Introduction:-

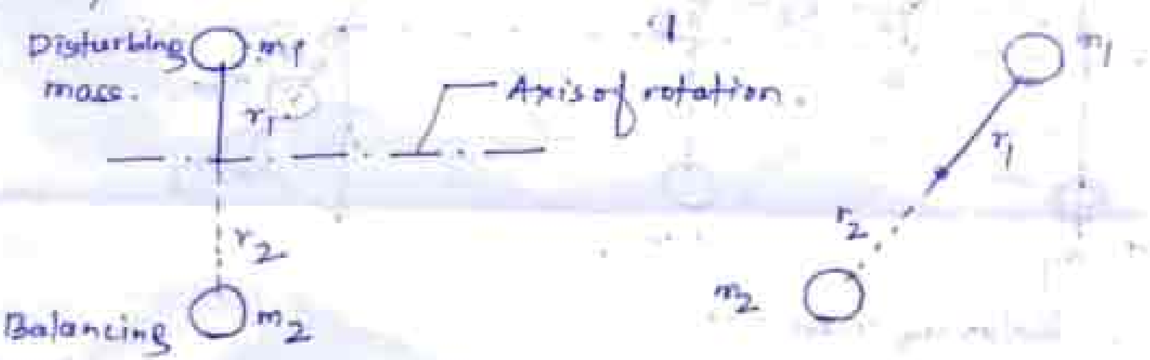
Machines have several rotating parts. Some of them have reciprocating motion e.g. piston and some of them have rotating motion e.g. crankshaft. If these moving parts are not in complete balance, inertia force generation would lead to vibration, noise, wear and tear of the parts.

- Balancing plays a major role in designing these systems to reduce unbalance to an acceptable limit.

Balancing of Single Revolving mass:-

(i) Balancing in same plane (ii) balancing in different plane.

(i) Balancing and disturbing mass revolve in same plane:-



Let m_1 = mass attached to the shaft

ω = angular velocity of the mass in rad/s.

r_1 = distance of CG of the mass from axis of rotation.

In order to counteract the disturbing force e.g. the centrifugal force due to m_1, a counter mass m_2 at a radius r_2 is placed in the same plane, such that the centrifugal forces due to the two masses are equal and opposite.

Mathematically, $F_{c_1} = m_1\omega^2 r_1$

balancing force $F_{c_2} = m_2\omega^2 r_2$

For balancing $F_{c_1} = F_{c_2}$

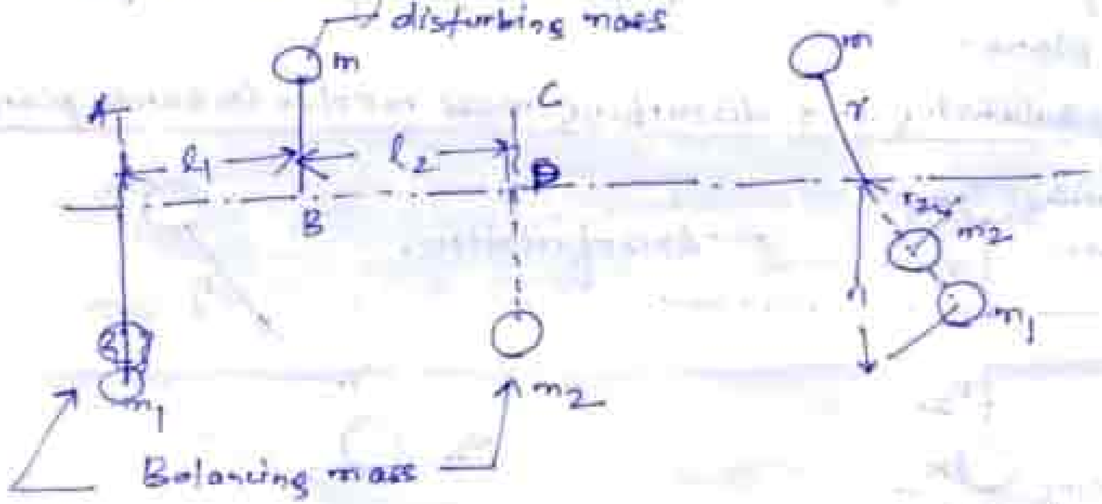
$\Rightarrow m_1\omega^2 r_1 = m_2\omega^2 r_2$

$\Rightarrow \boxed{m_1 r_1 = m_2 r_2}$

Generally the value of r_2 is kept is larger to reduce the value of balancing mass m_2

(ii) Balancing and Disturbing mass revolve in different Plane! —

In case the balancing and the disturbing mass lie in different planes, the disturbing mass can not be balanced by a single mass as there will be a couple left unbalanced. In such case at least two balancing masses are required for complete balancing. The three masses are arranged in such a way that the resultant force and couple on the shaft are zero.



