## 7.7. Block Diagram Representation for Discrete-Time LTI Systems

As a matter of fact, the $z$-transform is used to replace time-domain operations such as convolution and time-shifting with algebric operations for discrete-time signals and LTI systems. Further, we know that the difference equations of a discrete-time LTI system can be replaced by algebraic descriptions. Also, z-transform is used to convert system description to algebraic equations which are helpful in the analysis of inter-connections of LTI systems.

In addition to this, these algebraic equations are also helpful in representing and synthesizing LTI systems as inter-connections of basic system building blocks.

### 7.7.1. Transfer Function of Inter-Connection of Discrete-Time LTI Systems

To analyze discrete-time block diagrams such as series or cascade, parallel and feedback inter-connections, the transfer function algebra is exactly the same as that for corresponding continuous-time LTI systems. In this sub-article, let us go through the following inter-connections of discrete-time LTI systems :

(i) Series or cascade inter-connection of two discrete-time LTI systems.

(ii) Parallel inter-connection of two discrete-time LTI systems.

(iii) Feedback inter-connection of two discrete-time LTI systems.

#### 7.7.1.1. Series or Cascade Inter-connection of Two Discrete-Time LTI Systems

Figure (7.5), shows the series interconnection of two discrete-time LTI systems.

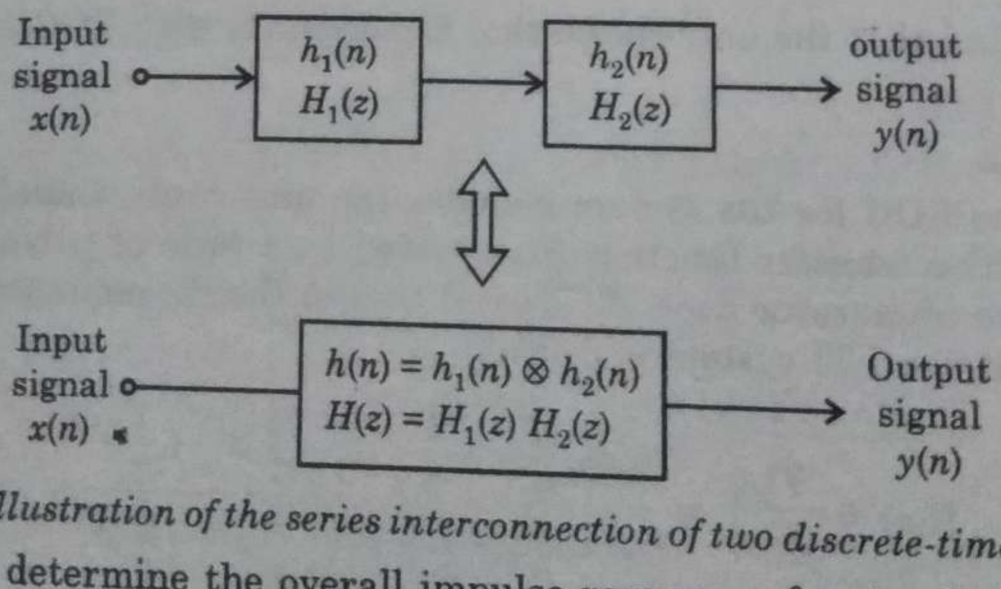


*Fig. 7.5. Illustration of the series interconnection of two discrete-time LTI systems.*

We can determine the overall impulse response of series inter-connection of two discrete-time LTI systems by taking convolution sum between $h_1(n)$ and $h_2(n)$. Here, $h_1(n)$ and $h_2(n)$ are the impulse responses of two discrete-time LTI systems.

Thus, we can express the overall impulse response of series interconnection of two discrete-time LTI systems as under:

$$h(n) = h_1(n) \otimes h_2(n) \quad ...(7.42)$$

Further, we can determine the transfer function of series interconnection of two discrete-time LTI systems simply by taking z-transform of both sides of the last equation as under:

$$Z[h(n)] = Z[h_1(n) \otimes h_2(n)]$$

Now, using convolution property of z-transform, the above expression becomes

$$H(z) = H_1(z)\, H_2(z) \qquad ...(7.43)$$

**Note :** *Hence, it may be observed that the transfer function H(z) for the cascade of two discrete-time LTI systems is the product of transfer functions of individual systems.*

#### 7.7.1.2. Parallel Inter-Connection of Two Discrete-Time LTI Systems

Figure (7.6) shows the parallel inter-connection of two discrete-time LTI systems.

