Imp.

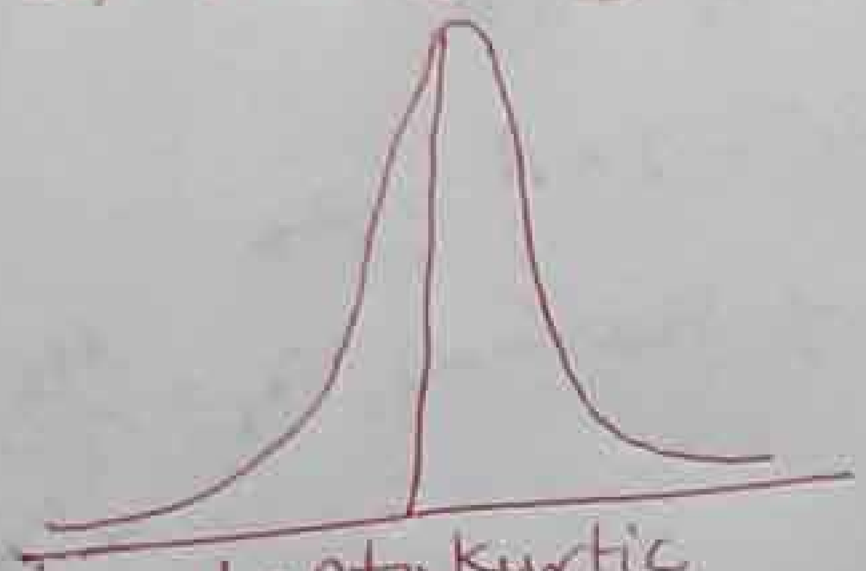
Measure of kurtosis :→ The measure of kurtosis is denoted by β_2 and is defined as $\boxed{\beta_2 = \frac{\mu_4}{\mu_2^2}}$

where μ_2 and μ_4 are respectively the second and fourth moments about mean of the distribution

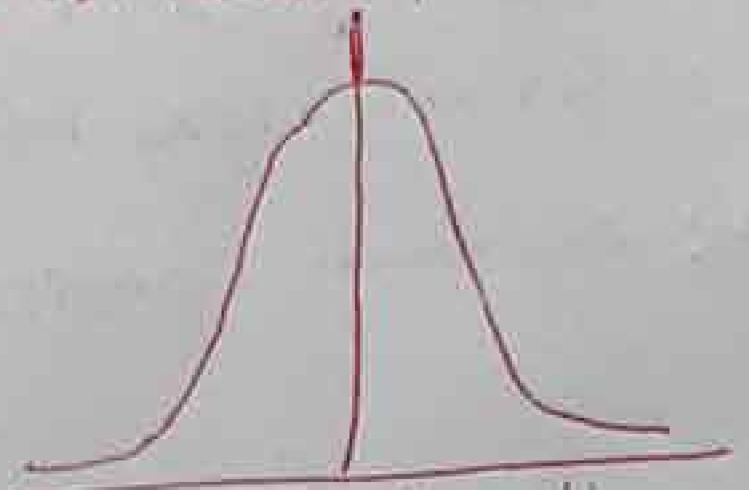
If $\beta_2 > 3$, ⇒ The distribution is leptokurtic

If $\beta = 3$ ⇒ The distribution is mesokurtic.

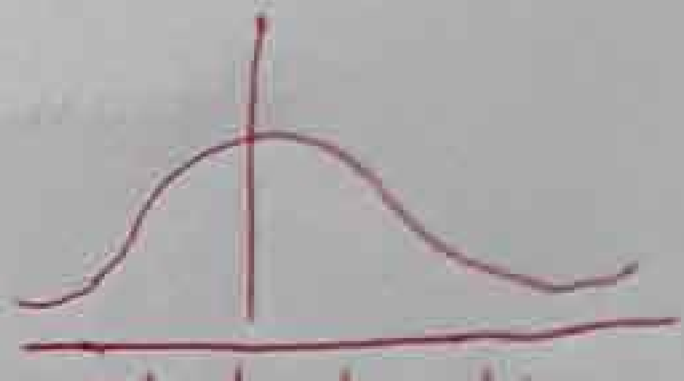
If $\beta < 3$ ⇒ The distribution is Platy kurtic.



