(V) Sol. of Difference Eq" using Z Transform —

(a) First order difference Eq"

(b) Second order difference Eq"

(a) First order difference Eq"

Question ⇒ $y[n] - \frac{1}{2}y[n-1] = 3\delta[n]$

Determine the value of $y[n]$ using Z Transform

Sol" ⇒ Taking Z-Transform both sides -

$$Y(z) - \frac{1}{2}z^{-1}Y(z) = 3 \cdot 1$$

$$Y(z)\left[1 - \frac{1}{2}z^{-1}\right] = 3$$

$$Y(z) = \frac{3}{1 - \frac{1}{2}z^{-1}}$$

Taking Inverse Z Transform both side -

$$y[n] = 3\left(\frac{1}{2}\right)^n u[n]$$

Question ⇒ $y[n] - \frac{1}{3}y[n-1] = 3\,u[n]$

Determine value of $y[n]$ using Z Transform -

Sol" ⇒ Taking Z Transform of both side

$$Y(z) - \frac{1}{3}z^{-1}Y(z) = 3 \cdot \frac{1}{(1 - z^{-1})}$$

$$Y(z)\left[1-\frac{1}{3}z^{-1}\right] = 3\cdot\frac{1}{(1-z^{-1})}$$

$$Y(z) = \frac{3}{(1-z^{-1})(1-\frac{1}{3}z^{-1})}$$

$$= \frac{A}{(1-z^{-1})} + \frac{B}{(1-\frac{1}{3}z^{-1})}$$

$$\Rightarrow 3 = A(1-\tfrac{1}{3}z^{-1}) + B(1-z^{-1})$$

Compare both side

$$A + B = 3$$

$$-\tfrac{1}{3}A - B = 0$$

$$\tfrac{2}{3}A = 3 \quad \text{or} \quad \boxed{A = \tfrac{9}{2}} \quad \boxed{B = -\tfrac{3}{2}}$$

$$Y(z) = \frac{9/2}{(1-z^{-1})} - \frac{3/2}{(1-\frac{1}{3}z^{-1})}$$

Taking Inverse Z Transform

$$y[n] = \frac{9}{2}u[n] - \frac{3}{2}\left(\frac{1}{3}\right)^n u[n]$$

Question $\Rightarrow$ $y[n] - \frac{1}{4}y[n-1] = \left(\frac{1}{4}\right)^n u[n]$

Determine value of $y[n]$

Sol:$\rightarrow$ Taking Z Transform of Both side

$$y(z) - \frac{1}{4}z^{-1}y(z) = \frac{1}{(1-\frac{1}{4}z^{-1})}$$

$$Y(z)\left[1 - \frac{1}{4}z^{-1}\right] = \frac{1}{1 - \frac{1}{4}z^{-1}}$$

$$Y(z) = \frac{1}{\left(1 - \frac{1}{4}z^{-1}\right)^2}$$

Taking Inverse Z Transform

$$Y[n] = \frac{\frac{1}{4}z^{-1}\,(4z)}{\left(1 - \frac{1}{4}z^{-1}\right)^2}$$

$$y[n] = 4(n+1)\left(\frac{1}{4}\right)^{n+1} u(n+1)$$

(b) Second order difference Eq^n →

Question ⇒ $y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = 3\delta[n]$

Determine Value of $y[n]$.

Sol^n ⇒ Taking Z Transform of both side

$$Y(z) - \frac{3}{4}z^{-1}Y(z) + \frac{1}{8}z^{-2}Y(z) = 3$$

$$Y(z)\left[1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}\right] = 3$$

$$Y(z) = \frac{3}{\left(1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}\right)} = \frac{3}{\left(\frac{1}{8}z^{-2} - \frac{3}{4}z^{-1} + 1\right)}$$

$$Y(z) = \frac{3}{\frac{1}{8}z^{-2} - \frac{1}{4}z^{-1} - \frac{1}{4}z^{-1} + 1}$$

$$Y(z) = \frac{3}{\frac{1}{2}z^{-1}\left(\frac{1}{4}z^{-1}-1\right) - 1\left(\frac{1}{4}z^{-1}-1\right)}$$

$$= \frac{3}{\left(\frac{1}{4}z^{-1}-1\right)\left(\frac{1}{2}z^{-1}-1\right)}$$

$$= \frac{3}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right)}$$

$$Y(z) = \frac{A}{\left(1-\frac{1}{4}z^{-1}\right)} + \frac{B}{\left(1-\frac{1}{2}z^{-1}\right)}$$

$$\Rightarrow 3 = A\left(1-\frac{1}{2}z^{-1}\right) + B\left(1-\frac{1}{4}z^{-1}\right)$$

Compair both side

$A + B = 3$

$-\frac{1}{2}A - \frac{1}{4}B = 0$

$\frac{1}{2}B = 3$ $\quad \boxed{B = 6}$ $\quad \boxed{A = -3}$

Then $Y(z) = \frac{-3}{\left(1-\frac{1}{4}z^{-1}\right)} + \frac{6}{\left(1-\frac{1}{2}z^{-1}\right)}$

Taking Inverse Z transform both side.

$$y[n] = -3\left(\frac{1}{4}\right)^n u[n] + 6\left(\frac{1}{2}\right)^n u[n]$$

Question → $y[n] - y[n-1] + \frac{1}{4}y[n-2] = 3u[n]$

Sol" → Taking Z Transform both side

$Y(z) - z^{-1}Y(z) + \frac{1}{4}z^{-2}Y(z) = \frac{3}{\left(1-z^{-1}\right)}$

$$Y(z)\left[1 - z^{-1} + \frac{1}{4}z^{-2}\right] = \frac{3}{(1-z^{-1})}$$

$$Y(z) = \frac{3}{(1-z^{-1})\left(\frac{1}{4}z^{-2} - z^{-1} + 1\right)}$$

$$= \frac{3}{(1-z^{-1})\left(\frac{1}{4}z^{-2} - \frac{1}{2}z^{-1} - \frac{1}{2}z^{-1} + 1\right)}$$

$$= \frac{3}{(1-z^{-1})\left[\frac{1}{2}z^{-1}\left(\frac{1}{2}z^{-1} - 1\right) - 1\left(\frac{1}{2}z^{-1} - 1\right)\right]}$$

$$= \frac{3}{(1-z^{-1})(1-\frac{1}{2}z^{-1})(1-\frac{1}{2}z^{-1})}$$

$$Y(z) = \frac{A}{1-z^{-1}} + \frac{B}{1-\frac{1}{2}z^{-1}} + \frac{C}{(1-\frac{1}{2}z^{-1})^2}$$

$$\Rightarrow 3 = A(1-\tfrac{1}{2}z^{-1})^2 + B(1-\tfrac{1}{2}z^{-1})(1-z^{-1}) + C(1-z^{-1})$$

$A = 6$
$B = -9$
$C = 6$

$$Y(z) = \frac{6}{(1-z^{-1})} - \frac{9}{(1-\frac{1}{2}z^{-1})} + \frac{6\times 2}{(1-\frac{1}{2}z^{-1})^2}$$

$$\therefore y[n] = 6\,u[n] - 9\left(\frac{1}{2}\right)^n u[n] + \frac{12\,z^{-1}}{(1-\frac{1}{2}z^{-1})} \quad (\because z)$$

$$y[n] = 6u[n] - 9\left(\frac{1}{2}\right)^n u[n] + 12(n+1)\left(\frac{1}{2}\right)^{n+1} u[n+1]$$