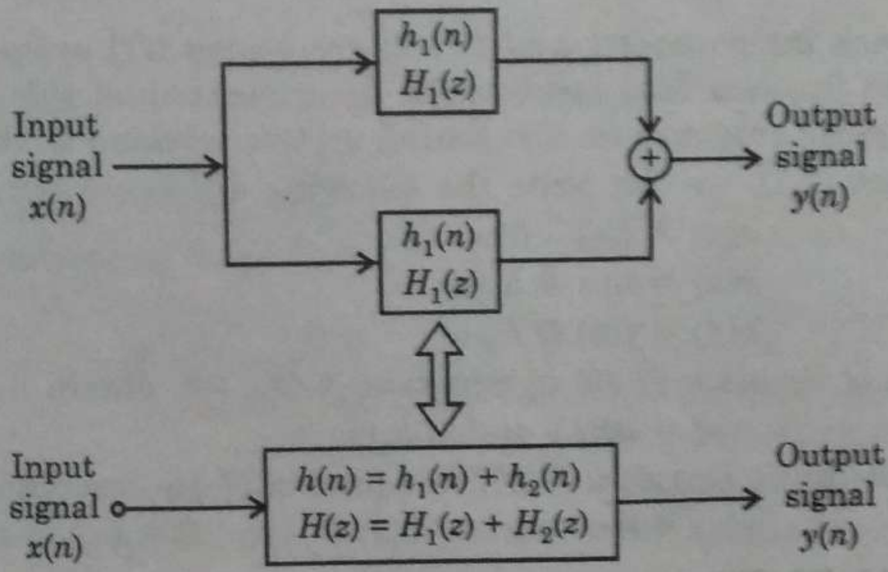


*Fig. 7.6. Illustration of parallel interconnection of two discrete-time LTI systems.*

We can determine the overall impulse response of a parallel inter-connection of two discrete-time LTI systems simply by addition of individual impulse-responses of the two systems.

Thus, the overall impulse response of parallel inter-connection of the above two discrete-time LTI systems can be expressed as under :

$$h(n) = h_1(n) + h_2(n) \qquad ...(7.44)$$

Further, the transfer function of parallel interconnection of two discrete-time LTI systems can be evaluated simply by taking z-transform of both sides of the last equation as under :

$$Z[h(n)] = Z[h_1(n) + h_2(n)]$$

Now, using the linearity property of z-transform, we get

$$Z[h(n)] = Z[h_1(n)] + Z[h_2(n)]$$

or

$$H(z) = H_1(z) + H_2(z) \qquad ...(7.45)$$

**Note :** *Thus, from equation (8.46), it may be observed that the transfer function H(z) for a parallel inter-connection of two discrete-time LTI systems is the sum of transfer functions of individual systems in parallel. Here, H(z) is the transfer function of parallel interconnection of two discrete-time LTI systems.*

#### 7.7.1.3. Feedback Interconnection of two Discrete-Time LTI Systems

Figure (7.7) shows the feedback inter-connection of two discrete-time LTI systems.