Leptokurtic
$\beta_2 < 3$
$\gamma_2 > 0$



Mesokurtic
$\beta_2 = 3$
$\gamma_2 = 0$



Platy kurtic
$\beta_2 < 3$
$\gamma_2 < 0$

where $\gamma_2 = \beta_2 - 3$

Imp.

Question :→ The first four moments about mean of a frequency distribution are 0, 100, −7 and 35000. Discuss the kurtosis of the distribution.

Solution : we have, $\mu_1 = 0$, $\mu_2 = 100$, $\mu_3 = -7$ and $\mu_4 = 35000$

$$\text{Now, } \beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{35000}{(100)^2} = 3.5 > 3$$

Since $\beta_2 > 3$ Hence the distribution is Leptokurtic.

M. Imp

Question:→ The first four moments of a distribution about $x=4$ are 1, 4, 10 and 45. obtain the various characteristics of the distribution on the basis of the given information. Comment upon the nature of the distribution.

Solution on next Page.

Solution:→ We have $A = 4,\ \mu_1' = 1,\ \mu_2' = 4,\ \mu_3' = 10$ and $\mu_4' = 45$

Moments about mean:

$\mu_1 = 0$ (always)

$\mu_2 = \mu_2' - \mu_1'^2 = 4 - (1)^2 = 3$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3 = 10 - 3\times 4\times 1 + 2(1)^3 = 0$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

$= 45 - 4\times 10\times 1 + 6\times 4\times 1^2 - 3\times 1^4 = 26$

Skewness: Moment coefficient of Skewness, $\gamma_1 = \frac{\mu_3}{\sqrt{\mu_2^3}}$

$= \frac{0}{\sqrt{3^3}} = 0$

∴ The distribution is symmetrical.

Kurtosis: $\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{26}{3^2} = 2.89 < 3$

∴ The distribution is Platy kurtic.

Note: Please See More Example of such type.

Question:→ Calculate the first four moments about the mean of the following distribution:

x	2	2.5	3	3.5	4	4.5	5
f	5	38	65	92	70	40	10

Also find the measures of Skewness and Kurtosis.

Sol. We have, $\sigma = 5 \Rightarrow \sigma^2 = 25 \Rightarrow \mu_2 = 25$ | ∵ $\mu_2 = \sigma^2$

Now, $\beta_2 = \dfrac{\mu_4}{\mu_2^2} = \dfrac{\mu_4}{625}$

Thus, the distribution will be

(*i*) Leptokurtic if $\beta_2 > 3 \Rightarrow \dfrac{\mu_4}{625} > 3 \Rightarrow \mu_4 > 1875$

(*ii*) Mesokurtic if $\beta_2 = 3 \Rightarrow \dfrac{\mu_4}{625} = 3 \Rightarrow \mu_4 = 1875$

(*iii*) Platykurtic if $\beta_2 < 3 \Rightarrow \dfrac{\mu_4}{625} < 3 \Rightarrow \mu_4 < 1875.$

Example 5. *The first four moments about the working mean 28.5 of a distribution are 0.294, 7.144, 42.409 and 454.98. Calculate the moments about the mean. Also evaluate* β_1, β_2 *and comment upon the skewness and kurtosis of the distribution.* (A.K.T.U. 2016)

Sol. We have, $\mu_1' = 0.294,\ \mu_2' = 7.144,\ \mu_3' = 42.409,\ \mu_4' = 454.98$

