AKTU-2018

Question :- A sample of 100 dry battery cells tested to find the length of life produced the following results.

$\bar{x} = 12$ hours, $\sigma = 3$ hours.

Assuming the data to be normally distributed, what percentage of battery cells are expected to have life.
(i) more then 15 hours (ii) Less then 6 hours
(iii) between 10 and 14 hours?

Sol:- Here x denotes the length of life of dry battery cells.

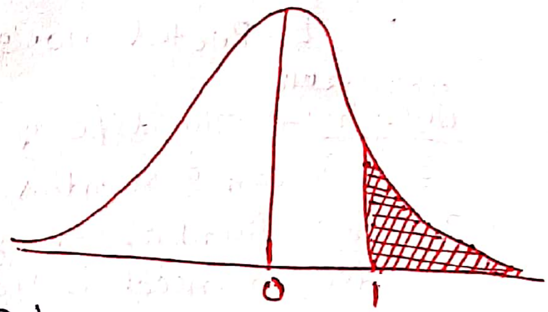
Also,
$$z = \frac{x-\bar{x}}{\sigma} = \frac{x-12}{3}$$

(i) when $x = 15$, $z = 1$

$\therefore$ $P(x>15) = P(z>1)$

$= P(0<z<\infty) - P(0<z<1)$

$= .5 - 0.3413 = 0.1587 = 15.87\,\%$

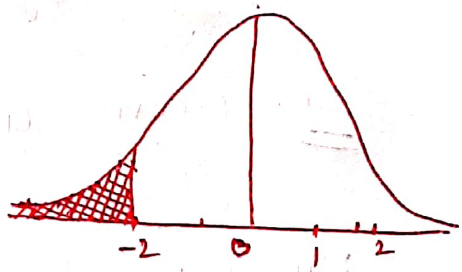


(ii) when $x = 6$, $z = \frac{6-12}{3} = -2$

$\therefore$ $P(x<6) = P(z<-2)$ $= P(z>2)$

$= P(0<z<\infty) - P(0<z<2)$

$= 0.5 - 0.4772 = 0.0228 = 2.28\,\%$



(iii) When $x = 10$, $z = \frac{10-12}{3} = -\frac{2}{3}$

$= -0.67$

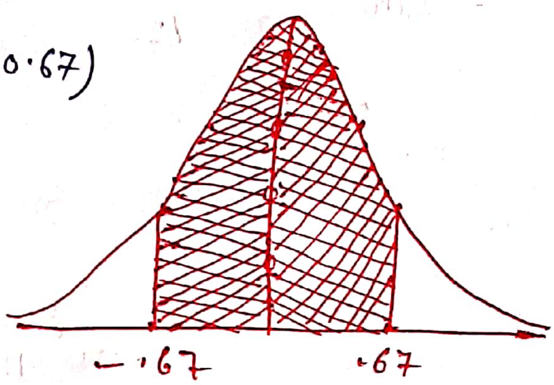
When $x = 14$, $z = \frac{2}{3} = 0.67$

$P(10<x<14) = P(-0.67<z<0.67)$

$= 2P(0<z<0.67)$

$= 2 \times 0.2485$

$= 0.4970 = 49.70\,\%$



(28)

GBTU 2011

Question :– Assume mean height of soldiers to be 68.22 inches with a variance of 10.8 inches square. How many soldiers in a regiment of 1000 would you expect to be over 6 feet tall, given that the area under the standard normal curve between $z=0$ and $z=0.35$ is 0.1368 and between $z=0$ and $z=1.15$ is 0.3746.

Solution :→ $x = 6$ feets $= 72$ inches.

$$z = \frac{x-\mu}{\sigma} = \frac{72-68.22}{\sqrt{10.8}} = 1.15$$

$$P(x>72) = P(z>1.15) = 0.5 - P(0 \le z \le 1.15)$$

$$= 0.5 - 0.3746 = 0.1254$$

∴ Expected no. of soldiers $= 1000 \times 0.1254 = 125.4 = 125$ App.

AKTU 2018

Question :→ The life of army shoes is normally distributed with mean 8 months and standard deviation 2 months. If 5000 pairs are insured, how many pairs would be expected to need replacement after 12 months?

[Given that $P(z \geq 2) = 0.0228$ and $z = \frac{x-\mu}{\sigma}$]

Sol. Mean $\mu = 8$,

Standard Deviation $(\sigma) = 2$

Number of pairs of shoes $= 5000$

Total months $(x) = 12$

When $x = 12$, $z = \frac{x-\mu}{\sigma} = \frac{12-8}{2} = 2$

Area $(z \geq 2) = 0.0228$

Number of pairs whose life is more than 12 months $= 5000 \times 0.0228$

$= 114$

Pair of shoes needing replacement after 12 months

$= 5000 - 114 = 4886$.