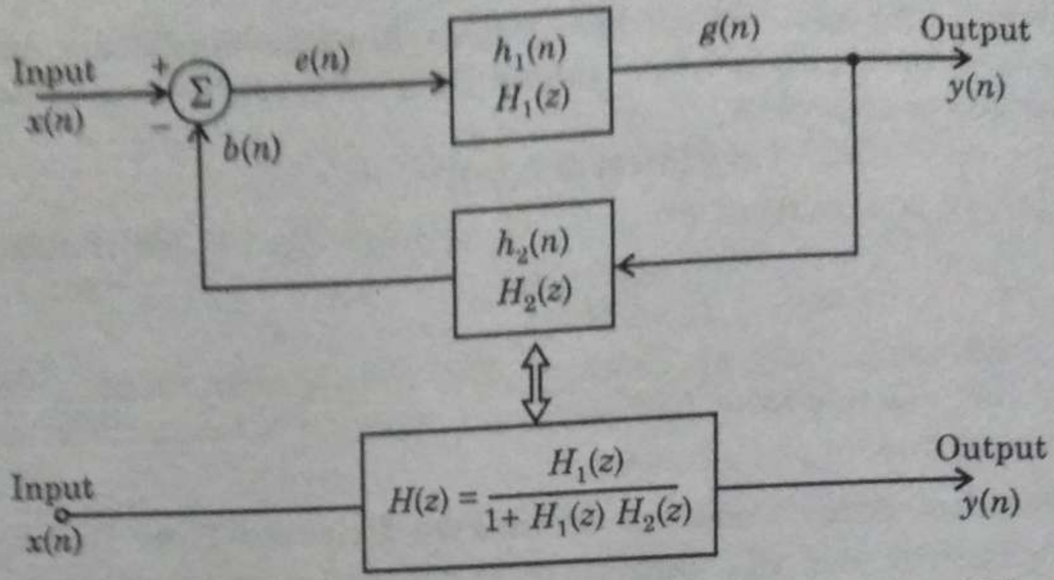


*Fig. 7.7. Illustration of feedback interconnection of two discrete-time LTI systems.*

For feedback interconnection of two discrete-time LTI systems, the evaluation of transfer function $H(z)$ involves the determination of the difference equation of the impulse response for the overall system working in the time-domain.

From figure (7.7), we can write the following difference equations :

$$e(n) = x(n) - b(n) \qquad ...(7.46)$$

$$y(n) = e(n) \otimes h_1(n) \qquad ...(7.47)$$

$$b(n) = y(n) \otimes h_2(n) \qquad ...(7.48)$$

Substituting equation (7.49) in equation (7.47), we obtain

$$e(n) = x(n) - y(n) \otimes h_2(n) \qquad ...(7.49)$$

Now, substituting equation (7.50) in equation (7.48), we obtain

$$y(n) = e(n) \otimes h_1(n) = [x(n) - y(n) \otimes h_2(n)] \otimes h_1(n)$$

Simplifying, we get

$$y(n) = x(n) \otimes h_1(n) - y(n) \otimes h_2(n) \otimes h_1(n) \qquad ...(7.50)$$

Further, taking z-transform of above equation, we get

$$Z[y(n)] = Z[x(n) \otimes h_1(n) - y(n) \otimes h_2(n) \otimes h_1(n)]$$

or $$Z[y(n)] = Z[x(n) \otimes h_1(n)] - Z[y(n) \otimes h_2(n) \otimes h_1(n)]$$

Now, using convolution property of z-transform, we get

$$Y(z) = X(z)\, H_1(z) - Y(z)\, H_1(z)\, H_2(z)$$

or $$Y(z) + Y(z)\, H_1(z)\, H_2(z) = X(z)\, H_1(z)$$

or $$\frac{Y(z)}{X(z)} = \frac{H_1(z)}{1 + H_1(z)\, H_2(z)}$$

or $$H(z) = \frac{Y(z)}{X(z)} = \frac{H_1(z)}{1 + H_1(z)\, H_2(z)} \qquad ...(7.51)$$

Hence, this is the overall transfer function of the feedback interconnection of the two discrete-time LTI systems.

### 7.7.2. Block Diagram Representation of Causal Discrete-Time LTI Systems described by Difference Equations and Rational Transfer Functions

As a matter of fact, causal discrete-time LTI systems described by linear constant-coefficient difference equations can be represented by block diagrams involving three basic operations. These operations are addition, multiplication by a constant-coefficient, and a unit delay. Further, we know that causal con-

...inuous-time LTI systems described by linear constant-coefficient difference equa-tions can also be represented by block diagrams involving the above three basic algebraic operations. Now, let us discuss some more examples to illustrate the basic ideals in constructing block diagram representation.

