Imp.

Moment Generating function: V.P.T.U. 2014, 2012, M.T.U. 2013

For certain theoretical developments, an indirect method for computing moments is used. The method depends on the finding of the moments generating function.

For continuous variable x, it is defined as

$$M(t) = \int_a^b e^{tx} f(x)\,dx \qquad \text{——— (1)}$$

where integral is a function of Parameter t only. The limits a, b can be $-\infty$ and ∞ respectively.

Let us see how $M(t)$ generates moments. For this, let us assume that $f(x)$ is a distribution function for which the integral given by (1) exists.

$$\therefore \quad M(t) = \int_a^b e^{tx} f(x)\,dx \qquad \text{——— (1)}$$

$$= \int_a^b \left[1 + tx + \frac{t^2}{\lfloor 2} x^2 + \cdots\right] f(x)\,dx$$

$$= \int_a^b f(x)\,dx + t\int_a^b x f(x)\,dx + \cdots$$

$$= V_0 + V_1 t + V_2 \frac{t^2}{\lfloor 2} + \cdots \qquad \text{——— (2)}$$

Obviously, the coefficient of $\frac{t^r}{\lfloor r}$ in (2) is the r^{th} moment about the origin.

Also, $$\left|\frac{d^r}{dt^r} M(t)\right|_{t=0} = \left|\frac{V_r}{\lfloor r} \lfloor r + V_{r+1} t + \cdots\right|_{t=0} = V_r \qquad \text{——— (3)}$$

Thus, V_r about origin $= r^{th}$ derivative of $M(t)$ with $t=0$.

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Although the moment generating function (m.g.f.) has been defined for the variable x only, the definition can be generalized so that it holds for a variable z where z is a function of x. for $z = x - m$ (m is mean)

the r^{th} moment about z will give r^{th} moment of x about the mean m.

$$\therefore M_z(t) = \int_a^b e^{tz} f(x)\,dx$$

$$M_{(x-m)}(t) = \int_a^b e^{t(x-m)} f(x)\,dx$$

$$= e^{-mt} \int_a^b e^{tx} f(x)\,dx = e^{-mt} M_x(t).$$

Properties of Moment Generating function.

① $M_{x+y}(t) = M_x(t) \cdot M_y(t)$, where x & y are independent radom variabes.

Proof:→ $M_{x+y}(t) = E[e^{t(x+y)}] = E[e^{tx} \cdot e^{ty}]$

$= E(e^{tx}) \cdot E\, e^{ty}$

$\Rightarrow M_{x+y}(t) = M_x(t) \cdot M_y(t)$

② Efect of change of origin and scale on m.g.f.

$M_u(t) = e^{-at/h} M_x(t/h)$, where $u = \frac{x-a}{h}$.

Proof:→ $u = \frac{x-a}{h} \iff x = a + hu$

Then $M_x(t) = E(e^{tx}) = E[e^{t(a+hu)}] = E(e^{at} e^{thu})$

$= e^{at} E(e^{thu}) = e^{at} M_u(th)$

$M_u(t) = E(e^{tu}) = E[e^{t\left(\frac{x-a}{h}\right)}]$

$= e^{-\frac{at}{h}} M_x\left(\frac{t}{h}\right)$

$\therefore M_{cx}(t) = M_x(ct)$, c be a const.