EOQ Model with Constant Demand and Shortages Allowed:

Case: When Order size Q and amount of the shortage S are the decision variable

This type of inventory problem is useful a company permits shortages or backorders to occur. However, in many situations shortages are economically desirable. Permitting shortages allows the manufacturer or retailer to increase the cycle time thereby spreading the setup or ordering cost over a longer time period.

The model is developed under following assumptions:

- The system deals with a single item.
- The demand rate of D units per time unit is known and constant.
- The item is produced in lots or purchases are made in orders.
- The inventory ordering cost C_o per order is known and constant during the period under review.
- Shortages are allowed. S represents the amount of the shortage (size of the back order) that has accumulated when the new shipment of size Q arrives,
- Lead time is zero.
- Order size Q and amount of the shortage S are the decision variable.
- T is the cycle time.
- The inventory holding cost C_h, per unit time is known and constant during the period under review.

The economic order quantity model with constant demand and permissible shortages has the graphic depiction as:

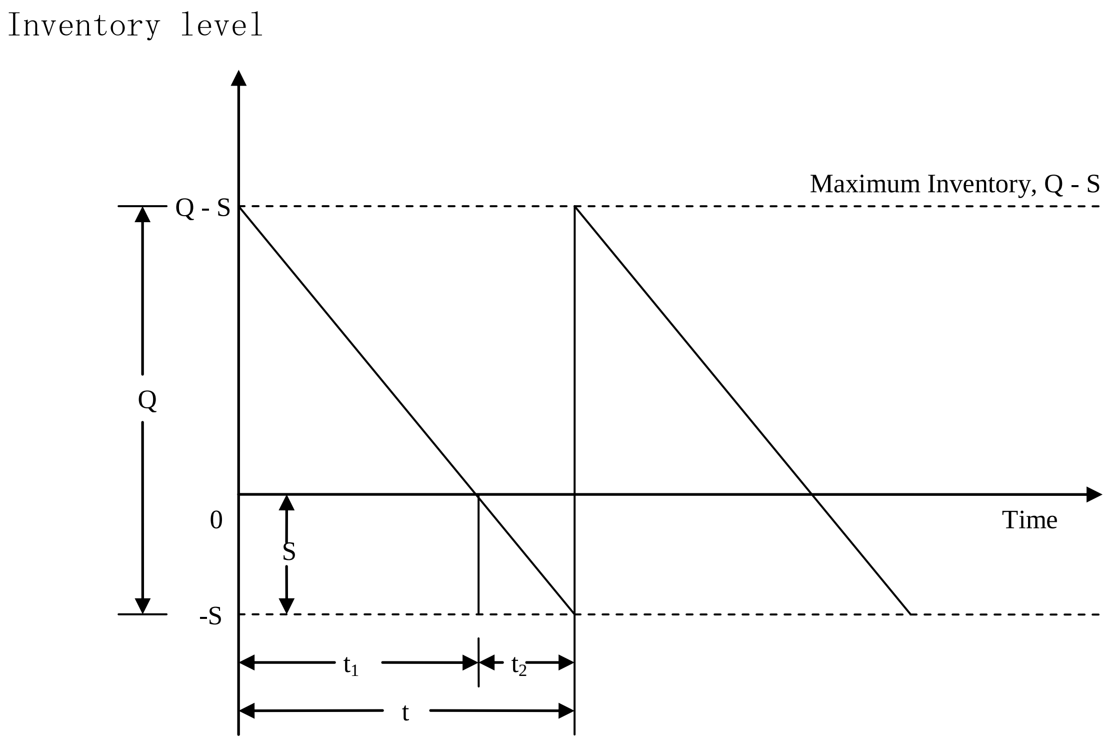

Fig 1 EOQ model with constant demand and shortages allowed

- When the new shipment of size Q arrives, the company immediately ships the back orders of size S to the customers. The remaining units Q-S immediately go into inventory.
- The inventory level will vary from a minimum of -S units to a maximum of Q-S units.
- The inventory cycle of T units is divided into two distinct parts: t_1 when inventory is available for filling orders and t_2 when inventory is not available, stock outs occur, and back orders are made.