**Example 7.35. Draw the block diagram representation in direct form, cascade form and parallel form for a discrete-time LTI system expressed by the following transfer function :**

$$H(z) = \frac{1}{\left(1+\frac{1}{3}z^{-1}\right)\left(1-\frac{1}{6}z^{-1}\right)}.$$

**Solution:** The given transfer function is

$$H(z) = \frac{1}{\left(1+\frac{1}{3}z^{-1}\right)\left(1-\frac{1}{6}z^{-1}\right)} \quad ...(i)$$

We know that transfer function $H(z)$ is given by

$$H(z) = \frac{Y(z)}{X(z)} \quad ...(ii)$$

Using equations (*i*) and (*ii*), we get

$$\frac{Y(z)}{X(z)} = \frac{1}{\left(1+\frac{1}{3}z^{-1}\right)\left(1-\frac{1}{6}z^{-1}\right)} = \frac{1}{1+\frac{1}{6}z^{-1}-\frac{1}{18}z^{-2}}$$

or

$$(z)\left[1+\frac{1}{6}z^{-1}-\frac{1}{18}z^{-2}\right] = X(z)$$

or

$$(z)+\frac{1}{6}z^{-1}Y(z)-\frac{1}{18}z^{-2}Y(z) = X(z) \quad ...(iii)$$

Now, taking the inverse z-transform of both sides of equation (*iii*) with zero initial conditions, we obtain

$$y(n)+\frac{1}{6}y(n-1)-\frac{1}{18}y(n-2) = x(n) \quad ...(iv)$$

Thus, the direct form block diagram representation of a discrete-time LTI system described by difference equation is given by equation (*iv*)

$$y(n)+\frac{1}{6}y(n-1)-\frac{1}{18}y(n-2) = x(n)$$

or

$$y(n) = -\frac{1}{6}y(n-1)+\frac{1}{18}y(n-2)+x(n) \quad ...(v)$$

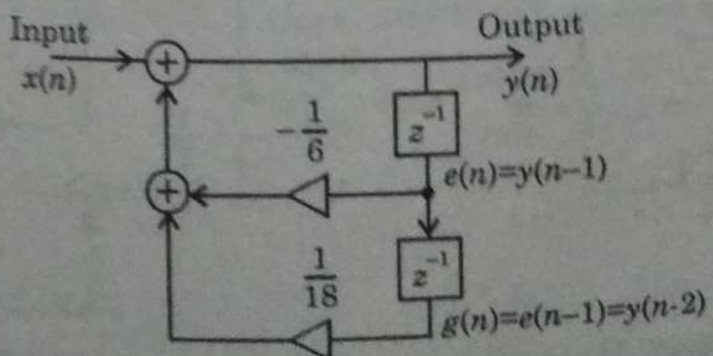


*Fig. 7.8. Direct form block diagram representation of a discrete-time LTI system described by transfer function given by equation (i).*

Also, cascade form block diagram representation of a discrete-time LTI system described by transfer function given by equation (*i*), is

$$H(z) = \frac{1}{\left(1+\frac{1}{3}z^{-1}\right)\left(1-\frac{1}{6}z^{-1}\right)}$$

or $$H(z) = \left[1+\frac{1}{3}z^{-1}\right]^{-1}\left[1-\frac{1}{6}z^{-1}\right]^{-1}$$

or $$H(z) = \left(1-\frac{1}{3}z^{-1}\right)\left(1+\frac{1}{6}z^{-1}\right) = H_1(z)\,H_2(z) \quad ...(vi)$$

