): This consists in relating the total no. of live births to the no. of females in the reproductive or child bearing ages and is given by the formula —— (8)

$$G.F.R = \frac{B^t}{\sum_{\alpha_1}^{\alpha_2} {}^fP_x} \times k$$

where B^t = no. of live births occurring among the popn of a given geographic area during a given period t,

${}^fP_x$ = female popn in the reproductive age in the given geographical region during the same time t.

α_1, α_2 = lower and upper limits of the female child bearing age, and ~~k =~~

k = a constant, usually 1000.

Thus general fertility rate may be defined as the no. of babies per k women in the reproductive age group

merits: 1. GFR is a probability rate since the denominator in (8) consists of the entire female popn which is exposed to the risk of producing children.

Demerits: GFR gives a heterogeneous figure since it overlooks the age composition of the female popn in the child-bearing age. Hence it suffers from the drawback of non-comparability in respect of time and country.

Specific Fertility Rate (S.F.R.): The concept of specific fertility rate originated from the fact that fertility is affected by a no of factors such as age, marriage, migration, state or region etc. The fertility rate computed with respect to any specific factor is called specific fertility rate (SFR) and is denoted by

$$SFR = \frac{\text{number of births to the female pop}^n \text{ of the specified section in a given period}}{\text{Total no. of female pop}^n \text{ in the specified section.}} \times k$$

where $k = 1000$, usually.

Age-specific Fertility Rate: In order to overcome the drawback of G.F.R. and get a better idea of the fertility situation prevailing in a community or locality it is necessary to compute the fertility rates for different age-groups of reproductive age separately. The fertility rate so computed on the basis of specification w.r.to age is called the age-specific-fertility rate. For its computation, the reproductive span is split into different subgroups and S.F.R is worked out for each subgroup.

Symbolically, the age-specific fertility rate for the age group x to $(x+n)$ is given by the formula

$$_n i_x = \frac{_nB_x}{_n^fP_x} \times k$$

where $_n i_x$ = age-SFR for age group x to $(x+n)$

$_nB_x$ = no of births to the females in age group $(x, x+n)$ i.e. age $\geq x$ but less than $x+n$ in the given geographic region during a period t.

$_n^fP_x$ = average female pop^n of age (x to $x+n$) in the given area during the period t, and

$k = 1000$, usually.

In particular if we take $n=1$ in above eq^n we get the so-called annual age-specific fertility rate given by

$$i_x = \frac{B_x}{^fP_x} \times k.$$

Remark: 1. In the computation of age-specific fertility rate, the female popn in the child bearing age group placed in small age groups so as to put them in common with other of the child bearing capacity. As already pointed out, the grouping of women of different ages is necessary since the capacity to bear children varies from age to age. eg., the women in the age group 20 to 25 are more liable to the risk of producing children than the women in the age group 40 to 45.

2. Age-specific fertility rate is a probability rate. It removes the drawback of GFR by taking into account the age-composition of the women in the child-bearing age group and is thus suitable for comparative studies.

Total Fertility Rate (T.F.R.): Age-specific fertility rate is not of much practical utility for comparative purposes. In order to arrive at more practical measure of the popn growth, the age specific fertility rates for different groups have to be combined together to give a single quantity. A simple technique is to obtain standardised fertility rate. This leads to TFR ~~which~~ which is obtained on adding the annual age-specific fertility rate.

Thus $$TFR = \sum_{\alpha_1}^{\alpha_2} i_x = \sum_{\alpha_1}^{\alpha_2} \frac{B_x}{{}^fP_x} \times K .$$

where i_x is age SFR for age group x to $(x+1)$ and α_1, α_2 are the lower and upper limits of female reproductive period. Thus TFR gives the no. of children born per K $(=1000$, usually$)$ females in the child bearing age divided into different age group. Thus TFR for a particular region during a given period may be regarded as an index of the overall fertility conditions operating in that region during the same period.

Usually $\alpha_1 = 15$ and $\alpha_2 = 49$. Thus in order to compute TFR from the above eqn, we shall have to calculate 34 age-specific fertility rates. The arithmetic may be reduced to a great extent by working with age groups, say x to $x+n$, where in general n, the width of interval may vary from one group to the other. In such a case, the TFR is approximately given by the formula

$$TFR = \sum_{x} n(n i_x).$$

Remarks: 1. TFR is hypothetical figure giving the no. of children born to a cohort of $R = 1000$ females (all born at the same time) assuming that :
(i) none of them dies before reaching the end of the child bearing age
(ii) at each age group (in the child bearing ages) they are subject to the fertility condition given by the observed age-specific fertility rate.