Moments about mean

$$\mu_1 = 0$$

$$\mu_2 = \mu_2' - \mu_1'^2 = 7.144 - (.294)^2 = 7.0576$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$

$$= 42.409 - 3(7.144)(.294) + 2(.294)^3 = 36.1588$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$= 454.98 - 4(42.409)(.294) + 6(7.144)(.294)^2 - 3(.294)^4$$

$$= 408.7896$$

Calculation of β_1 and β_2

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = 3.7193 \qquad \beta_2 = \frac{\mu_4}{\mu_2^2} = 8.2070$$

Skewness

Since β_1 is positive, $\gamma_1 = 1.9285$ | μ_3 is positive

∴ The distribution is **positively skewed.**

Kurtosis

Since $\beta_2 = 8.2070 > 3$

∴ The distribution is **leptokurtic.**

Example 6. *The first four moments of a distribution about the value '0' are – 0.20, 1.76, – 2.36 and 10.88. Find the moments about the mean and measure the kurtosis.*

Sol. We have, $\mu'_1 = -0.20,\ \mu'_2 = 1.76,\ \mu'_3 = -2.36,\ \mu'_4 = 10.88$

Moments about the mean:

$$\mu_1 = 0$$

$$\mu_2 = \mu'_2 - \mu_1'^2 = 1.76 - (-0.20)^2 = 1.72$$

$$\mu_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3$$

$$= -2.36 - 3(1.76)(-0.20) + 2(-0.20)^3 = -1.32$$

$$\begin{aligned}\mu_4 &= \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2\mu_1'^2 - 3\mu_1'^4\\ &= 10.88 - 4(-2.36)(-0.20) + 6(1.76)(-0.20)^2 - 3(-0.20)^4\\ &= 9.4096\end{aligned}$$

Kurtosis:

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = 3.180638$$

Since, $\beta_2 > 3$ hence the distribution is **leptokurtic.**

Example 7. *The following table represents the height of a batch of 100 students. Calculate kurtosis.* (A.K.T.U. 2018)

Height (in cm)	59	61	63	65	67	69	71	73	75
No. of students	0	2	6	20	40	20	8	2	2

Sol. To calculate β_2, we will have to first find the values of μ_2 and μ_4.

Moments about 67

$$\mu_1' = \left(\frac{\Sigma fu}{N}\right)h = \left(\frac{12}{100}\right)(2) = 0.24$$

$$\mu_2' = \left(\frac{\Sigma fu^2}{N}\right)h^2 = \left(\frac{164}{100}\right)(4) = 6.56$$

Height (cm) x	*No. of students f*	$u = \frac{x-67}{2}$	fu	fu^2	fu^3	fu^4
59	0	−4	0	0	0	0
61	2	−3	−6	18	−54	162
63	6	−2	−12	24	−48	96
65	20	−1	−20	20	−20	20
67	40	0	0	0	0	0
69	20	1	20	20	20	20
71	8	2	16	32	64	128
73	2	3	6	18	54	162
75	2	4	8	32	128	512
	$N = \Sigma f = 100$		$\Sigma fu = 12$	$\Sigma fu^2 = 164$	$\Sigma fu^3 = 144$	$\Sigma fu^4 = 1100$

$$\mu_3' = \left(\frac{\Sigma fu^3}{N}\right)h^3 = \frac{144}{100}\times 8 = 11.52$$

$$\mu_4' = \left(\frac{\Sigma fu^4}{N}\right)h^4 = \frac{1100}{100}\times 16 = 176$$

(ii) Find all four central moments and discuss skewness and kurtosis for the frequency distribution given in the following table:

Range of Expenditure (in ₹ 100 per month)	2–4	4–6	6–8	8–10	10–12
No. of families	38	292	389	212	69

[G.B.T.U. 2013; M.T.U. 2012]

(i) Find the measures of skewness and kurtosis on the basis of moments for the following distribution:

x	1	3	5	7	9
f	1	4	6	4	1

[G.B.T.U. (C.O.) 2011]

(ii) Find the measure of skewness and kurtosis on the basis of moments for the following distribution and draw your conclusion: [A.K.T.U. 2018]

Marks	5–15	15–25	25–35	35–45	45–55
No. of Students	1	3	5	7	4

Calculate β_1 and β_2 from the following data:

Profit (in lakhs of ₹)	10–20	20–30	30–40	40–50	50–60
No. of companies	18	20	30	22	10

Indicate the nature of frequency curve.