Let m = mass of disturbing body ~~acting~~ acting in plane B

m_1 = mass of balancing weight acting in plane A

m_2 = mass of balancing weight acting in plane B

l_1 = distance betn plane A and B

l_2 = distance betn plane B and C

$l = l_1 + l_2$

$r, r_1, r_2 \rightarrow$ distances of CG of m, m_1, m_2 respectively

Now $F_c = m\omega^2 r$

$F_{c_1} = m_1\omega^2 r_1$

$F_{c_2} = m_2\omega^2 r_2$

For balancing the centrifugal force of ~~balancing~~ disturbing mass must be equal to the sum of centrifugal force of balancing mass

$F_c = F_{c_1} + F_{c_2}$

or $m\omega^2 r = m_1\omega^2 r_1 + m_2\omega^2 r_2$

$\Rightarrow \boxed{mr = m_1 r_1 + m_2 r_2}$

For complete balance, sum of moments should be zero.

Taking moment about B, D

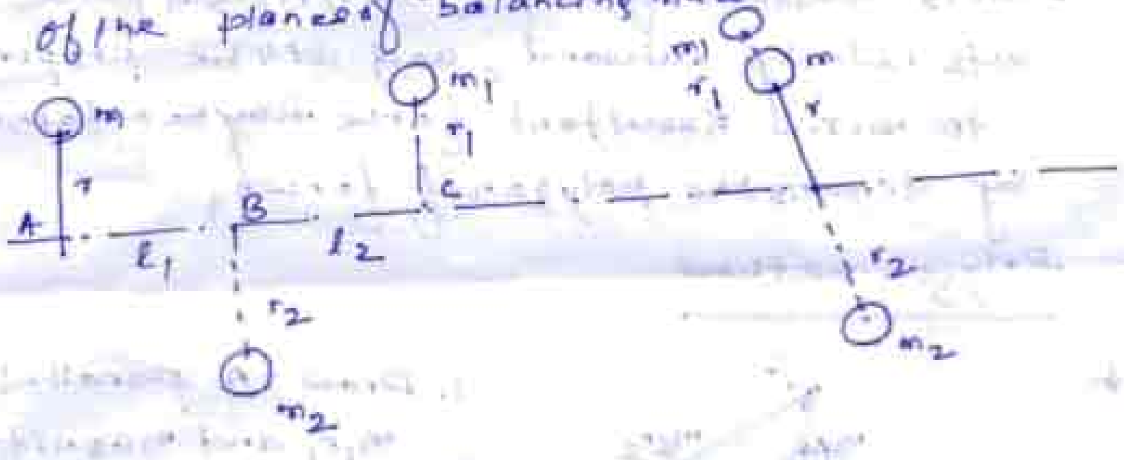
$(l_1+l_2) F_{c_1} = F_c \cdot l_2$

$\Rightarrow l\, m_1\omega^2 r_1 = m\omega^2 r \cdot l_2$

$\Rightarrow \boxed{m_1 r_1 = \frac{m r l_2}{l}}$

where $\boxed{l = l_1 + l_2}$

Case-II → Plane of disturbing mass lies on one side of the plane of balancing mass.



We have ~~$F_c = F_{c_1} + F_{c_2}$~~

$F_c + F_{c_1} = F_{c_2}$

or $m\omega^2 r + m_1\omega^2 r_1 = m_2\omega^2 r_2$

or $mr + m_1 r_1 = m_2 r_2$

couple equation can be written by taking moment at B

$F_c \cdot l_1 = F_{c_1} \cdot l_2$

or $\boxed{m r l_1 = m_1 r_1 l_2}$

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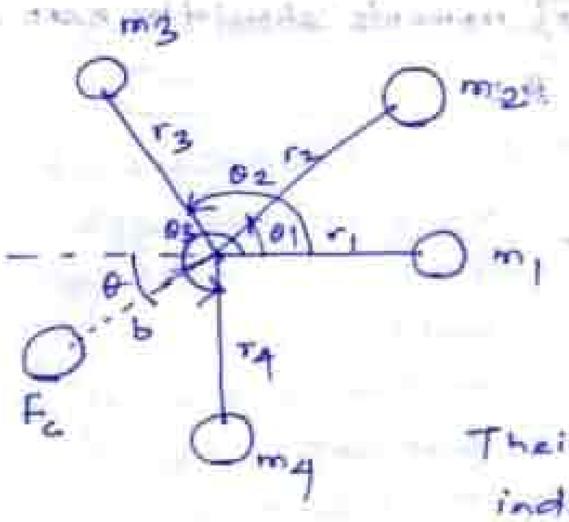
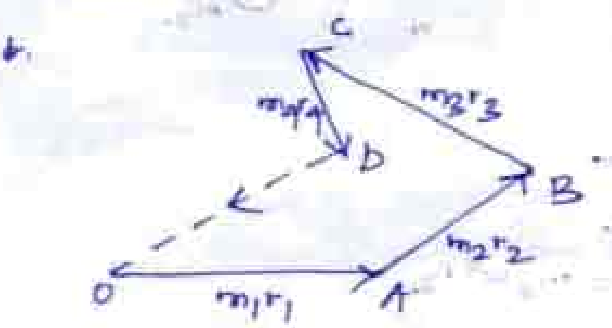
Balancing of several masses revolving in same plane :-



Consider any no of masses say four m_1, m_2, m_3 and m_4 rigidly attached to the shaft and lie in same plane. Let r_1, r_2, r_3 and r_4 be the radii of rotation of masses. Their relative positions are indicated by angles $\theta_1, \theta_2, \theta_3$ and θ_4

During rotation of shaft, centrifugal force acts radially outward, and will be proportional to $m.r.$ Resultant force may be obtained by drawing the polygon of forces

Polygon method



1. Draw OA parallel to m_1r_1 and magnitude $OA = m_1r_1$
2. from A draw AB parallel and equal to m_2r_2.
3. From B draw BC parallel and equal to m_3r_3
4. from C draw CD parallel and equal to m_4r_4
5. Join D with O, DO represents the direction and magnitude of balanced force.

