$Q_{ic} = mC_p(T_c - T_d)$.

8.6. CONDITIONS FOR MINIMUM WORK (MAXIMUM EFFICIENCY)

Consider a two stage compressor with complete intercooling. The total work required,

$$W = p_1V_1\left(\frac{n}{n-1}\right)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 2\right]$$

If p_1 and p_3 are fixed, the intermediate pressure, p_2 can be found out form the condition, $dW/dp_2 = 0$. This value of p_2 will give minimum compressor work.

Now

$$p_1 = \text{Constant}$$

$$V_1 = \text{Constant}$$

$$\frac{n-1}{n} = \text{Constant} = a \text{ (say)}$$

$$\therefore \quad W = \text{Constant} \times a\left[\left(\frac{p_2}{p_1}\right)^a + \left(\frac{p_3}{p_2}\right)^a - 2\right]$$

$$= \text{Constant} \times a\left[p_2^a p_1^{-a} + p_3^a p_2^{-a} - 2\right]$$

$$\frac{dW}{dp_2} = a\, p_1^{-a} . p_2^{a-1} - a\, p_2^{-a-1} p_3^a = 0$$

$$\therefore \quad a\, p_2^{a-1} p_1^{-a} = a\, p_2^{-a-1} p_3^a$$

$$\frac{p_2^a}{p_2} \cdot \frac{1}{p_1^a} = \frac{p_3^a}{p_2^a p_2}$$

$$\left(\frac{p_2}{p_1}\right)^a = \left(\frac{p_3}{p_2}\right)^a$$

$$\frac{p_2}{p_1} = \frac{p_3}{p_2}$$

$$\therefore \quad p_2 = \sqrt{p_1 p_3}$$

For minimum work of compressor (or maximum efficiency) the pressure ratio in each cylinder of compressor is the same. Therefore, work done in each cylinder is the same.

$$\therefore \quad W = 2p_1V_1\left(\frac{n}{n-1}\right)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

Conditions for minimum work of compressor are :

1. The intercooling is complete.
2. The pressure ratio in each stage is same.
3. The work required for each stage is same.

8.7. ISOTHERMAL AND ADIABATIC EFFICIENCY OF COMPRESSOR

Isothermal work required per cycle

$$= p_1V_1 \ln r$$

Isothermal power required

$$= \frac{p_1V_1 \ln r \times N}{60 \times 1000} [\text{kW}]$$

Where N = No. of cycles per minute.

Indicated power is obtained from the actual indicator diagram during a test on the compressor.

Compressor efficiency,

$$\eta_c = \frac{\text{Isothermal power}}{\text{Indicated power}}$$

The shaft power is the brake power required to drive the compressor and can be found out by a dynamometer.

Isothermal efficiency,

$$\eta_{iso} = \frac{\text{Isothermal power}}{\text{Shaft power}} \approx 0.70.$$

Adiabatic power can be calculated from an hypothetical indicator diagram with adiabatic compression process.

$$\text{Adiabatic power} = \frac{\gamma}{\gamma - 1} p_1V_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]\frac{N}{60 \times 1000}[\text{kW}]$$

Adiabatic efficiency is the ratio of adiabatic power and shaft power.

Adiabatic efficiency,

$$\eta_{adia.} = \frac{\text{Adiabatic power}}{\text{Shaft power}}$$

8.8. VOLUMETRIC EFFICIENCY

The volumetric efficiency of an air compressor is the ratio of the actual volume of free air inhaled during suction stroke (reduced to STP conditions of 1.01325 bar and 15°C) to the swept volume of the piston.

