Model IV (M/M/S): (∞/FCFS)

In this model first M denotes the Poisson arrival or exponential inter arrival time,

Second M denotes the Poisson departure or exponential service time and

S denotes the multiple server or channels. Service rate at each channel is the same as μ.

In this model the length of the waiting line will depend on the number of occupied channels.

If no. of customer less than no. of server i.e. **n < S** then there will be no customer waiting in queue.

If no. of customer is equal to no. of server i.e. **n = S** then all service channel will be busy.

If no. of customer is greater than no. of server i.e. **n > S** then all service channel will be busy, while n-S will be waiting in queue.

Mathematical derivation for different queuing characteristics is similar to the previous three models. In step one first we find the system of steady state equations. Thereafter in step two we solve the steady state equations and find out the result for probability function.

We can obtain the expressions as

Probability that the system has no customer $$P_0 = \frac{1}{\left[\sum_{n=0}^{S-1} \frac{(S\rho)^n}{n!} + \frac{(S\rho)^S}{S!(1-\rho)}\right]}$$

Probability that the system has n customer $$P_n = \begin{bmatrix} \frac{(n\rho)^n}{n!} P_0, 1 \le n \le S \\ \frac{S^S \rho^n}{S!} P_0, \quad n \ge S \end{bmatrix}$$ Where $\rho = \frac{\lambda}{\mu S}$

(a) Expected queue length (average number of customers in the queue)

If n>S, then n-S customer will be waiting in queue.

$$E\left(L_q\right)=\sum_{n=S}^{\infty}(n-S)P_n=\sum_{n=S}^{\infty}(n-S)\frac{S^S\rho^n}{S!}P_0=\frac{(S\rho)^S}{S!}P_0\sum_{n=S}^{\infty}(n-S)\rho^{n-S}$$

$$=P_S\left[0+\rho+2\rho^2+........\right] \qquad \text{Since } P_S=\frac{(S\rho)^S P_0}{S!}$$

$$=\quad \rho P_S(1-\rho)^{-2}$$

$$E\left(L_q\right)=\frac{\rho P_S}{(1-\rho)^2}$$

(b) Expected (or average) number of customer in the system

$$E(W_S)=E\left(W_q\right)+\frac{1}{\mu}$$

$$\lambda E(W_S)=\lambda E\left(W_q\right)+\frac{\lambda}{\mu}$$

$$E(L_S)=E\left(L_q\right)+\frac{\lambda}{\mu}$$

$$E(L_S)=\frac{\rho P_S}{(1-\rho)^2}+S\rho$$

(c) Expected waiting time in the system (including service time)

$$\frac{1}{\lambda}E(L_S)=E(W_S)$$

$$E(W_S)=\frac{\rho P_S}{\lambda(1-\rho)^2}+\frac{S\rho}{\lambda}$$

(d) **Expected waiting time in the queue (excluded service time)**

$$E\left(W_q\right)=\frac{1}{\lambda}E\left(L_q\right)$$

$$E(W_q) = \frac{\rho P_S}{\lambda(1-\rho)^2}$$

Model V : (M/ E_k /1) : (∞/FCFS)

Here M represents the Poisson arrival,

E_k Represents the Erlangian service time

1 represents the single service channel,

∞ Represents the infinite capacity of the system and

FCFS means first come first served.

In this model each customer is served in k-phases one by one and a new service does not start until all k-phases have been completed , therefore each arrival increase the number of phases by k in the system.

If μ is the number of units served per unit time, then k μ will be number of phases served per unit time. Thus, here we take,

$$\lambda_n = \lambda, \text{ arrival per unit time}$$

$$\mu_n = \mu, \mu k \text{ phases served per unit time}$$

We directly write the result for different characteristics of queue

Probability that the system has no customer $P_0 = 1 - \rho k$

Probability that the system has n customer $P_n = (1-\rho k)\sum_{m,r,s} \rho^m (-1)^r \binom{m}{r}\binom{m+s-1}{s}$

Where $\rho = \frac{\lambda}{\mu k}$

(c) Expected queue length (average number of customers in the queue)

If n>S, then n-S customer will be waiting in queue.

$$E\left(L_q\right)=\frac{(k+1)}{2k}\frac{\lambda^2}{\mu(\mu-\lambda)}$$

(d) Expected (or average) number of customer in the system

$$E\left(L_S\right)=E\left(L_q\right)+\frac{\lambda}{\mu}$$

$$E\left(L_S\right)=\frac{(k+1)}{2k}\frac{\lambda^2}{\mu(\mu-\lambda)}+\frac{\lambda}{\mu}$$

(c) expected waiting time in the system (including service time)

$$\frac{1}{\lambda}E\left(L_S\right)=E\left(W_S\right)$$

$$E\left(W_S\right)=\frac{(k+1)}{2k}\frac{\lambda}{\mu(\mu-\lambda)}+\frac{1}{\mu}$$

(e) **expected waiting time in the queue (excluded service time)**

$$E\left(W_q\right)=\frac{1}{\lambda}E\left(L_q\right)$$

$$E\left(W_q\right)=\frac{(k+1)}{2k}\frac{\lambda}{\mu(\mu-\lambda)}$$

Model V : (M/E_k/1) : (1/FCFS)

In this model capacity of the system is one unit only which is served through k phases.

After this completion another unit arrives to the system and repeats the process.

Try this model yourself with the help of book.