(iv) Property of Z Transformation ⟹

(a) Linear Property ⟹ If $x_1[n]$ & $x_2[n]$ are Two discrete Time signal

$$Z[x_1[n]] = X_1(z)$$
$$Z[x_2[n]] = X_2(z)$$

$$\boxed{Z[a x_1[n] + b x_2[n]] = a X_1(z) + b X_2(z)}$$

Proof ⟹

we know that, $Z[x[n]] = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$

Similerly,

$$Z[a x_1[n] + b x_2[n]] = \sum_{n=-\infty}^{\infty} [a x_1[n] + b x_2[n]] z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} a x_1[n] z^{-n} + \sum_{n=-\infty}^{\infty} b x_2[n] z^{-n}$$

$$\boxed{Z[a x_1[n] + b x_2[n]] = a X_1(z) + b X_2(z)}$$

(b) Time Shifting Property ⟹

If $x[n]$ is Discrete Time signal.

$$Z[x[n]] = X(z)$$

$$\boxed{Z[x[n-n_0]] = z^{-n_0} X(z)}$$

Proof →

$$Z[x[n-n_0]] = \sum_{n=-\infty}^{\infty} x[n_1] Z^{-n_1} Z^{-n_0}$$

$$= Z^{-n_0} \sum_{n=-\infty}^{\infty} x[n_1] Z^{-n_1}$$

$$\boxed{Z[x[n-n_0]] = Z^{-n_0} X(z)}$$

(6) Differentiation Property :-
(Multiplication by n Property)

If $Z[x[n]] = X(z)$

Then; $\boxed{Z[n\,x[n]] = -z \frac{d}{dz} X(z)}$

Proof ⇒ We know that, $Z[x[n]] = X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$ — (i)

Differentiating Eqⁿ (i) w.r.t z

$$\frac{d}{dz} X(z) = \frac{d}{dz}\left\{\sum_{n=-\infty}^{\infty} x[n] z^{-n}\right\}$$

$$= \sum_{n=-\infty}^{\infty} x[n] \left\{\frac{d}{dz} z^{-n}\right\}$$

$$= \sum_{n=-\infty}^{\infty} x[n] \left\{-n z^{-n-1}\right\}$$

$$= -\sum_{n=-\infty}^{\infty} n\,x[n] z^{-n} z^{-1}$$

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$$\Rightarrow \frac{d}{dz}[X(z)] = -z^{-1}\sum_{n=-\infty}^{\infty} n\, x[n] z^{-n}$$

$$-z\frac{d}{dz}X(z) = \sum_{n=-\infty}^{\infty} n\, x[n]\cdot z^{-n}$$

or $$\boxed{Z\{n\,x[n]\} = -z\frac{d}{dz}X(z)}$$

Example $\Rightarrow$ $Z\left[n\left(\frac{1}{2}\right)^n u[n]\right]$ solve it by using Property?

Solⁿ $\Rightarrow$ By Differentiation Property -

$$= -z\frac{d}{dz}\left[\frac{1}{1-\left(\frac{1}{2}\right)z^{-1}}\right]$$

$$= -z\frac{d}{dz}\left[\frac{2z}{2z-1}\right]$$

$$= -z\left[\frac{(2z-1)\cdot 2 - 2z\cdot 2}{(2z-1)^2}\right]$$

$$= -z\left[\frac{4z-2-4z}{(2z-1)^2}\right]$$

$$= -z\left[\frac{-2}{(2z-1)^2}\right]$$

$$= \frac{2z}{(2z-1)^2}, = \frac{1}{2z\left(1-\frac{1}{2z}\right)^2}$$

(d) Convolution Property ⇒

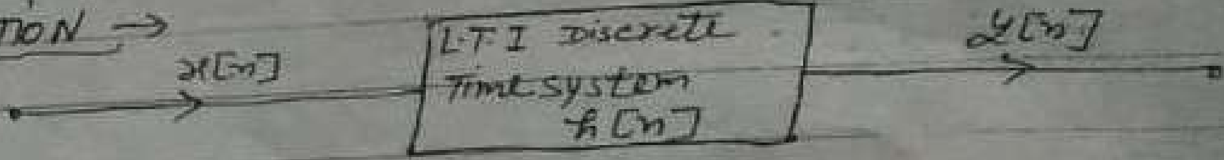
If $x_1[n]$ and $x_2[n]$ are two D.T. Time Signal.

