

For certain theoretical developments, an indirect method for computing moments is used. The method depends on the finding of the moments generating function.

For continuous variable x , it is defined as

$$M(t) = \int_a^b e^{tx} f(x) dx \quad \text{--- (1)}$$

Where integral is a function of Parameter t only. The limits a, b can be $-\infty$ and ∞ respectively.

Let us see how $M(t)$ generates moments. For this, let us assume that $f(x)$ is a distribution function for which the integral given by (1) exists.

$$\therefore M(t) = \int_a^b e^{tx} f(x) dx \quad \text{--- (1)}$$

$$= \int_a^b \left[1 + tx + \frac{t^2}{2} x^2 + \dots \right] f(x) dx$$

$$= \int_a^b f(x) dx + t \int_a^b x f(x) dx + \dots$$

$$= V_0 + V_1 t + V_2 \frac{t^2}{2} + \dots \quad \text{--- (2)}$$

Obviously, the coefficient of $\frac{t^r}{r!}$ in (2) is the r^{th} moment about the origin.

$$\text{Also, } \left| \frac{d^r}{dt^r} M(t) \right|_{t=0} = \left| \frac{V_r}{r!} r + V_{r+1} t + \dots \right|_{t=0} = V_r$$

Thus, V_r about origin = r^{th} derivative of $M(t)$ with $t=0$.

Thus, V_r about origin = r^{th} derivative of $M(t)$ with $t=0$
 Although the moment generating function (m.g.f.) has been defined for the variable x only, the definition can be generalized so that it holds for a variable z where z is a function of x . For $z = x - m$ (m is mean), the r^{th} moment about z will give r^{th} moment of x about the mean m .

$$\therefore M_z(t) = \int_a^b e^{tz} f(x) dx$$

$$M_{(x-m)}(t) = \int_a^b e^{t(x-m)} f(x) dx$$

$$= e^{-mt} \int_a^b e^{tx} f(x) dx = e^{-mt} M_x(t).$$

Properties of Moment Generating function.

① $M_{x+y}(t) = M_x(t) \cdot M_y(t)$, where x & y are independent random variables.

Proof: $\rightarrow M_{x+y}(t) = E[e^{t(x+y)}] = E[e^{tx} \cdot e^{ty}]$
 $= E[e^{tx}] \cdot E[e^{ty}] \Rightarrow M_{x+y}(t) = M_x(t) \cdot M_y(t)$

② Effect of change of origin and scale on m.g.f.

$M_u(t) = e^{-at/h} M_x(t/h)$, where $u = \frac{x-a}{h}$.

Proof: $\rightarrow u = \frac{x-a}{h} \Leftrightarrow x = a + hu$
 Then $M_x(t) = E(e^{tx}) = E(e^{t(a+hu)}) = E(e^{at} \cdot e^{thu})$
 $= e^{at} E(e^{thu}) = e^{at} M_u(t/h)$
 $M_u(t/h) = E(e^{t/h u}) = E(e^{t(\frac{x-a}{h})})$
 $= e^{-\frac{at}{h}} M_x(\frac{t}{h})$
 $\therefore M_{cu}(t) = M_x(ct), \quad c \text{ be a const.}$