Measurement of Population Growth : Having obtained the measures of mortality and fertility, our next objective is to find out if the given popn has a tendency to increase, decrease or remain stable. Fertility rates are inadequate to give us any idea about the rate of popn growth since they ignore the sex of the newly born children and their mortality. Obviously the popn increases through female births. Thus if a majority of births are those of girls, the popn is bound to increase while it will have a downward trend if the majority of births are boys. Similarly if we ignore the mortality of the newly born children we can not form a correct idea of the rate of growth of the popn, since it is possible that a number of female children may die before reaching the reproductive age. In the following sections we shall study some measurements of the growth of popn under the assumption that ~~if~~ in future also it is subject to the current fertility and mortality rates.

Crude Rate of Natural Increase and Pearle's Vital Index

The simplest measure of the popn growth known as Crude rate of natural increase is defined as the difference between the crude birth rate (per thousand) and the crude death rate and is given by

Crude Rate of natural increase = CBR - CDR

Since CBR(CDR) gives the proportion by which popn increases (decreases) through births (deaths) the above formula gives the net increase or decrease in popn through births and deaths taken together.

Another indicator of popn growth based on births and deaths taken together is provided by R. Pearle's Vital Index, defined as follows:

$$\text{Pearle's Vital Index} = 100 \times \frac{\text{No. of births in the given period } t}{\text{No. of deaths in the given period } t}$$

$$= \frac{B^t}{D^t} \times 100 = \frac{CBR}{CDR} \times 100$$

Remarks: 1. Both these measures are simple and easy to calculate. Pearle's index is regarded as a fairly reliable statistical constant reflecting the net biological status of the popn as a whole. If the ratio B^t/D^t is greater than one then the popn is regarded as having good medical care and if the ratio is less than one then the popn is not holding its own.

2. It certainly fails to give us any idea about the trend in the popn growth.

3. Both these measures suffers from the drawback of CBR or CDR and as such are not suitable for comparative studies.

Gross Reproduction Rate (G.R.R.):

In order to have a better idea about the rate of popⁿ growth, in addition to the age and sex composition of the popⁿ we must take into account the sex of the new-born children since it is ultimately the female births who are the potential future mothers and result in an increase in the popⁿ. The GRR is a step in this direction and is defined as the sum of age-specific fertility rates calculated from female births for each year of reproductive period. Symbolically, if ^fB_x is the no. of female births to the women of age x during the given period in the given region, then in the usual notations, we have

$$GRR = \sum_{\alpha_1}^{\alpha_2} \frac{^fB_x}{^fP_x} \times k = \sum_{\alpha_1}^{\alpha_2} {^fi_x} \qquad \text{---} (*)$$

where $^fi_x = \frac{^fB_x}{^fP_x} \times k$ is termed as the female age-specific fertility rate and $k = 1000$, usually. More precisely formula (*) gives female gross reproduction rate.

GRR is thus a modified form of total fertility rate and gives the no. of females expected to be born to k newly born daughters if

(i) none of them is subject to risk of mortality till attaining the age α_2, the upper limit of reproductive period, and

(ii) all of them experience, throughout the reproductive period, the current level of fertility as represented by ^fi_x.

In other words, GRR exhibits the rate at which mothers would be replaced by daughters and the old generation by the new, under the above two assumptions.

Suppose now that instead of annual data, we are given the figures for different age groups of reproductive period. Let ${}_n^fB_x$ be the no. of female babies born to the women, in the age group x to $x+n$, then in the usual notation, we get

$$GRR = \sum_{\lambda_1}^{\lambda_2} n\left(\frac{{}_n^fB_x}{{}_n^fP_x}\right) \times k = \sum_{\lambda_1}^{\lambda_2} n\left({}_n^fi_x\right)$$

where $\quad {}_n^fi_x = \dfrac{{}_n^fB_x}{{}_n^fP_x} \times k \quad$ is the age specific fertility rate for the age group x to $x+n$ based on female births.