Analytical method

Resolving each force horizontally and vertically

Resultant vertical component is

$$F_v = m_1r_1 \sin\theta + m_2r_2 \sin\theta_1 + m_3r_3 \sin\theta_2$$

$F_H = m_1 r_1 \cos\theta + m_2 r_2 \cos\theta_1 + m_3 r_3 \cos\theta_2 + m_4 r_4 \cos\theta_3$ —— (2)

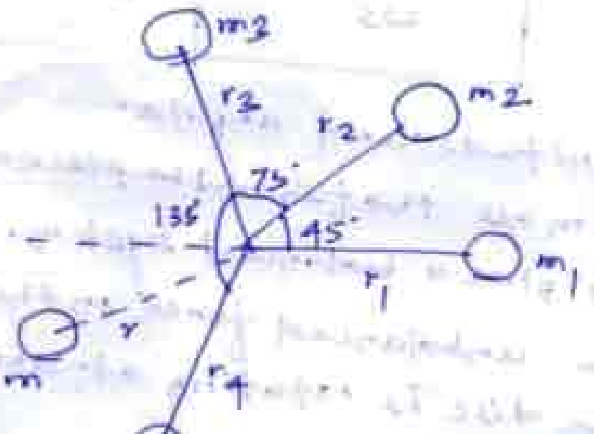
The resultant B.b may be written as

$$\boxed{B.b = \sqrt{(F_V^2 + F_H^2)}}$$

And its direction $\tan\theta = \frac{F_V}{F_H}$

$$\Rightarrow \boxed{\theta = \tan^{-1}\left(\frac{F_V}{F_H}\right)}$$

Q.1 Four masses m_1, m_2, m_3 and m_4 having their radii of rotations as 200 mm, 150 mm, 250 mm and 300 mm are 200 kg, 300 kg, 240 kg and 260 kg. The angle betn the successive masses are 45°, 75° and 135°. Find position and magnitude of the balance mass required if its radius of rotation is 200 mm.



We have

$m_1 = 200$ kg $m_2 = 300$ kg

$m_3 = 240$ kg $m_4 = 260$ kg.

$\theta_1 = 0$ $\theta_2 = 45°$

$\theta_3 = 45 + 75 = 120°$

$\theta_4 = 120° + 135 = 255°$

$r_1 = .2$ m $r_2 = 0.15$ m $r_3 = .25$ m

$r_4 = 0.3$ m.

$r = 0.2$ m

Analytical method

$\Sigma F_V = m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3 + m_4 r_4 \sin\theta_4$

$= 40 \sin 0° + 45 \sin 45° + 60 \sin 120° + 78 \sin 255°$

$= 8.439$ kg-m.

$\Sigma F_H = m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3 + m_4 r_4 \cos\theta_4$

$= 40 \cos 0 + 45 \cos 45 + 60 \cos 120 + 78 \cos 255 = 21.62$ kg m.

Resultant force

$$F = \sqrt{F_V^2 + F_H^2} = 23.2 \text{ Kg-m.}$$

Now $m \cdot r = 23.2$ Kg.m

$$\Rightarrow m = \frac{23.2}{0.2} = \boxed{116 \text{ kg.}}$$

Direction

$$\tan\theta = \frac{\Sigma F_V}{\Sigma F_H}$$

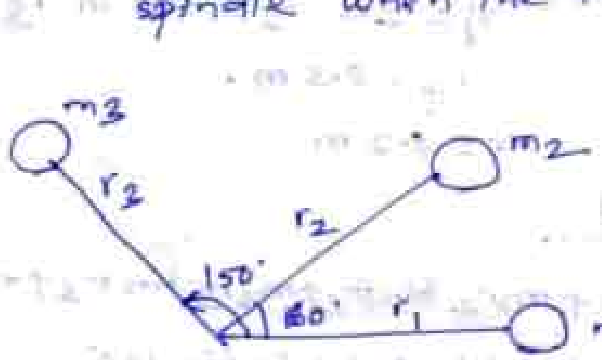
$$\Rightarrow \theta = \tan^{-1}\left(\frac{8.43}{21.65}\right) = \boxed{\cancel{201.5^\circ}\ 21.29^\circ}$$

Direction from m_1 = $180 + 21.29 = 201.5^\circ$

Q.2 A circular disc rotating around a vertical spindle, has the following masses placed on it.

mass	θ, wrt X-X	Distance from centre (mm)	Magnitude
m_1	0	260	2.5
m_2	60	300	3.5
m_3	150°	225	5.0

Determine the magnitude and angular position of a ~~ba~~ mass that should be placed at 262.5 mm to give a balanced system. Also determine the unbalanced force on the spindle when the disc is rotating at 950 rpm.



$$\Sigma F_V = m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3$$

$$= 0.65 \sin 0 + 1.05 \sin 60^\circ + 1.125 \sin 150^\circ$$

$$= 1.471 \text{ Kg m.}$$

Resultant force $F = 1.484$ kgm.

