(i) Z Transform of Standard Signal.
Imp (ii) Concept of ROC [Region of Convergence]
(iii) Finite & Infinite Sequence Signal
(iv) Property of Z Transformation

(v) Solⁿ of Difference Eqⁿ using Z Transformation

(vi) Analysis of LTI System using Z Transformation

(vii) Inverse Z-Transformation.

(viii) Initial & Final Value Theorem.

(ix) Block diagram Representation of System.

(i) Z-Transform of Standerd Signal ÷

Z-Transform is a summition Transformation which Transform a discrete Time domain signal into Z domain signal.

mathematically. $\boxed{Z[x[n]] = X(z)}$

where,

$Z$ = Z Transform operator

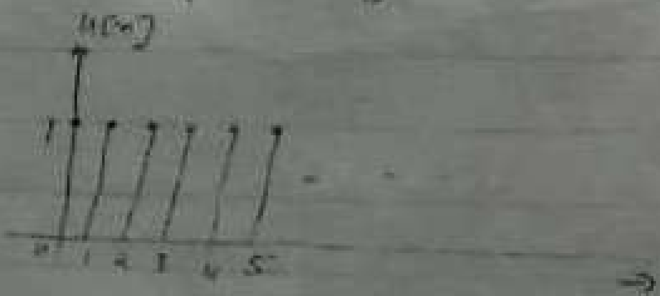
$x[n]$ = Discrete Time domain signal

$X(z)$ = Z domain signal.

Definition. $\boxed{Z[x[n]] = X(z) = \sum_{n=-\infty}^{\infty} x[n] Z^{-n}}$

* Z Transformation of Standerd Signal →

(i) Unit step Signal [sequence.



$$u[n] = \begin{cases} 0 & , n < 0 \\ 1 & , n \geq 0 \end{cases}$$

$$z[x[n]] = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

similarly,



$$Z[u[n]] = \sum_{n=-\infty}^{\infty} u[n]\, z^{-n}$$

$$Z[u[n]] = \sum_{n=-\infty}^{-1} u[n]\, z^{-n} + \sum_{n=0}^{\infty} u[n]\, z^{-n}$$

$$= 0 + \sum_{n=0}^{\infty} 1 \cdot z^{-n}$$

$$= z^{0} + z^{-1} + z^{-2} + \cdots$$

$$= 1 + z^{-1} + z^{-2} + \cdots$$

$$\boxed{Z[u[n]] = \frac{1}{(1-z^{-1})}}$$

$\left\{ S_\infty = \frac{a}{1-r} \right\}$

(ii) Ramp Signal :-

$$x[n] = n\, u[n] = \begin{cases} 0 & , n < 0 \\ n & , n \geq 0 \end{cases}$$


$$Z[n\, u[n]] = \sum_{n=-\infty}^{\infty} n\, u[n]\, z^{-n}$$

$$= \sum_{n=-\infty}^{-1} n\, u[n]\, z^{-n} + \sum_{n=0}^{\infty} n\, u[n]\, z^{-n}$$

$$Z[n\,u(n)] = 0 + \sum_{n=0}^{\infty} n\, z^{-n}$$

$$X(z) = 0 + 1z^{-1} + 2z^{-2} + 3z^{-3} \cdots$$

$$X(z)\, z^{-1} = 0 + z^{-2} + 2z^{-3} + 3z^{-4} \cdots$$

Eq$^n$ (1) - (2)

