Normal Distribution: → The normal distribution is a continuous distribution. It can be derived from the binomial distribution in the limiting case when n, the number of trials is very large and p, the Probability of a success, is close to $\frac{1}{2}$. The general equation of the normal distribution is given by

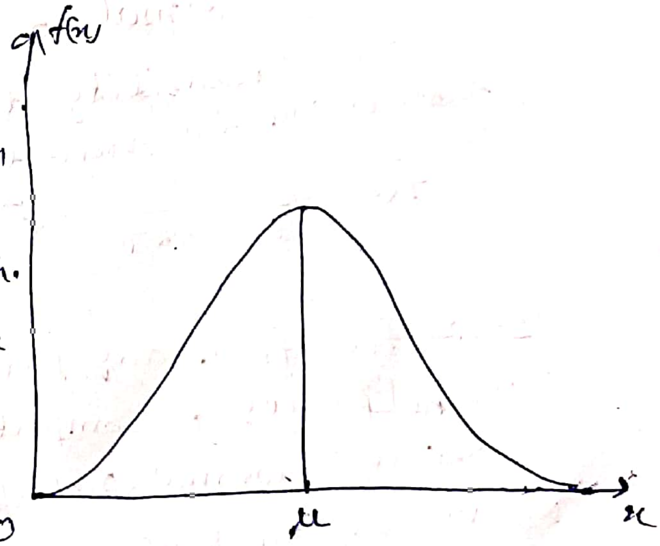
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad \text{where } -\infty < x < \infty$$

μ = mean

σ = S.D. and $\sigma > 0$

x is called the normal variate.

$f(x)$ is called Probability density function of the normal distribution.



A bell-shapped and symmetrical about the mean μ which the normal curve is called the normal distribution graph.

The line $x = \mu$ divides the area under the normal curve above x-axis into two equal Parts. Thus, the median and mode of the distribution also coincides with its mean. The area under the normal curve between any two given ordinates $x = x_1$ and $x = x_2$ represents the Probability of values falling into the given interval. The total area under the normal curve above the x-axis is 1.

Basic Properties of the Normal Distribution

The Probability density function of the normal distribution is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

(i) $f(x) \geq 0$ (ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

i.e. the total area under the normal curve above the x-axis is 1.

(iii) The normal distribution is symmetrical about its mean.

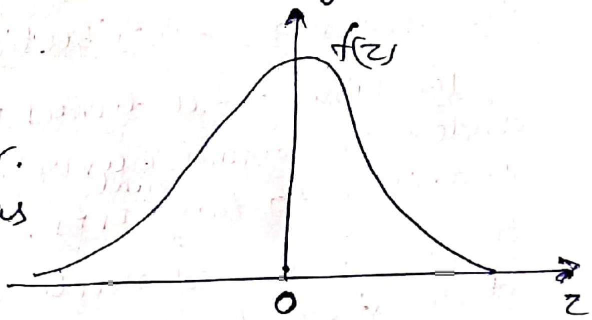
(iv) It is a unimodal distribution. The mean, mode and the median of this distribution coincide.

→ Standard form of the normal Distribution:—

If X is a normal random variable with mean μ and S. D. σ, then random variable $Z = \frac{X-\mu}{\sigma}$ has the normal distribution with mean 0 and Standard Deviation 1. The random variable Z is called the standard normal random Variable.

The Probability density function for the normal distribution in standard form is given by

$$f(z) = \frac{1}{2\pi} e^{-\frac{1}{2}z^2}$$



$(-\infty < z < \infty)$

It is free from any Parameter. This helps us to compute areas under the normal probability curve by making use of Standard tables.

Note 1. If $f(z)$ is the Probability density function for the normal distribution, then

$$P(z_1 \leq Z \leq z_2) = \int_{z_1}^{z_2} f(z)dz = F(z_2) - F(z_1),$$

where $F(z) = \int_{-\infty}^{z} f(z)\,dz = P(Z \leq z)$

where $F(z)$ is called normal distribution function.

Note 2:→ $P(z_1 \leq Z \leq z_2) = P(z_1 < Z \leq z_2) = P(z_1 \leq Z < z_2) = P(z_1 < Z < z_2)$

Note 3:— $F(-z_1) = 1 - F(z_1)$.