$$Z[x_1[n]] = X_1(z)$$

$$Z[x_2[n]] - X_2(z)$$

then, $$\boxed{Z[x_1[n] \otimes x_2[n]] = X_1(z)\, X_2(z)}$$

APPLICATION →

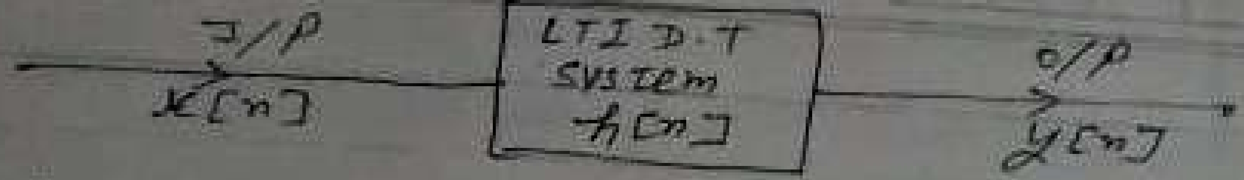


We Know That, $y[n] = x[n] \otimes h[n]$

taking Z transform both side

then. $$\boxed{Y(z) = X(z) \cdot H(z)}$$

Question :



given.

$$x[n] = \left(\tfrac{1}{2}\right)^n u[n]$$
$$h[n] = 5\cdot\left(\tfrac{1}{3}\right)^n u[n]$$

Determine O/P of the given System :

Sol.^n : We know that, $y[n] = x[n] \otimes h[n]$
By taking Z Transform

$$Y[z] = X(z)\cdot H(z)$$

$$X(z) = Z[x[n]] = Z\left[\left(\tfrac{1}{2}\right)^n u[n]\right]$$
$$= \frac{1}{\left(1-\left(\tfrac{1}{2}\right)z^{-1}\right)}$$

$$H(z) = Z[h[n]] = Z\left[5\cdot\left(\tfrac{1}{3}\right)^n u[n]\right]$$
$$= \frac{5}{\left(1-\tfrac{1}{3}z^{-1}\right)}$$

then, $Y(z) = \frac{1}{\left(1-\frac{1}{2z}\right)}\cdot\frac{5}{\left(1-\frac{1}{3z}\right)}$

By Partial fraction

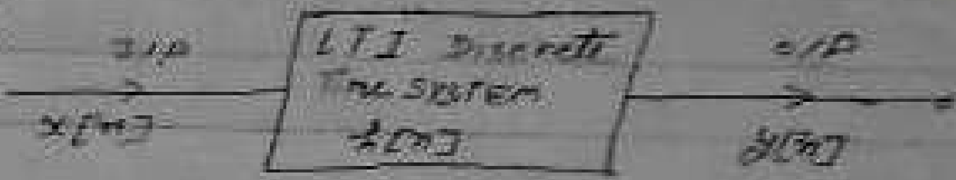
$$Y(z) = \frac{A}{\left(1-\frac{1}{2z}\right)} + \frac{B}{\left(1-\frac{1}{3z}\right)}$$

$$= \frac{15}{\left(1-\frac{1}{2}z^{-1}\right)} - \frac{10}{\left(1-\frac{1}{3}z^{-1}\right)}$$

Now By Taking Inverse Z Transform

$$y[n] = 15\left(\frac{1}{2}\right)^n u[n] - 10\left(\frac{1}{3}\right)^n u[n]$$

Question ⇒



The system is defined as -

$$y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = 5x[n]$$

Determine —

(i) Transfer function of given System

(ii) Step Response of given System.

