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Subt Signal & System.
Class: B.tech ECE (4th sem.)
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Unit - V

Topic: Properties of Z-transform

2. Time Shifting:

Statement:

If $x(n) \xleftrightarrow{Z} X(Z)$.

then $x(n-k) \xleftrightarrow{Z} Z^{-k} X(Z)$

2. Time Shifting :-

Statement :-

If $x(n) \xleftrightarrow{Z} X(Z)$

then $x(n-k) \xleftrightarrow{Z} Z^{-k} X(Z)$

Proof :- Acc. to defenition of Z-transform

$$Z\{x(n)\} = X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \quad \text{—— (1)}$$

then $Z\{x(n-k)\}$ can be written as :-

$$\therefore Z\{x(n-k)\} = \sum_{n=-\infty}^{\infty} x(n-k) z^{-n}$$

Now z^{-n} can be written as $z^{-n} = z^{-(n-k)} z^{-k}$

$$Z\{x(n-k)\} = \sum_{n=-\infty}^{\infty} x(n-k) z^{-(n-k)} z^{-k}$$

$$\therefore Z\{x(n-k)\} = z^{-k} \sum_{n=-\infty}^{\infty} x(n-k) z^{-(n-k)}$$

Put $n-k = m$ on R.H.S.

Replace $n = m$.

$$\therefore Z\{x(n-k)\} = Z^{-k} \sum_{m=-\infty}^{\infty} x(m) Z^{-m} \quad \text{--- (2)}$$

compare the eq. (2) with (1)

$$\therefore Z\{x(n-k)\} = Z^{-k} \cdot X(z).$$

$$\therefore x(n-k) \overset{Z}{\longleftrightarrow} Z^{-k} \cdot X(Z).$$

Similarly it can be written

$$\boxed{x(n+k) \overset{Z}{\longleftrightarrow} Z^{+k} \cdot X(Z).}$$

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Ex-(1) Find Z-transform

$$x(n) = \delta(n-k).$$

Sol:- We know that $\delta(n)$ is unit impulse
and $Z\{\delta(n)\} = 1$.
that means $\delta(n) \xleftrightarrow{Z} 1$

$$x(n-k) \xleftrightarrow{Z} Z^{-k} \cdot X(Z).$$

$$\therefore \delta(n-k) \xleftrightarrow{Z} Z^{-k} \cdot 1$$

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Ex-2: Find z-transform of :-

$$x(n) = \delta(n+2)$$

Sol:- We have.

$$\delta(n) \xleftrightarrow{Z} 1.$$

Using time shifting property.

$$x(n+k) \xleftrightarrow{Z} Z^{+k} \cdot X(Z)$$

$$\therefore \delta(n+2) \xleftrightarrow{Z} Z^{2}$$

ROC:- Entire z-plane except $z = \infty$

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Ex-3 Find y(z) for following causal System

$$y(n) - \frac{3}{4} y(n-1) + \frac{1}{8} y(n-2) = \delta(n-2).$$

Sol:-

Given eq. is

$$y(n) - \frac{3}{4} y(n-1) + \frac{1}{8} y(n-2) = \delta(n-2) \quad \text{--- (1)}$$

We have $\delta(n) \xleftrightarrow{z} 1$ and using time shifting

$$\delta(n-2) \xleftrightarrow{z} z^{-2} \cdot 1.$$

Taking z- transform of eq. (1) we get

$$Y(z) - \frac{3}{4} z^{-1} Y(z) + \frac{1}{8} z^{-2} Y(z) = z^{-2}$$

$$\therefore \quad Y(z) \left[1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2} \right] = z^{-2}$$

$$\therefore \quad Y(z) = \frac{z^{-2}}{1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}}$$

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Ex-4 Find z-transform of

$$\left\{ \cos\left\{ \frac{n\pi}{4} + \alpha \right\} \right\} \quad n \geq 0.$$

