Binomial Probability Distribution: [A.K.T.U. 2018] V. Imp.

Let there be n independent trials in an experiment. Let a random variable X denote the number of successes in these n trials. Let p be the probability of a successes and q that of a failure in a single trial so that $p+q=1$. Let the trials be independent and p be constant for every trial.

Let us find the probability of r successes in n trials.

r = successes can be obtained in n trials in ${}^nC_r$ ways

$$\therefore P(r) = P(X=r) = {}^nC_r \underbrace{P(S)\cdot P(S)\cdot P(S)\cdots}_{r\text{-times}} \underbrace{P(F)\cdot P(F)\cdots}_{(n-r)\text{ times}}$$

$$= {}^nC_r\, p\cdot p\cdot p\cdots p \text{ (r-times)} \cdot q\,q\,q\cdots q \text{ (n-r) times.}$$

$$\boxed{P(r) = {}^nC_r\, p^r q^{n-r}} \quad \text{where } p+q=1 \text{ and } r=0,1,2,\ldots,n \qquad (1)$$

Called Binomial Prob. distribution

Note: where ${}^nC_0 q^n,\ {}^nC_1 p q^{n-1},\ {}^nC_2 p^2 q^{n-2}, \ldots, {}^nC_n p^n$ called successive terms of the binomial distributions for $r=1,2,3\ldots$

Recurrence Formula for the Binomial distribution:

We know that $P(r) = {}^nC_r\, p^r q^{n-r} = \dfrac{\lfloor n}{\lfloor n-r \lfloor r} p^r q^{n-r}$

$$P(r+1) = {}^nC_{r+1}\, p^{r+1} \cdot q^{n-r-1} = \frac{\lfloor n}{\lfloor n-r-1 \lfloor r+1} \cdot p^{r+1} \cdot q^{n-r-1}$$

$$\frac{P(r+1)}{P(r)} = \frac{(n-r)\cdot \lfloor n-r-1}{\lfloor n-r-1} \cdot \frac{\lfloor r}{(r+1)\lfloor r} \cdot \frac{p}{q} = \left(\frac{n-r}{r+1}\right) \cdot \frac{p}{q}$$

$$\therefore P(r+1) = \frac{n-r}{r+1} \cdot \frac{p}{q} \cdot P(r)$$

Which is the required recurrence formula. By this formula we can find $P(1), P(2), P(3) \ldots$ If $P(0)$ is given.

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Mean and Variance of the Binomial Distribution : (A.K.T.U. 2018)

We know that Binomial distribution,

$$\therefore P(r) = {}^nC_r\, q^{n-r}.p^r$$

$$\text{Mean } \mu = \sum_{r=0}^{n} r\,P(r) = \sum_{r=0}^{n} r.\, {}^nC_r\, q^{n-r}\, p^r$$

$$= 0 + 1.\, {}^nC_1 .q^{n-1}.p + 2\, {}^nC_2 . q^{n-2}.p^2 + 3.\,{}^nC_3\, q^{n-3}.p^3 + \cdots\cdots$$

$$+ n.\, {}^nC_n\, p^n$$

$$= n q^{n-1} p + 2\frac{n(n-1)}{2.1}.q^{n-2}p^2 + 3\frac{n(n-1)(n-2)}{3.2.1} q^{n-3}.p^3 + \cdots + np^n$$

$$= n q^{n-1}.p + n(n-1)q^{n-2}p^2 + \frac{n(n-1)(n-2)}{2.1} q^{n-3}p^3 + \cdots + np^n$$

$$= np\,[\,{}^{n-1}C_0\, q^{n-1} + {}^{n-1}C_1\, q^{n-2}.p + {}^{n-1}C_2\, q^{n-3}.p^2 + \cdots + {}^{n-1}C_{n-1}\, p^{n-1}$$

$$= np\,(q+p)^{n-1} = np \times 1 = np \qquad (\because p+q=1)$$

Hence $\boxed{\text{Mean } \mu = np}$

$$\text{Variance } \sigma^2 = \sum_{r=0}^{n} r^2 P(r) - \mu^2 = \sum_{r=0}^{n} [r + r(r-1)]\,P(r) - \mu^2$$

$$= \sum_{r=0}^{n} r P(r) + \sum_{r=0}^{n} r(r-1)\,P(r) - \mu^2 = \mu + \sum_{r=2}^{n} r(r-1)\, {}^nC_r\, q^{n-r} p^r - \mu^2$$

For $r=0$, and $r=1$, contribution is zero

$$= \mu + [2.1.\, {}^nC_2\, q^{n-2}.p^2 + 3.2.\, {}^nC_3\, q^{n-3}.p^3 + \cdots + n(n-1)\, {}^nC_n\, p^n] - \mu^2$$

$$= \mu + [2.1.\frac{n(n-1)}{2.1}.q^{n-2}.p^2 + 3.2\frac{n(n-1)(n-2)}{3.2.1} q^{n-3}p^3 + \cdots + n(n-1)p^n] - \mu^2$$

$$= \mu + [n(n-1)\, q^{n-2}p^2 + n(n-1)(n-2)\, q^{n-3}p^3 + \cdots + n(n-1)p^n] - \mu^2$$

$$= \mu + n(n-1)\,p^2\,(q+p)^{n-2} - \mu^2 = \mu + n(n-1)p^2 - \mu^2 \qquad \because p+q=1,\ \because \mu = np$$

$$= np + n(n-1)p^2 - n^2p^2 = np(1-p) = npq$$

Hence the Variance of binomial distribution is npq

$\therefore$ Standard deviation of the binomial distribution is $\sqrt{npq}$

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