So $m \cdot r = 1.484$ kgm

$\Rightarrow \boxed{m = 5.653 \text{ kg.}}$

$\theta = \tan^{-1}\left(\frac{1.471}{0.2007}\right) = \boxed{82.23^\circ}$

Direction from $m_1 = \boxed{262.23^\circ}$

Magnitude of Resultant force.

$m\omega^2 r = 5.653 \times \left(\frac{2\pi \times 750}{60}\right)^2 \times .2625$

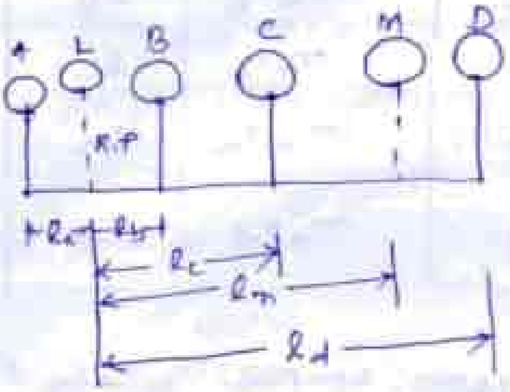
$= \boxed{1817 \text{ N.}}$

Balancing of Several masses Revolving in Different Planes :-

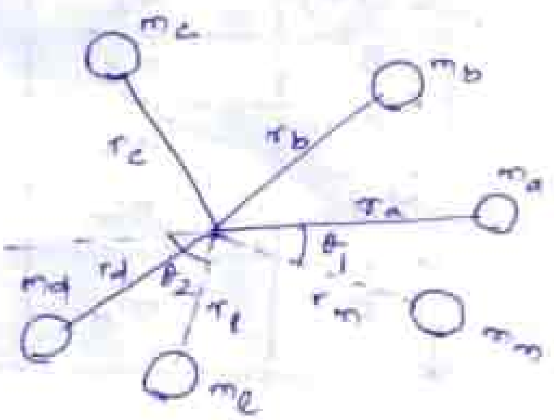
- Balancing of several masses revolving in different planes is done by transfer of the centrifugal force acting in different planes to a single plane, known as reference plane, thereby masses rotating in different planes are transferred to reference plane.

The effect of transferring the rotating mass m in the reference plane is to generate a centrifugal force $F_c = m\omega^2 r$ and a couple $C = F_c \cdot l$ in the reference plane where l = distance betn the reference plane and rotating.

- For complete balancing of such system, two conditions must be satisfied.
  1. Resultant centrifugal force must be zero
  2. Resultant couple must be zero.



(position of planes)



(Angular position of masses)

Let's consider several masses m_a, m_b, m_c and m_d revolving in planes A, B, C and D respectively.

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Two masses for balancing are used because of the following use!

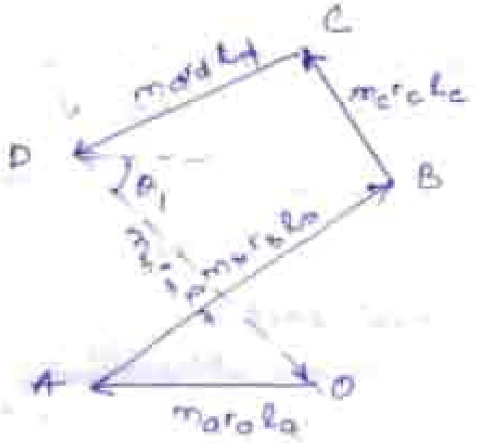
1. If a single mass is used the system will be difficult to handle.
2. If more than two masses are used, no of unknown parameters will be more than no of equations.

Procedure:-

1. Take one plane L as the reference plane. Distance to the left of this plane are taken with minus sign and those to right with +ve sign.
2. Tabulate the forces and couples as shown in the table.

Plane (1)	Mass (m) (2)	radius (r) (3)	centrifugal force ÷ ω^2 (mr) (4)	Distance from R.P. (l) (5)	couple ÷ ω^2 (mrl) (6)
A	m_a	r_a	$m_a r_a$	$-l_a$	$-m_a r_a l_a$
L (RP)	m_l	r_l	$m_l r_l$	0	0
B	m_b	r_b	$m_b r_b$	l_b	$m_b r_b l_b$
C	m_c	r_c	$m_c r_c$	l_c	$m_c r_c l_c$
M	m_m	r_m	$m_m r_m$	l_m	$m_m r_m l_m$
D	m_d	r_d	$m_d r_d$	l_d	$m_d r_d l_d$

3. Draw the couple polygon. Couple $m_a r_a l_a$ is -ve wrt RP. so the couple $(-m_a r_a l_a)$ is drawn radially inwards as it's in reverse direction of Om_a. couple $m_b r_b l_b$ is +ve wrt RP so it's drawn in the direction of Om_b. Similarly couples $m_c r_c l_c$ and $m_d r_d l_d$ are drawn in the directions of Om_c and Om_d respectively.