Sol: We have standard z-transform pair;

$$\cos \omega_0 n\, u(n) = \frac{z^2 - z\cos\omega_0}{z^2 - 2z\cos\omega_0 + 1}$$

consider the term $\cos\left(n\frac{\pi}{4}\right)$.

here $\omega_0 = \frac{\pi}{4}$.

$$\therefore \cos\left(n\frac{\pi}{4}\right) = \frac{z^2 - z\cos\left(\frac{\pi}{4}\right)}{z^2 - 2z\cos\left(\frac{\pi}{4}\right) + 1}$$

$$= \frac{z^2 - 0.707z}{z^2 - 1.414z + 1}$$

According to time shifting property

$$x(n+k) = z^k \cdot X(z).$$

$$\cos\left(\frac{n\pi}{4} + \alpha\right) = z^{\alpha} \cdot \left[\frac{z^2 - 0.707z}{z^2 - 1.414z + \ldots} \right.$$

Ex. 3.3.11 : Express the Z transform of

$$y(n) = \sum_{k=-\infty}^{n} x(k) \text{ in terms of } X(Z).$$

Soln. : The given expression is,

$$y(n) = \sum_{k=-\infty}^{n} x(k)$$

Let us expand the summation

$$\therefore\ y(n) = x(-\infty) + \ldots + x(0) + x(1) + \ldots + x(n-1) + x(n)$$

Replace 'n' by 'n – 1' in Equation (1)

$$\therefore\ y(n-1) = \sum_{k=-\infty}^{n-1} x(k)$$

Expanding Equation (3) we get,

$$y(n-1) = x(-\infty) + \ldots + x(0) + x(1) + \ldots + x(n$$

Subtracting Equation (4) from Equation (2) we get,

$$y(n) - y(n-1) = x(n)$$

Taking Z transform of both sides,

$$Y(Z) - Z^{-1} Y(Z) = X(Z)$$

$$\therefore Y(Z)[1 - Z^{-1}] = X(Z)$$

$$\boxed{\therefore\ X(Z) = Y(Z)[1 - Z^{-1}]}$$

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Ex. 3.3.9 : *It is given that :*

$$x_1(n) = \{1, 2, 3, 4, 0, 1\}$$
$$\uparrow$$

Using time shifting property find Z-transform of $x_2(n)$ where :

$$x_2(n) = \{1, 2, 3, 4, 0, 1\}$$
$$\uparrow$$

Soln. :

Comparing the given sequences, we can conclude that $x_2(n)$ is advanced version of $x_1(n)$ and the advance is by two units.

$$\therefore \quad x_2(n) = x_1(n+2)$$

According to time shifting property

$$x(n+k) \overset{Z}{\longleftrightarrow} Z^k X(Z)$$

$$\therefore \quad x(n+2) \overset{Z}{\longleftrightarrow} Z^2 X(Z)$$

Now we will obtain $X_1(Z)$

$$\text{Here } x_1(n) = \{1, 2, 3, 4, 0, 1\}$$
$$\uparrow$$

According to definition of Z-transform.

$$X_1(Z) = \sum_{n=-\infty}^{\infty} x(n) Z^{-n}$$

Here range of sequence x (n) is from n = 0 to 5

$$\therefore \quad X_1(Z) = \sum_{n=0}^{5} x(n) Z^{-n}$$

Expanding the summation and putting the values of x (n) we get,

$$X_1(Z) = 1Z^0 + 2Z^{-1} + 3Z^{-2} + 4Z^{-3} + 0 + 1Z^{-5}$$

Using Equations (1) and (2) we can write,

$$Z\{x_2(n)\} = Z^2 [1 + 2Z^{-1} + 3Z^{-2} + 4Z^{-3} + Z^{-5}]$$

$$\therefore \quad X_2(Z) = Z^2 + 2Z + 3 + 4Z^{-1} + Z^{-3}$$

OC :

This is two sided finite duration sequence. Thus ROC is entire Z-plan

∞.

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