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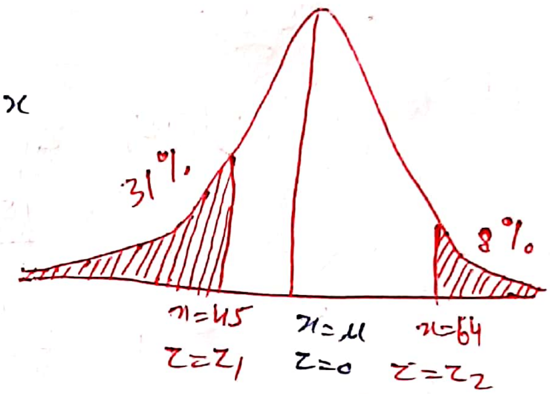
Question: In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and Standard deviation of the distribution. It is given that if $f(t) = \frac{1}{\sqrt{2\pi}} \int_0^t e^{-\frac{1}{2}x^2} dx$ then $f(0.5) = 0.19$ and $f(1.4) = 0.42$

[A.K.T.U. 2019, 2018]

Sol: Let μ and σ be the mean and Standard deviation respectively.

31% of the items are under 45.

$\Rightarrow$ Area to the left of the ordinate $x = 45$ is 0.31



when $x = 45$, Let $z = z_1$

$P(z_1 < z < 0) = 0.5 - 0.31 = 0.19$

From the tables, the value of z corresponding to this area is 0.5

$\therefore$ $z_1 = -0.5$ $[z_1 < 0]$

when $x = 64$, Let $z = z_2$

$P(0 < z < z_2) = 0.5 - 0.08 = 0.42$

From the tables, the value of z corresponding to this area is 1.4. Hence $z_2 = 1.4$

$\therefore$ Since $z = \frac{x - \mu}{\sigma} \Rightarrow 0.5 = \frac{45 - \mu}{\sigma}$ and

$1.4 = \frac{64 - \mu}{\sigma}$

$\Rightarrow 45 - \mu = 0.5\sigma$ ——— (1) and $64 - \mu = 1.4\sigma$ ——— (2)

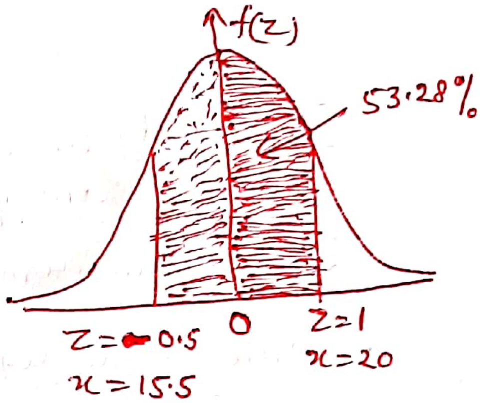
Subtracting $-19 = -1.9\sigma \Rightarrow \boxed{\sigma = 10}$

From (1), $45 - \mu = -0.5 \times 10 = -5$

$\therefore \boxed{\mu = 50}$

Question: The mean yield per plot of a crop is 17 kg and standard deviation is 3 kg. If distribution of yield per plot is normal, find the percentage of plot giving yield : (i) Between 15.5 kg and 20 kg : and (ii) More than 20 kg

Solution:



Mean $= \mu = 17$ kg

S.D. $= \sigma = 3$ kg

Standard Normal Variable

$$Z = \frac{x-\mu}{\sigma}$$

(i) when $x_1 = 15.5$, $Z_1 = \frac{x_1-\mu}{\sigma}$

$$= \frac{15.5-17}{3} = -0.5$$

when $x_2 = 20$, $Z_2 = \frac{x_2-\mu}{\sigma} = \frac{20-17}{3} = 1$

$\therefore$ $P(15.5 < x < 20) = P(0.5 < z < 1)$

$$= P(0 < z < -0.5) + P(0 < z < 1)$$

$$= 0.1915 + 0.3413 = \underline{0.5328} \text{ Ans.}$$

(ii) when $x = 20$, $Z = \frac{20-17}{3} = 1$

$P(x > 20) = P(z > 1)$

$= 0.5 - P(0 < z < 1)$

$= 0.5 - 0.3413$

$= 0.1587$ Ans.