(couple polygon)

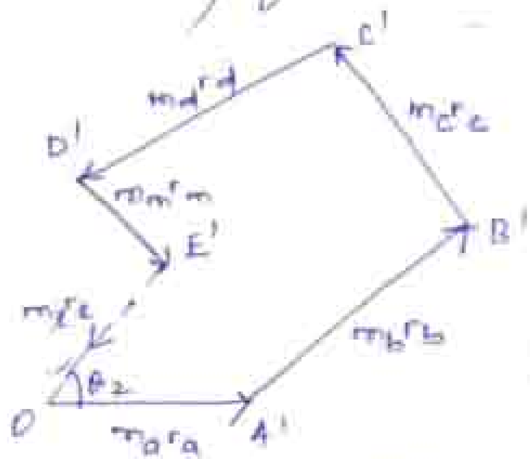
couple $m_m r_m l_m$ is the closing side. The balancing couple OD is proportional to $m_m r_m l_m$.

If the balancing radius r_m is known, balancing mass m_m can be obtained in magnitude and direction.

$$\boxed{m_m = \frac{OD}{r_m l_m}} \qquad \boxed{OD = m_m r_m l_m}$$

Thus m_m in plane M can be determined and angle θ_1 can be measured.

4. We can find other balancing mass m_l in plane L with the help of force polygon tabulated in column (4) of the table.



If the radius of 2nd balancing mass m_l is known m_l can be found in plane L and its angle of inclination θ_2 with horizontal may be measured.

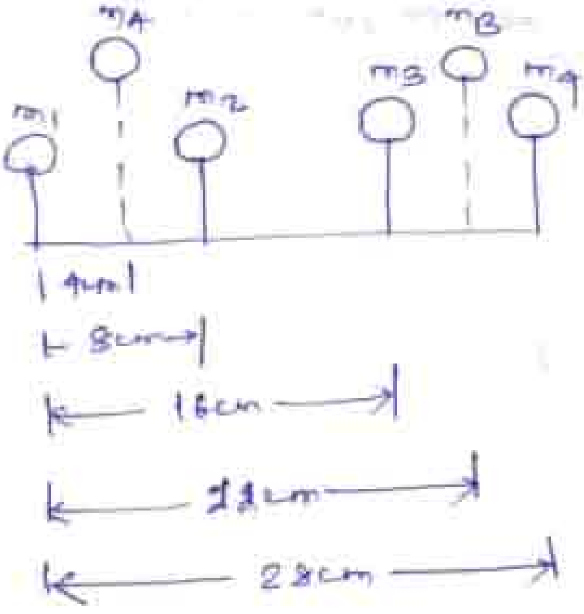
Q1 A rotating shaft carries four unbalanced masses 18 kg, 14 kg, 16 kg and 12 kg at radii 5cm, 6cm, 7cm and 6cm respectively. The 2nd, 3rd and 4th masses revolve in planes 8cm, 16cm and 28cm respectively measured from the plane of 1st mass and are angularly located at 60°, 135°, 270° respectively measured anticlockwise from 1st mass. The shaft is dynamically balanced by two masses both located at 5cm radii and revolving in planes mid way between those of 1st and 2nd masses and 3rd and 4th masses. Determine graphically the magnitude of the masses and their respective angular positions.

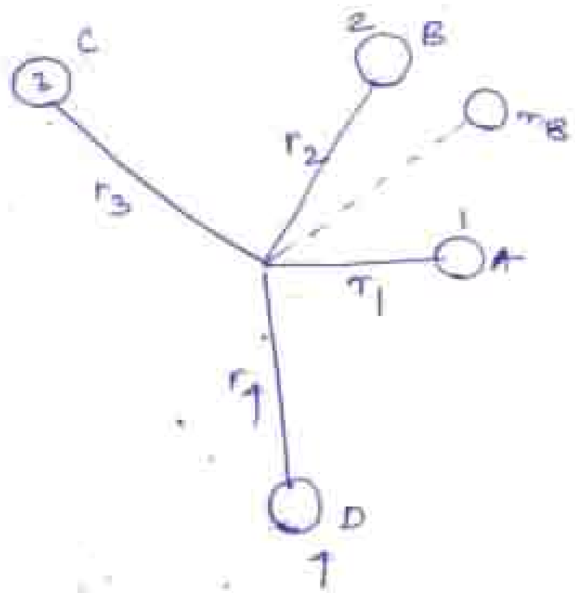
Given data :- $m_1 = 18$ kg $m_2 = 14$ kg $m_3 = 16$ kg $m_4 = 12$ kg.

$r_1 = 5$cm $r_2 = 6$cm $r_3 = 7$cm $r_4 = 6$cm.