Prove that the frequency distribution curve of the following frequency distribution is leptokurtic.

Class	10–15	15–20	20–25	25–30	30–35	35–40	40–45	45–50	50–55
Frequency	1	4	8	19	35	20	7	5	1

Calculate the first four moments about the mean of the following distribution:

x	2	2.5	3	3.5	4	4.5	5
f	5	38	65	92	70	40	10

Also find the measures of skewness and kurtosis. (M.T.U. 2012)

Calculate the first four moments about the mean for the following frequency distribution and hence find the coefficient of skewness and kurtosis and comment upon the nature of the distribution.

Class-interval	5–10	10–15	15–20	20–25	25–30	30–35	35–40
Frequency	6	8	17	21	15	11	2

(G.B.T.U. 2013)

[M.T.U. 2014; G.B.T.U. (C.O.) 2011]

Define the coefficients of kurtosis.

What do you mean by kurtosis? Explain in brief.

Define kurtosis of a distribution.

Answers

1. $\beta_2 = 2.2278$, Platykurtic
2. $\beta_2 = 3$, Mesokurtic
3. (i) $\beta_2 = 3$, Mesokurtic (ii) 0.17708, 2.9936
4. $\mu_4 = 768$
5. (i) $\beta_2 = 0.8889$, Platykurtic
   (ii) $\mu_1 = 0, \mu_2 = 16, \mu_3 = -64, \mu_4 = 162$
   $\gamma_1 = -1, \beta_2 = 0.6328$; Negatively skewed and platykurtic
6. (i) $\beta_2 = 2.3216$, Platykurtic (ii) $\beta_2 = 2.7240$, Yes
7. (i) $\mu_1 = 0, \mu_2 = 36.66, \mu_3 = -85.104, \mu_4 = 4373.3832, \gamma_1 = -0.3834, \beta_2 = 3.2541$
   (ii) $\mu_1 = 0, \mu_2 = 37267.04, \mu_3 = 1746530.688, \mu_4 = 3567851989$
   $\gamma_1 = 0.24275, \beta_2 = 2.5689$, positively skewed and platykurtic.
8. (i) $\gamma_1 = 0, \beta_2 = 2.5$
   (ii) $\mu_1 = 0, \mu_2 = 125, \mu_3 = -600, \mu_4 = 37625, \gamma_1 = -0.4293, \beta_2 = 2.408$, negatively
   platykurtic.
9. $\beta_1 = 0.0001, \beta_2 = 2.047$, Platykurtic.
11. $\mu_1 = 0, \mu_2 = 0.45328125, \mu_3 = 0.009890625, \mu_4 = 0.502111743, \gamma_1 = 0.0324, \beta_2 = 2.4$
    positively skewed and platykurtic.
12. $\mu_1 = 0, \mu_2 = 56, \mu_3 = -176.5625, \mu_4 = 7502.9375, \gamma_1 = -0.4213, \beta_2 = 2.3925$; negatively
    platykurtic.

3.20 CURVE FITTING

Let there be two variables x and y which give us a set of n pairs of numerical valu
(x_2, y_2), ..., (x_n, y_n). In order to have an approximate idea about the relationship
variables, we plot these n paired points on a graph thus, we get a diagram
simultaneous variation in values of both the variables called *scatter or dot diag*
scatter diagram, we get only an approximate non-mathematical relation between
Curve fitting means an exact relationship betw
this relationship is the equation of the cu
of the curve from the given data.
the point of view of theor

Theoretically
enables us to