(iii) Response of system when $x[n] = \left(\frac{1}{4}\right)^n u[n]$

Solⁿ ⇒ (i) $y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = 5x[n]$

by Taking Z Transform

$$Y(z) - \frac{5}{6}z^{-1}Y(z) + \frac{1}{6}z^{-2}Y(z) = 5X(z)$$

$$Y(z)\left\{1 - \frac{5}{6z} + \frac{1}{6z^2}\right\} = 5X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{5}{1 - \frac{5}{6z} + \frac{1}{6z^2}}$$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{5}{\frac{1}{6z^2} - \frac{5}{6z} + 1}$$

$$= \frac{5}{\frac{1}{6z^2} + \left(-\frac{1}{2} - \frac{1}{3}\right)\frac{1}{z} + 1}$$

$$= \frac{5}{\frac{1}{2z}\left(\frac{1}{3z} - 1\right) - 1\left(\frac{1}{3z} - 1\right)}$$

$$H(z) = \frac{5}{\left(\frac{1}{2z} - 1\right)\left(\frac{1}{3z} - 1\right)}$$

$$H(z) = \frac{5}{\left(1 - \frac{1}{2z}\right)\left(1 - \frac{1}{3z}\right)}$$

By Partial fraction

$$H(z) = \frac{A}{1 - \frac{1}{2z}} + \frac{B}{1 - \frac{1}{3z}}$$

$$\Rightarrow 5 = A\left(1 - \frac{1}{3z}\right) + B\left(1 - \frac{1}{2z}\right)$$

$$A + B = 5 \quad \text{—(1)}$$

$$-\frac{A}{3} - \frac{B}{2} = 0 \quad \text{—(2)}$$

$$2A + 3B = 0$$

$$2A + 2B = 10$$

$$\boxed{B = -10} \quad \text{then} \quad \boxed{A = 15}$$

then $$H(z) = \frac{15}{1 - \frac{1}{2z}} + \frac{-10}{1 - \frac{1}{3z}}$$

Taken Inverse Z transform,

$$h[n] = 15\left(\frac{1}{2}\right)^n u[n] - 10\left(\frac{1}{3}\right)^n u[n]$$

(ii) Step Response of System -

$$y[n] = x[n] \otimes h[n]$$

$$Y(z) = X(z) \cdot H(z)$$

If $x[n] = u[n]$

then $X(z) = \frac{1}{1-z^{-1}}$

$$H(z) = \frac{5}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{3}z^{-1})}$$

$$y(z) = X(z)\, H(z)$$

$$= \frac{1}{(1-\frac{1}{z})} \cdot \frac{5}{(1-\frac{1}{2z})(1-\frac{1}{3z})}$$

$$y(z) \Rightarrow \frac{A}{1-\frac{1}{z}} + \frac{B}{1-\frac{1}{2z}} + \frac{C}{1-\frac{1}{3z}}$$

$$\Rightarrow A(1-\tfrac{1}{2z})(1-\tfrac{1}{3z}) + B(1-\tfrac{1}{z})(1-\tfrac{1}{3z}) + C(1-\tfrac{1}{z})(1-$$

when $\frac{1}{z} = 1$ or $z = 1$ then $5 = 0 + 0 + A(\frac{1}{2})(\frac{2}{3})$, $A = 15$

when $\frac{1}{2z} = 1$ or ... $5 = 0 + B(1-2)(1-\frac{2}{3})$, $5 = -B(\frac{1}{3})$, $B = -15$

when $\frac{1}{3z} = 1$ or $z = \frac{1}{3}$: $5 = 0 + 0 + C(1-3)$, $5 = C(1-3)(-\frac{1}{2})$, $C = 5$

then $y(z) = \frac{15}{1-\frac{1}{z}} - \frac{15}{1-\frac{1}{2z}} + \frac{5}{1-\frac{1}{3z}}$

Taking Inverse Z Transform

$$y[n] = 15\, u[n] - 15\left(\tfrac{1}{2}\right)^n u[n] + 5\left(\tfrac{1}{3}\right)^n u[n]$$

$15 - \frac{15 \cdot 2}{?} + 5 \times 2$