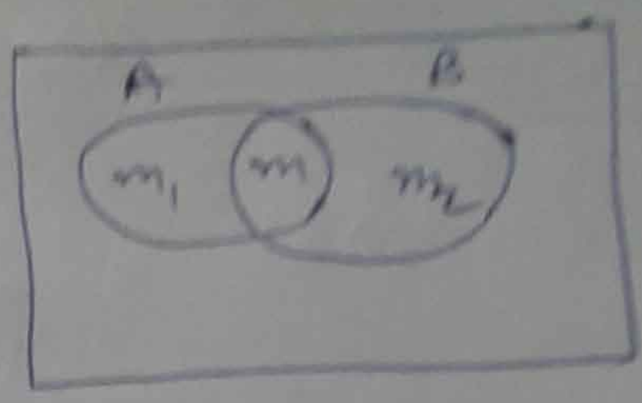
$$P(A) = \frac{m_1}{n}, \quad P(B) = \frac{m_2}{n}, \quad P(A \cap B) = \frac{m}{n}$$

The favourable outcomes to the event A only $= m_1 - m$

The favourable outcomes to the event B only $= m_2 - m$



The favourable outcomes to the event $A \cap B$ only $= m$

The favourable outcomes to the events A or B or both

i.e. $A \cup B = (m_1 - m) + (m_2 - m) + m$

$= m_1 + m_2 - m$

So $P(A \cap B) = \frac{m_1 + m_2 - m}{n} = \frac{m_1}{n} + \frac{m_2}{n} - \frac{m}{n}$

$P(A \cap B) = P(A) + P(B) - P(A \cap B)$ Proved

Theorem:→ If $P_1, P_2, \ldots P_n$ be separate Probabilities of mutually exclusive events, then the Probability P, that any of these events will happen is given by

$P = P_1 + P_2 + P_3 \ldots + P_n$

Multiplication Law of Probability :→ If there are two independent events the respective Probabilities of which are known, then the Probability that both will happen is the Product of the Probabilities of their happening respectively.

$P(A B) = P(A) \times P(B)$

Proof:→ Suppose A and B are two independent events. Let A happen in m_1 ways and fail in n_1 ways.

$\therefore P(A) = \frac{m_1}{m_1 + n_1}$

Also let B happen in m_2 ways and first in n_2 ways.

$$P(B) = \frac{m_2}{m_2 + n_2}$$

$$\therefore P(AB) = \frac{m_1 m_2}{(m_1+n_1)(m_2+n_2)} = \frac{m_1}{m_1+n_1} \cdot \frac{m_2}{m_2+n_2}$$

$$P(AB) = P(A) \cdot P(B)$$

Question: → An urn contains 9 balls, two of which are red three blue and four black. Three balls are drawn from the urn at random. What is the Probability that (i) The three balls are of different colours? (ii) The three balls are of the same colour?

Solution: → Urn contains 2 Red balls, 3 Blue balls and 4 black balls.

(i) Three balls will be of different colours if one red, one blue and one black ball are drawn.

$$\therefore \text{Required Probability} = \frac{{}^2C_1 \times {}^3C_1 \times {}^4C_1}{{}^9C_3} = \frac{2}{7} \text{ Ans.}$$

(ii) Three balls will be of the same colour if either 3 blue balls or 3 black balls are drawn.

$\Rightarrow P$ (3 blue balls or 3 Black balls) =

$$= P(\text{3 Blue balls}) + P(\text{3 Black balls})$$

$$= \frac{{}^3C_3}{{}^9C_3} + \frac{{}^4C_3}{{}^9C_3} = \frac{1}{\frac{9}{3}\times\frac{8}{2}\times\frac{7}{1}} + \frac{4}{\frac{9}{3}\times\frac{8}{2}\times\frac{7}{1}}$$

$$= \frac{1+4}{\frac{9\times8\times7}{3\times2\times1}} = \frac{5}{84} \text{ Ans.}$$