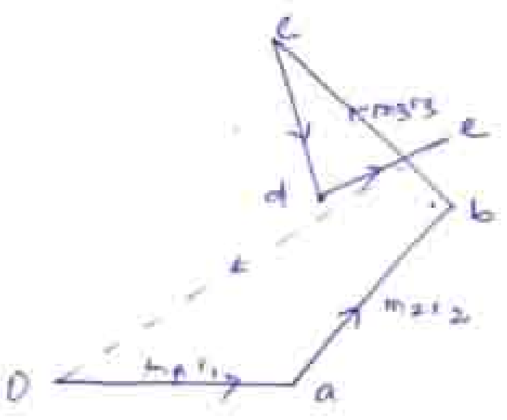
$\theta_1 = 0$ $\theta_2 = 60°$ $\theta_3 = 135°$ $\theta_4 = 270°$

Let the two balancing masses are m_A and m_B





plane	mass (m) Kg)	Radius (r) m)	centrifugal force ÷ w^2 (mr)	Distance from RP (l)	couple ÷ w^2 (mrl)
1	18	0.05	0.9	-.04	-0.036
A	m_A	0.05	$0.05m_A$	0	0
2	14	.06	0.84	.04	0.0336
3	16	.07	1.12	.12	0.1344
B	m_B	.05	$0.05m_B$	.18	$.009m_B$
4	12	.06	0.72	.24	0.1728

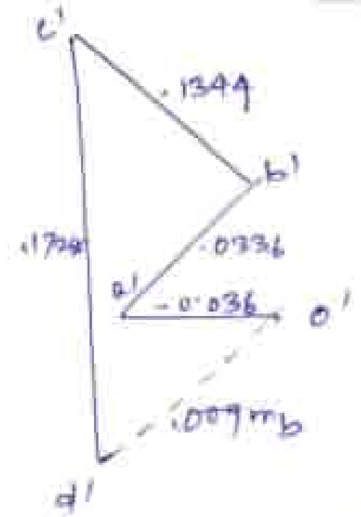


(force polygon)

$m_A = \frac{1.575}{0.05} = 31.5$ kg.

$\theta_A = 220°$ (from plane 1)

(2nd)



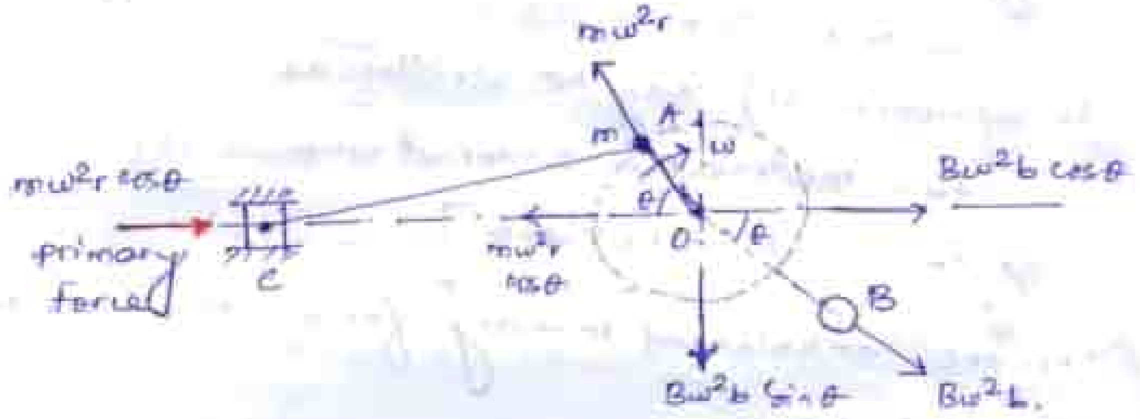
$m_B = \frac{o'd'}{.009}$

$= \frac{.12}{.009} = 13.33$ kg.

$\theta_B = 25°$

(1st)

Partial Primary Balancing :-



Consider a slider crank mechanism OAC. A primary unbalanced force $m\omega^2 r\cos\theta$ is required to accelerate the reciprocating mass, which acts along the direction from O to C. So balancing of primary force is considered equivalent to the component and parallel to the line of stroke, of the centrifugal force produced by an equal mass 'm' attached to the crank end and rotating at 'r' radius. To balance this force a rotating counter mass B is placed at a radius b, directly opposite to crank.

For complete balancing

$$B\omega^2 b\cos\theta = m\omega^2 r\cos\theta$$

$$\Rightarrow \boxed{B \cdot b = m \cdot r}$$

- However the vertical component of rotating mass B, of magnitude $B\omega^2 b\sin\theta$ remains unbalanced

Now the resultant disturbing force parallel to the line of stroke is

$$F_H = m\omega^2 r\cos\theta - B\omega^2 b\cos\theta$$

$$\Rightarrow \boxed{F_H = (m \cdot r - B \cdot b)\,\omega^2\cos\theta} \quad \text{--- (i)}$$

If $m \cdot r = B \cdot b$, the primary disturbing force is zero and the system will be unbalanced because of the vertical component of force.

Practically, a compromise is made and only a fraction c of reciprocating mass is balanced i.e,

$c \cdot m r = B \cdot b$.

so equation (1) may be written as

$F_H = m\omega^2 r\cos\theta - c \cdot m r\omega^2 \cos\theta$

$$\Rightarrow \boxed{F_H = (1-c)\, m r\omega^2 \cos\theta} \quad \text{--- (2)}$$

This is the unbalanced primary force acting along the line of stroke.