$$\eta_{vol.} = \frac{\text{Effective swept volume}}{\text{Swept volume}}$$

$$= \frac{V_1 - V_4}{V_1 - V_3} \approx 0.85$$

$$\text{Clearance ratio} = \frac{\text{Clearance vol.}}{\text{Swept vol.}}$$

$$k = \frac{V_3}{V_1 - V_3} = \frac{V_c}{V_s} \simeq 0.03$$

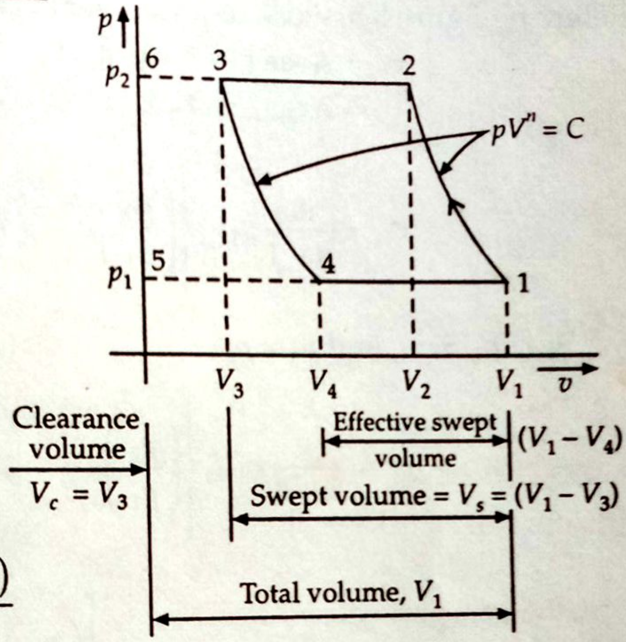

Fig. 8.5. Volumetric Efficiency

$$\eta_{vol.} = \frac{V_1 - V_4}{V_1 - V_3}$$

$$= \frac{(V_1 - V_3) + (V_3 - V_4)}{(V_1 - V_3)}$$

$$= 1 + \frac{V_3}{V_1 - V_3} - \frac{V_4}{V_1 - V_3}$$

$$= 1 + \frac{V_3}{V_1 - V_3} - \frac{V_4}{V_1 - V_3} \cdot \frac{V_3}{V_3}$$

$$= 1 + \frac{V_3}{V_1 - V_3} - \frac{V_3}{V_1 - V_3} \cdot \frac{V_4}{V_3}$$

$$\eta_{vol.} = 1 + k - k\frac{V_4}{V_3}$$

Now $\frac{V_4}{V_3} = \left(\frac{p_3}{p_4}\right)^{\frac{1}{n}}$

$$\therefore \quad \eta_{vol.} = 1 + k - k\left(\frac{p_3}{p_4}\right)^{\frac{1}{n}}$$

But $p_3 = p_2$ and $p_4 = p_1$

$$\therefore \quad \eta_{vol.} = 1 + k - k\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} = 1 + k - k\left(\frac{V_1}{V_2}\right)$$

The volumetic efficiency is lowered by the following factors :

1. Very high speed of the compressor.
2. Leakage of air past the piston.
3. Large clearance volume.
4. Obstruction in inlet valves.
5. Overheating of air by contact with hot cylinder walls.
6. Inertia effect of air in suction pipe.

8.9. EFFECT OF CLEARANCE SPACE ON COMPRESSOR WORK

Referring figure 8.5, work required per cycle,

$$W = \text{Area } (1 - 2 - 3 - 4 - 1)$$
$$= \text{Area } (5 - 1 - 2 - 6 - 5) - \text{Area } (5 - 4 - 3 - 6 - 5)$$

$$W = \frac{n}{n-1} p_1 V_1 \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right\} - \frac{n}{n-1} p_4 V_4 \left\{ \left(\frac{p_3}{p_4} \right)^{\frac{n-1}{n}} - 1 \right\}$$

But $p_4 = p_1$ and $p_3 = p_2$

$$\therefore \quad W = \frac{n}{n-1} p_1 V_1 \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right\} - \frac{n}{n-1} p_1 V_4 \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right\}$$

$$\therefore \quad W = \frac{n}{n-1} p_1 (V_1 - V_4) \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right\}$$

Now $V_1 - V_4 = V_a$ = Actual volume of free air delivered per cycle

$$\therefore \quad W = \frac{n}{n-1} m_1 R T_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

Therefore, there is no effect of clearance on work required. Work spent in compressing the clearance volume air is theoretically regained during its expansion form V_3 and V_4.

NUMERICAL EXAMPLES (RECIPROCATING COMPRESSORS)

Example 8.1. *A single acting, single cylinder air compressor has a cylinder of 150 mm diameter and a piston stroke of 225 mm. Air is drawn into the cylinder at a pressure of 1 bar and a temperature of 15° C. It is then compressed adiabatically to a pressure of 6 bar. Find the theoretical power required to drive the compressor if its speed is 100 rpm. Also find the mass flow rate of air compressed.*

Solution : State point a

$$p_a = 1 \text{ bar}$$
$$T_a = 15 + 273 = 288 \text{ K}$$