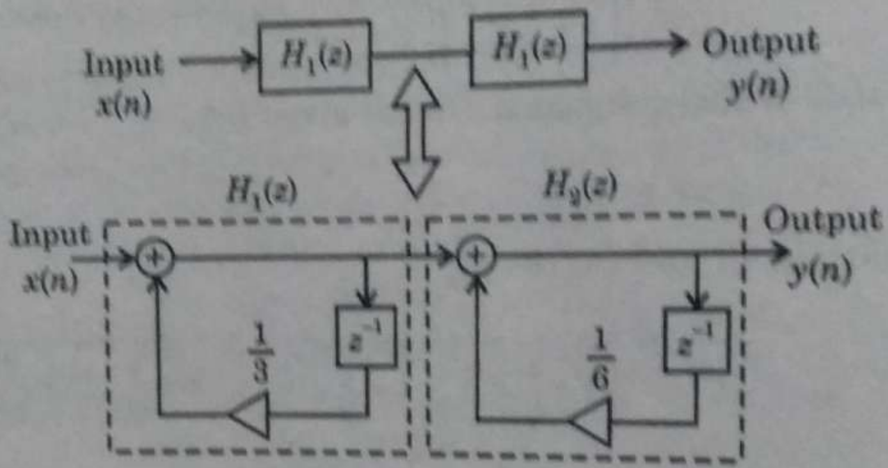


*Fig. 7.9. Cascade form block diagram representation of a discrete-time LTI system described by transfer function given by equation (vi).*

Further, parallel form block diagram representation of a discrete-time LTI system described by equation (*i*), will be

$$H(z) = \frac{1}{\left(1+\frac{1}{3}z^{-1}\right)\left(1-\frac{1}{6}z^{-1}\right)} = \frac{\alpha_1}{\left(1+\frac{1}{3}z^{-1}\right)} + \frac{\alpha_2}{\left(1-\frac{1}{6}z^{-1}\right)}$$

(using partial fraction expansion)

or $$H(z) = \frac{2/3}{\left(1+\frac{1}{3}z^{-1}\right)} + \frac{1/3}{\left(1-\frac{1}{6}z^{-1}\right)} = H_1(z) + H_2(z) \quad ...(vii)$$

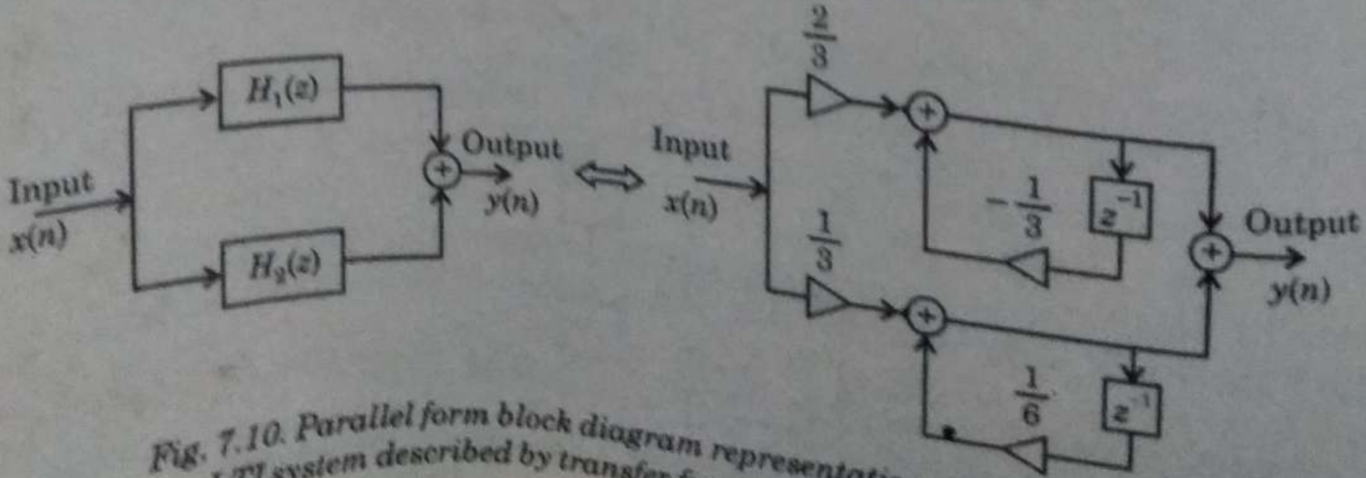


*Fig. 7.10. Parallel form block diagram representation of a di... LTI system described by transfer function...*