Remarks: 1. The computation of GRR requires the availability of the following data

(i) the classification of the births according to the age of the mother at the time of birth and the age of the new-born babies.

(ii) the sex of the new-born babies.

Usually such data are not available. In that case, however, an approximate value of GRR may be obtained under the assumption that sex-ratio at birth remains more or less constant at all the ages of the women in the reproductive period. Then

$$\text{Sex ratio} = \frac{\text{No. of male births}}{\text{No. of female births}} = \text{constant}$$

Finally. $\quad GRR = \dfrac{\text{No of female births}}{\text{Total no of births}} \times TFR.$

2. As a measure of fertility, GRR is quite useful for comparing the fertility in different regions or in the same region at different periods of time.

3. GRR is computed on the hypothesis that none of the newly born female babies is subject to the risk of mortality till the end of the reproductive period of life. This is very serious limitation of

GRR. Since all the girls born do not survive till the end of the child-bearing span. Accordingly GRR leads to fallacious conclusion as it inflates the no. of potential mothers. The drawback is overcome in net reproduction rate.

Net Reproduction Rate (N.R.R.): As already pointed out, the principal limitation of GRR is that it completely ignores the current mortality and takes into account only the current fertility. NRR is nothing but GRR adjusted for the effects of mortality. According to Benjamin, " NRR measures the extent to which mothers produce female infants who survive to replace them. It measures the extent to which a generation of girl babies survive to reproduce themselves as they pass ~~the~~ through the child-bearing age group".

Let us now take into consideration the factor of mortality of mothers also in measuring the growth of pop^n. To formulate our ideas mathematically, to start with we construct a life table for females on the basis of age-specific death rates for females f_{m_x}. The values in the L_x column of the table, denoted by ${}_nL_x^f$, gives the mean size of the cohort of l_0^f females in the age-interval x to $x+n$. In the usual notations let ${}_nB_x^f$ be the no. of female births of the women in the age group x to $x+n$ at any-period t (say). Then

$$\frac{{}_nL_x^f}{l_0^f} \times {}_nB_x^f$$

gives the average no. of female children that would be born to the choort l_0^f in the age group x to $x+n$. The quantity ${}_n\pi_x^f = \dfrac{{}_nL_x^f}{l_0^f}$ gives the life table probability of survival of a female to the age-interval x to $x+n$ and is called the survival rate. This implies that out of k newly born female babies $k \times ({}_{n}^{f}\Pi_x)$ will enter into the child bearing age-interval x to $x+n$; $k \times ({}_{n}^{f}\Pi_{x+n})$ into the age group $x+n$ to $x+2n$ and so on.

Hence instead of multiplying ${}_{n}^{f}B_x / {}_{n}^{f}P_x$ by k alone as in GRR, we multiply it by the factor $k({}_{n}^{f}\Pi_x)$ for each age interval x to $x+n$. Finally a new measure of pop^n growth, known as (female) NRR is given by

$$NRR = k \sum_x n \left[\frac{{}_{n}^{f}B_x}{{}_{n}^{f}P_x} \times {}_{n}^{f}\Pi_x \right]$$

From practical point of view formula can be rewritten as

$$NRR = k \sum_x \left[n ({}_{n}^{f}i_x) \times {}_{n}^{f}\Pi_x \right]$$

$$= k \sum_x \left[n \times (\text{Female Age-SFR}) \times (\text{survival factors}) \right]$$

Summation being taken over all the age groups of reproductive span.

Remarks: 1. Since NRR takes into account the mortality of the new born (female), we get

$$NRR \le k \sum_x n \left[\left(\frac{{}_{n}^{f}B_x}{{}_{n}^{f}P_x} \right) \right] \qquad (\because {}_{n}^{f}\Pi_x \le 1)$$

$$\Rightarrow \quad NRR \le GRR$$

the sign of equality holding iff all the new born girls survive at least till the end of the reproductive period.

2. It should be clearly born in mind that the use of NRR for pop^n projections, i.e., for forecasting future pop^n changes is not desirable at all because of the following two reasons:

(i) It assumes that current mortality and fertility rates prevail in future, an assumption which is not true since in practice both these rates go on changing from time to time.

(ii) It overlooks the factor of migration. The pop^n of a given region in any given period may be depleted more by emigration rather than by declining birth rate or it may increase as a result of fresh stock of immigrants who might be more virile.

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