The unbalanced force ⊥r to the line of stroke is

$$F_V = B\omega^2 b\sin\theta = c m\omega^2 r\sin\theta \quad \text{--- (3)}$$

so the resultant unbalanced force

$F = \sqrt{F_H^2 + F_V^2}$

$$\Rightarrow \boxed{F = m\omega^2 r\sqrt{(1-c)^2\cos^2\theta + c^2\sin^2\theta}} \quad \text{--- (4)}$$

The value of c is kept betn $1/2$ to $3/4$.

The value of unbalanced force is minm when

$c = \frac{1}{2}$

$$F_{min} = m\omega^2 r\sqrt{\left(\tfrac{1}{2}\right)^2\cos^2\theta + \left(\tfrac{1}{2}\right)^2\sin^2\theta}$$

$$\Rightarrow \boxed{F_{min} = \frac{m\omega^2 r}{2}}$$

Q1 The following data relate to a single-cylinder reciprocating engine:

mass of reciprocating parts = 40 kg.

mass of revolving part = 30 kg at 180 mm radius.

speed = 150 rpm

stroke = 350 mm.

If 60% of reciprocating parts and all revolving parts are to be balanced, determine

(i) balance mass required at 320 mm radius.

∴ turned 45° from the TDC.

We have $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 150}{60} = 15.7 \text{ rad/s}$.

$r = \frac{350}{2} = 175 \text{ mm}$,

(i) mass to be balanced $= c \cdot m + m_p$

where m_p = mass of crankpin

m = reciprocating mass

c = fraction of reciprocating mass

So total mass to be balanced $= 0.6 \times 40 + 30$
$= 54 \text{ kg}$.

Now $B \cdot b = m \cdot r$

$B \times 320 = 54 \times 180$

Complete Balancing of Reciprocating Parts of an engine:-

For complete balancing of reciprocating parts of an engine, the following conditions must be satisfied:

- Primary force polygon must close
- Primary couple polygon must close
- secondary force polygon must close
- secondary couple polygon must close.

Partial Balancing of Locomotives :-

Most of the locomotives have two cylinders of same dimension and placed symmetrically either inside or outside the wheels.

The ratio of length of connecting rod to crank radius $\left(\frac{l}{r} = n\right)$ is generally large, so the effect of secondary unbalanced force

i.e. $\cos 2\theta$ are neglected

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In the partial balancing, two sets of unbalanced force exists

(i) an unbalanced force along the line of stroke

(ii) an unbalanced force ⊥r to the line of stroke.

The effect of (i) is to produce variation of tractive force along the line of stroke and unbalanced couple which is known as swaying couple.

- The effect of (ii) is to produce the variation of pressure on the rails which cause hammering action on rails. The max^m magnitude of unbalanced force ⊥r to the line of stroke is called Hammer blow.

Variation of Tractive force:-

The resultant unbalanced primary force due to two cylinders along line of stroke is called tractive force.

Let the crank of 1st cylinder be inclined at an angle θ with the line of stroke. Crank of 2nd cylinder will be inclined at angle $(90+\theta)$ with the line of stroke.

The unbalanced force along line of stroke for cylinder 1 is

$$F_1 = (1-c)\, m\omega^2 r \cos\theta$$

Unbalanced force along the line of stroke for cylinder 2 is

$$F_2 = (1-c)\, m\omega^2 r \cos(90+\theta)$$
$$= -(1-c)\, m\omega^2 r \sin\theta$$

Tractive force $F_T = F_1 + F_2$

$$= (1-c)\, m\omega^2 r \cos\theta - (1-c)\, m\omega^2 r \sin\theta$$
$$(1-c)\, m\omega^2 r\, (\cos\theta - \sin\theta)$$

for tractive force F_T to be max^m and min^m,
depends upon the value of $(\cos\theta - \sin\theta)$

$$\frac{d}{d\theta}(1-c)m\omega^2 r(\cos\theta - \sin\theta) = 0$$

$$-\sin\theta - \cos\theta = 0$$

$$\text{or } \tan\theta = -1$$

$$\text{or } \theta = 135^\circ \text{ or } 315^\circ$$

so the tractive force is max^m or min^m when
θ is 135° or 315°.

$$\text{so } F_{T\,max} = (1-c)m\omega^2 r(\cos 315 - \sin 315)$$

$$F_{T\,max} = \sqrt{2}(1-c)m\omega^2 r$$

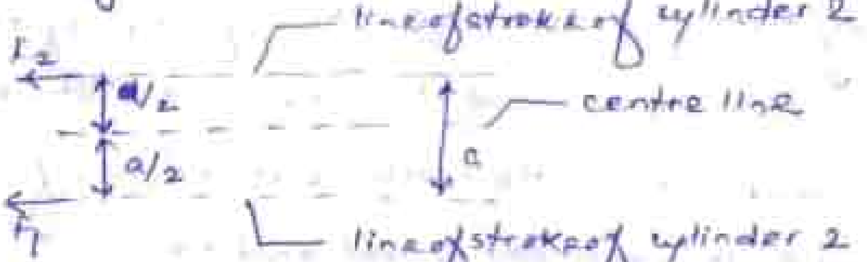
$$F_{T\,min} = (1-c)m\omega^2 r(\cos 135 - \sin 135)$$

$$\Rightarrow F_{T\,min} = -\sqrt{2}(1-c)m\omega^2 r$$

Thus $\boxed{F_T = \pm\sqrt{2}(1-c)m\omega^2 r}$

Swaying couple :-

The unbalanced force acting at a distance betn the line of stroke of two cylinders, constitute a couple in the horizontal direction. This couple is called as swaying couple.



