Discrete Random Variables: → If a random variable can take only a finite number of distinct values, then it must be discrete. For example: No of Voters, No. of students, No. of children playing in a Park..etc.

Continuous Random Variable: Random variables represent counted data while continuous random variables represent measured data. For example, Random variables like Length, Thickness, Weights and Temprature are called continuous variables.

Probability Mass Function: → (P.M.F.)

Let $x_1, x_2, \ldots x_n = x_i$, $\{i = 1, 2, \ldots n\}$ be the values of a random variable X, which is associate a number $P_i = P(X = x_i)$; $i = 1, 2, 3 \ldots n$ which is known as a probability of x_i and satisfies the following conditions

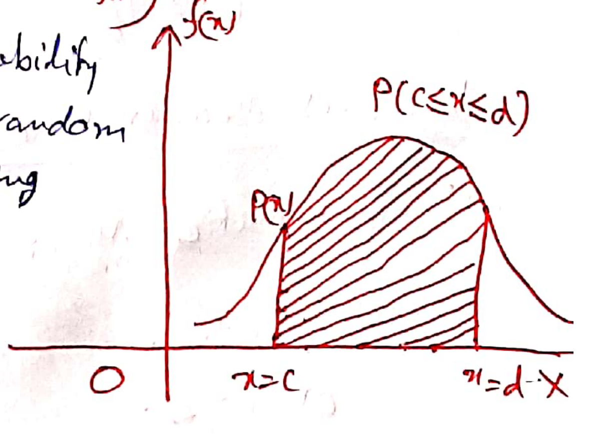
(i) $P_i = P(X = x_i) \geq 0$

(ii) $\Sigma P_i = P_1 + P_2 + \cdots P_n = 1$

The set of all possible ordered pairs $\{x, P(x)\}$, is called probability distribution of the random variable X.

Probability Density function (P.d.f.)

A function $f(x)$ is said to be the probability density function of the continuous random variable X if it satisfies the following properties.



(i) $f(x) \geq 0 \ \forall\, x \in [c, d]$

(ii) For any two distinct numbers c and d

$P(c \le X \le d)$ represents the area under the Probability curve between the ordinates at $x = c$ and $x = d$.

(iii) The total area under the Probability curve is 1

i.e. $\int_c^d f(x)\,dx = 1 \Rightarrow P(c \le X \le d) = 1$

The Probability density function $f(x)$ satisfies the following conditions

(i) $f(x) \geq 0 \;\forall x$ (ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Question :- If $f(x)$ has Probability density cx^2, $0 < x < 1$, determine c and find the Probability that $\frac{1}{3} < x < \frac{1}{2}$

i.e. $P\left[\frac{1}{3} < x < \frac{1}{2}\right]$

Solution :- $f(x)$ will have a probability density function

if $\int_0^1 f(x)\,dx = 1 \Rightarrow \int_0^1 cx^2\,dx = 1$

$\Rightarrow \left[c\frac{x^3}{3}\right]_0^1 = 1 \Rightarrow \boxed{c = 3}$

Now $P\left[\frac{1}{3} < x < \frac{1}{2}\right] = \int_{1/3}^{1/2} cx^2\,dx = \int_{1/3}^{1/2} 3x^2\,dx$

$= \left[\frac{\cancel{3}x^3}{\cancel{3}}\right]_{1/3}^{1/2} = \left(\frac{1}{8} - \frac{1}{27}\right) = \frac{19}{216}$

Various Measures for Continuous Probability Distributions

(i) Airthmatic Mean $\bar{x} = \int_a^b x f(x)\,dx$, where $f(x)$ be the P.d.f. of a random variable X, defined from a to b.