Here apart from the notations introduced in the previous model we introduce two new notations as follows:

C_s: cost of back order, per unit per unit time

S: the number of units short or back ordered.

Now,

The inventory ordering cost is a function of the number of orders made, *D/Q*, and the inventory ordering cost per order, C_o.

$$OC = \text{No. of orders} \times \text{Cost per order} = (D/Q)\, C_o$$

Also we know that $t_1 = (Q\text{-}S)/D$ and $t_2 = S/D$

The inventory holding cost can be calculated from the Fig 1 as:

The average inventory for the time period, t = [(Avg. inventory over t_1) + (Avg. inventory over t_2)] / t

The positive inventory level ranges from *Q-S* to *0*. This means that the average inventory level is *(Q-S)/2* for the time period t_1. For t_2 it is *0*.

Inventory holding cost (*IHC)* = Average inventory × cost of holding one unit

$$= \left(\frac{\frac{\text{Q-S}}{2} t_1 + 0 t_2}{t} \right) C_h = \frac{(\text{Q-S})^2}{2\text{Q}} C_h \quad \text{putting the value of } t_1$$

The backordering cost is computed in a similar way. From the figure we can see that the shortage ranges from *0* units to *S* units. This means that the average shortage is *S/2* while there are shortages i.e. during the time period t_2.

Shortage cost *(SC)* = Average number of units short × cost of one unit being short

$$= \frac{\frac{\text{S}}{2} t_2}{t} C_s = \frac{\text{S}^2}{2\text{Q}} C_s \quad \text{putting the value of } t_2$$

Since *TC* is a function of two variables *Q* and *S*, therefore to determine the optimal order size and the optimal shortage level *S*, we need to differentiate the total variable cost function with respect to *Q* and *S*, set the two resulting equations equal to zero and solve them simultaneously. By doing so, we get the following results

$$\text{Q}^* = \sqrt{\frac{2DC_0}{C_h} \frac{C_s + C_h}{C_s}}$$

$$S^* = Q^* \frac{C_h}{C_s + C_h}$$

The number of orders for the planning horizon = *D/Q**

The maximum inventory level = *Q* - S**

The average positive inventory level is = *(Q* - S*) / Q**

The length of time during which there are no shortages, $t_1^* = (Q^* - S^*) / D$

The length of time during which there are shortages, $t_2^* = S^* / D$

The length of the inventory cycle, $t^* = Q^* / D$

The minimum total inventory cost during the planning horizon

$$= TC^* = \frac{D}{Q^*} C_0 + \frac{(Q^* - S^*)^2}{2Q^*} C_h + \frac{S^{*2}}{2Q^*} C_s$$

When values of *Q** and *S** are substituted, we get $TC^* = \sqrt{2DC_0C_h\left(\frac{C_s}{C_s + C_h}\right)}$

EXAMPLE

The demand for an item each costing Re 1, is 10000 units per year. The ordering cost is Rs. 10. Inventory carrying charge is 20% based on the average inventory per year. Stock-out cost is Rs. 5 per unit of shortage incurred. Find various parameters.

Solution : Here $D = 10{,}000$, $C_o = 10$, $C_h = 20\%$ of Re $1 = 0.2$, $C_s =$ Rs. 5

$$\text{Therefore, } Q^* = EOQ = \sqrt{\left(\frac{2\times10\times10000}{0.20}\right)\left(\frac{0.20+5}{5}\right)} = 1020 \text{ units}$$

$$\text{The Inventory level } I^* = \sqrt{\left(\frac{2\times10\times10000}{0.20}\right)\left(\frac{5}{0.20+5}\right)} = 980 \text{ units}$$

$$\text{The shortage level} = Q^*-I^* = 1020-980 = 40 \text{ units}$$

$$\text{Cycle period, } t^* = \frac{1020}{10{,}000} = 37.23 \text{ days or } 37 \text{ days}$$

$$\text{Number of orders/yr} = \frac{10{,}000}{1020} = 9.8 \text{ (or 10)}$$

$$\text{Total Cost} = \sqrt{\frac{2\times10\times0.20\times5\times10000}{(0.20+5)}} = \text{Rs. } 400.04$$