Karl Pearson's co-Efficient of correlation (or Product Moment correlation coefficient)

correlation co-efficient between two variables x and y, usually denoted by $r(x,y)$ or r_{xy} is a numerical measure of linear relationship between them and is defined as

$$r_{xy} = \frac{\Sigma (x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\Sigma (x_i-\bar{x})^2 \Sigma (y_i-\bar{y})^2}} = \frac{\frac{1}{n}\Sigma (x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\frac{1}{n}\Sigma (x_i-\bar{x})^2 \cdot \frac{1}{n}\Sigma (y_i-\bar{y})^2}}$$

$$= \frac{1}{n} \frac{\Sigma (x_i-\bar{x})(y_i-\bar{y})}{\sigma_x \sigma_y} \Rightarrow \boxed{r_{xy} = \frac{\Sigma (x_i-\bar{x})(y_i-\bar{y})}{n \sigma_x \sigma_y}}$$

Alternate form of r_{xy}

$$r_{xy} = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\sqrt{n\Sigma y^2 - (\Sigma y)^2}}$$

Note: → Correlation co-efficient is independent of change of origin and scale. Let us use new variables u and v two defined.

$u = \frac{x-a}{h}$, $v = \frac{y-b}{k}$, where a, b, h, k are constants, then $r_{xy} = r_{uv}$

Then $r_{(u,v)} = \frac{n\Sigma uv - \Sigma u \Sigma v}{\sqrt{n\Sigma u^2 - (\Sigma u)^2}\sqrt{n\Sigma v^2 - (\Sigma v)^2}}$

Question:→ Find the co-efficient of correlation for the following table:

x :	10	14	18	22	26	30
y :	18	12	24	6	30	36

Solution:→ Let $u = \frac{x-22}{4}$, $v = \frac{y-24}{6}$

x	y	u	v	u^2	v^2	uv
10	18	-3	-1	9	1	3
14	12	-2	-2	4	4	4
18	24	-1	0	1	0	0
22	6	0	-3	0	9	0
26	30	1	1	1	1	1
30	36	2	2	4	4	4
		$\Sigma u = -3$	$\Sigma v = -3$	$\Sigma u^2 = 19$	$\Sigma v^2 = 19$	$\Sigma uv = 12$

Here $n = 6$

$\bar{u} = \frac{\Sigma u}{n} = \frac{-3}{6} = -\frac{1}{2}$

$\bar{V} = \frac{\Sigma v}{n} = \frac{-3}{6} = -\frac{1}{2}$

$$r_{uv} = \frac{n\Sigma uv - \Sigma u \Sigma v}{\sqrt{n\Sigma u^2 - (\Sigma u)^2}\sqrt{n\Sigma v^2 - (\Sigma v)^2}} = \frac{6\times 12 - (-3)(-3)}{\sqrt{6\times 19 - (-3)^2}\sqrt{6\times 19 - (-3)^2}}$$

$$= \frac{63}{\sqrt{105}\sqrt{105}} = 0.6$$

$\therefore r_{xy} = r_{uv} = 0.6$ Ans.

Question:→ Find the coefficient of correlation between the values of x and y.

x :	1	3	5	7	8	10
y :	8	12	15	17	18	20

$n = 6$

Hint.

x	y	x^2	y^2	xy
1	8	1	64	8
3	12	9	144	36
5	15	25	225	75
7	17	49	289	119
8	18	64	324	144
10	20	100	400	200

$\Sigma x = 34$, $\Sigma y = 90$, $\Sigma x^2 = 248$, $\Sigma y^2 = 1446$

$\Sigma xy = 582$

$$\therefore r(x,y) = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\sqrt{n\Sigma y^2 - (\Sigma y)^2}}$$

$$= \frac{6\times 582 - (34\times 90)}{\sqrt{6\times 248 - (34)^2}\sqrt{6\times 1446 - (90)^2}}$$