$$X(z)(1-z^{-1}) = z^{-1} + z^{-2} + z^{-3} + z^{-4} \cdots$$

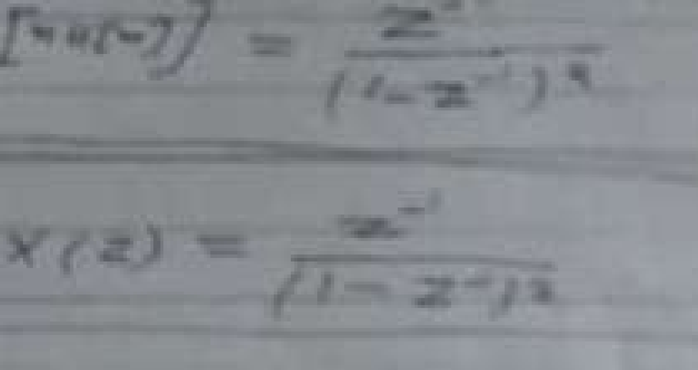
$$X(z) = \frac{z^{-1}}{(1-z^{-1})} \cdot \frac{1}{(1-z^{-1})}$$

$$\boxed{Z[n\,u(n)] = \frac{z^{-1}}{(1-z^{-1})^2}}$$

or $$\boxed{X(z) = \frac{z^{-1}}{(1-z^{-1})^2}}$$

(iii) Exponential Signal →

$$x[n] = a^n u(n) = \begin{cases} 0 & n<0 \\ a^n & n \geq 0 \end{cases}$$



$$z[a^n u[n]] = \sum_{n=-\infty}^{\infty} a^n u[n] \cdot z^{-n}$$

$\{ u[n] = 1$

$$= 0 + \sum_{n=0}^{\infty} a^n z^{-n}$$

$$= 1 + a z^{-1} + a^2 z^{-2} + a^3 z^{-3} \dots$$

$$z[a^n u[n]] = \frac{1}{1 - a z^{-1}}$$

$\cancel{\text{Ex}}$

* $x[n] = \cos an\, u[n]$

$$= \left( \frac{e^{ian} + e^{-ian}}{2} \right) u[n]$$

$$= \frac{1}{2} \left\{ (e^{ia})^n u[n] + (e^{-ia})^n u[n] \right\}$$

$$= \frac{1}{2} \left\{ (K_1)^n u[n] + (K_2)^n u[n] \right\}$$

$$= \frac{1}{2} \left\{ \frac{1}{1 - K_1 z^{-1}} + \frac{1}{1 - K_2 z^{-1}} \right\}$$

$$= \frac{1}{2} \left\{ \frac{1}{1 - e^{ia} z^{-1}} + \frac{1}{1 - e^{-ia} z^{-1}} \right\} \cancel{\tfrac{1}{2}}$$

* $x[n] = \sin an\, u[n]$

$$= \left( \frac{e^{ian} - e^{-ian}}{2i} \right) u[n]$$

$$= \frac{1}{2i} \left[ e^{ian} u[n] - e^{-ian} u[n] \right]$$

$$= \frac{1}{2i} \left[ (K_1)^n u[n] - (K_2)^n u[n] \right]$$

$$= \frac{1}{2j}\left[\frac{1}{1-K_1 z^{-1}} + \frac{1}{1-K_2 z^{-1}}\right]$$

$$= \frac{1}{2j}\left[\frac{1}{1-e^{j\omega_0} z^{-1}} + \frac{1}{1-e^{-j\omega_0} z^{-1}}\right]$$

Question → $a^{|n|}$ Find Z transform

Sol$^n$ → $Z[a^{|n|}] = \sum_{n=-\infty}^{\infty} a^{|n|} z^{-n}$

$$= \sum_{n=-\infty}^{-1} a^{|n|} z^{-1} + \sum_{n=0}^{\infty} a^{|n|} z^{-n}$$

$$= \sum_{n=-\infty}^{-1} a^{-n} z^{-n} + \sum_{n=0}^{\infty} a^{n} z^{-n}$$

$$= (a\, z^{1} + a^2 z^2 + a^3 z^3 + \cdots) + \frac{1}{1-az^{-1}}$$

$$= \frac{az}{1-az} + \frac{1}{1-az^{-1}}$$

$$\frac{1}{1-az^{-1}} - \frac{1}{1-a^{-1}z^{-1}}$$