Economic production quantity

- The **economic production quantity** model (also known as the **EPQ model**) determines the quantity a company or retailer should order to minimize the total inventory costs.
- This method is an extension of the economic order quantity model (also known as the EOQ model).
- The difference between these two methods is that the EPQ model assumes the company will produce its own quantity. While the EOQ model assumes the order quantity arrives complete and immediately after ordering, meaning that the parts are produced by another company and are ready to be shipped when the order is placed.
- In some literature, "economic manufacturing quantity" model (EMQ) is used for "economic production quantity" model (EPQ).
- We want to determine the optimal number of units of the product to order so that we minimize the total cost associated.

Inventory Model with Finite Replenishment Rate (Production Rate), Constant Demand and No Shortages

Let us consider the situation in which a company supplies units to inventory at a uniform replenishment rate over time rather than in economic order quantities at specific points of time. The amount ordered is not delivered all at once, but ordered quantity is sent or received gradually over a length of time at a finite rate per unit of time. In some cases usage and production (or delivery rates) are equal and inventory will not build up because the company will use all items immediately.

More typically, the production or delivery rate p, will exceed the demand or usage rate u ($p > u$).

The model is developed under following assumptions:

- The system deals with a single item.
- The demand rate of u units per time unit is known and constant.
- The item is produced with constant production rate p.
- The inventory ordering cost C_o per order is known and constant during the period under review.
- Shortages are not allowed.
- Order size Q and amount of the shortage S are the decision variable.
- T is the cycle time.
- The inventory holding cost C_h, per unit time is known and constant during the period under review.

The economic order quantity model with constant demand and permissible shortages has the graphic depiction as:

The graphic depiction of this situation is shown in the Fig 1

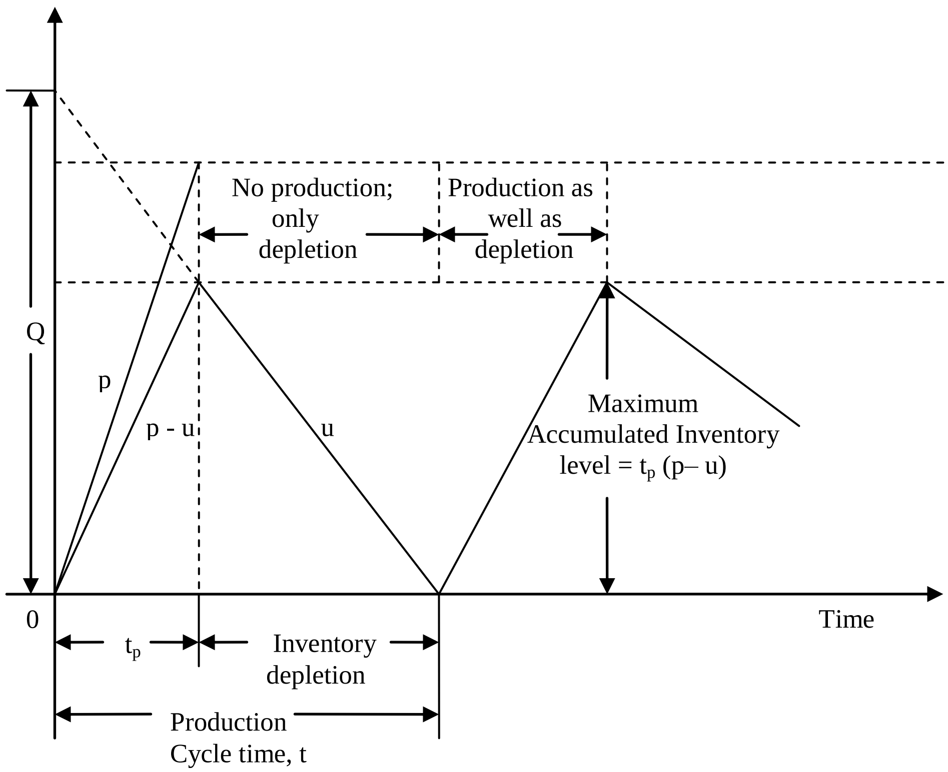

Fig 1 EPQ model with constant demand and shortages allowed

We start at zero inventory level. If t_p is the time period required to produce one entire batch amount Q at the rate p, then the rate at which the stocks arrive is $p = Q / t_p$.

The company uses and produces items during the first part of the inventory cycle i.e. during the time t_p the company produces a lot size of Q units. Since there is also a simultaneous usage, the inventory level builds gradually at the rate $p\text{-}u$ units during this time. This production stops when a batch size of Q units is produced. When production ceases, the second part of the inventory cycle begins. The company will deplete its inventory level at the usage or demand rate u.

The maximum inventory level reached at the end of t_p = inventory accumulation rate × production time

$$= (\text{p-u})\,\text{t}_\text{p} = (\text{p-u})\frac{\text{Q}}{\text{p}} = \left(1 - \frac{u}{p}\right)Q$$

The inventory holding cost IHC = Average inventory × cost of holding one unit $= \left(1 - \frac{u}{p}\right)\frac{Q}{2}C_h$

The annual set up or the ordering cost = $(D/Q)\, C_o$

Total Inventory Cost TC = Ordering cost + holding cost = $\frac{D}{Q}C_0 + \left(1-\frac{u}{p}\right)\frac{Q}{2}C_h$

Differentiating this equation of TC with respect to Q and solving for Q, we get the optimal lot size Q^* as

$$Q^* = \sqrt{\frac{2DC_0}{C_h}\left(\frac{p}{p-u}\right)}$$

The length of time required to produce a lot, $t_p^* = Q^* / p$

The maximum inventory level = $Q^*\left(1-\frac{u}{p}\right)$

The length of time required to deplete the maximum on-hand inventory, $t_d^* = \frac{Q^*}{u}\left(1-\frac{u}{p}\right)$

The length of an inventory cycle, $t = t_p^* + t_d^* = Q^*/p + \frac{Q^*}{u}\left(1-\frac{u}{p}\right)$

The minimum total inventory cost, $TC^* = \frac{D}{Q^*}C_0 + \left(1-\frac{u}{p}\right)\frac{Q^*}{2}C_h$

When the value of Q^* is substituted, we get $TC^* = \sqrt{2DC_0C_h\left(1-\frac{u}{p}\right)}$