Let a = distance betn the centre lines of the two cylinders.

F_1, F_2 = unbalanced forces for cylinder 1 and 2 respectively

$$F_1 = (1-c)\, m\omega^2 r \cos\theta$$

$$F_2 = (1-c)\, m\omega^2 r \cos(90+\theta)$$

The forces differ in phase by 90°.

swaying couple

$$T = F_1 \cdot \frac{a}{2} - F_2 \times \frac{a}{2}$$

$$= (1-c)\, m\omega^2 r \cos\theta \cdot \frac{a}{2} - (1-c)\, m\omega^2 r \cos(90+\theta) \cdot \frac{a}{2}$$

$$= (1-c)\, m\omega^2 r \cos\theta \, \frac{a}{2} + (1-c)\, m\omega^2 r \sin\theta \cdot \frac{a}{2}$$

$$= (1-c)\, m\omega^2 r \,(\cos\theta + \sin\theta) \cdot \frac{a}{2}$$

couple will be maximum or minimum when $(\cos\theta + \sin\theta)$ is maxm or minm.

Thus $\frac{d}{d\theta}(\cos\theta + \sin\theta) = 0$

$\Rightarrow -\sin\theta + \cos\theta = 0$

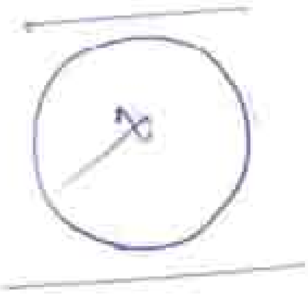
$\Rightarrow \tan\theta = 1$

or $\theta = 45°$ or $135°$

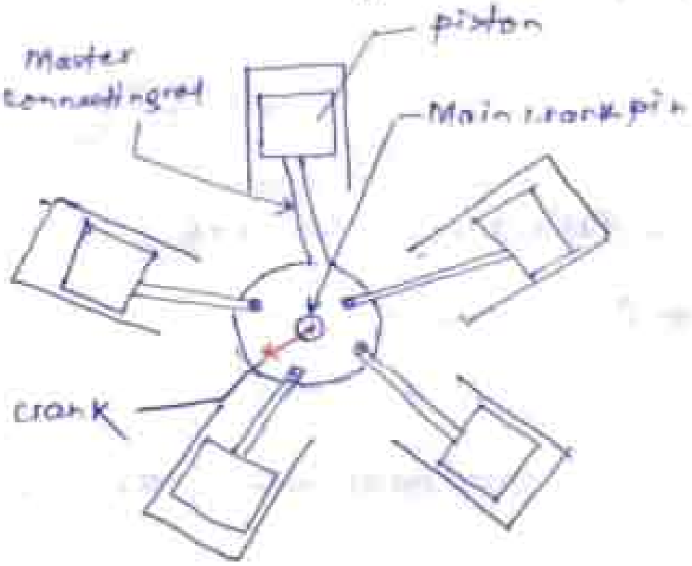
So the maxm and minm value of swaying couple

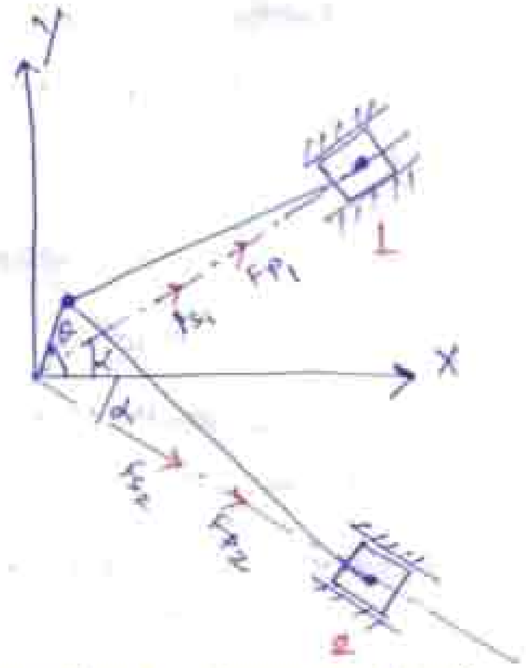
Hammer Blow:-

The max^m value of unbalanced force perpendicular to the line of stroke is called hammer blow. With very high speed this unbalance force may be very harmful causing the lifting of the wheels from rails and hitting on it. The effect of hammer blow is to cause variation of pressure between the wheel and rail.



Balancing of V- Engines:-





V-engines are also known as radial engine as their cylinders are arranged along radial lines. The centre lines of the cylinder form the shape of letter V. These cylinder have a common crank.