SIR CHHOTU RAM INSTITUTE OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF MECHANICAL ENGINEERING

ENGINEERING MECHANICS (BT-419)

TOPIC: ANALYSIS OF PERFECT FRAMES

11.2. Types of Frames

Though there are many types of frames, yet from the analysis point of view, the frames may be classified into the following two groups:

1. Perfect frame. 2. Imperfect frame.

11.3. Perfect Frame

A perfect frame is that, which is made up of members just sufficient to keep it in equilibrium, when loaded, without any change in its shape.

The simplest perfect frame is a triangle, which contains three members and three joints as shown in Fig. 11.1. It will be intersting to know that if such a structure is loaded, its shape will not be distorted. Thus, for three jointed frame, there should be three members to prevent any distortion. It will be further noticed that if we want to increase a joint, to a triangular frame, we require two members as shown by dotted lines in Fig. 11.1. Thus we see that for every additional joint, to a triangular frame, two members are required.

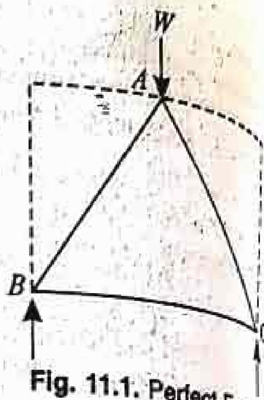


Fig. 11.1. Perfect Frame.

The no. of members, in a perfect frame, may also be expressed by the relation :

$$n = (2j - 3)$$

n = No. of members, and

j = No. of joints.

11.4. Imperfect Frame

An imperfect frame is that which does not satisfy the equation :

$$n = (2j - 3)$$

Or in other words, it is a frame in which the no. of members are *more* or *less* than $(2j - 3)$. The imperfect frames may be further classified into the following two types :

1. Deficient frame. 2. Redundant frame.

11.5. Deficient Frame

A deficient frame is an imperfect frame, in which the no. of members are less than $(2j - 3)$.

11.6. Redundant Frame

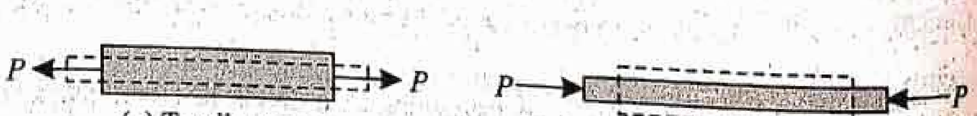
A redundant frame is an imperfect frame, in which the no. of members are more than $(2j - 3)$. In this chapter, we shall discuss only perfect frames.

11.7. Stress

When a body is acted upon by a force, the internal force which is transmitted through the body is known as stress. Following two types of stress are important from the subject point of view :

1. Tensile stress. 2. Compressive stress.

11.8. Tensile Stress



(*a*) Tensile stress (*b*) Compressive stress

Fig. 11.2.

Sometimes, a body is pulled outwards by two equal and opposite forces and the body tends to extend, as shown in Fig 11.2. (*a*). The stress induced is called tensile stress and corresponding force is called tensile force.

11.9. Compressive Stress

Sometimes, a body is pushed inwards by two equal and opposite forces and the body tends to shorten its length as shown in Fig. 11.2 (*b*). The stress induced is called compressive stress and the corresponding force is called compressive force.

11.10. Assumptions for Forces in the Members of a Perfect Frame

Following assumptions are made, while finding out the forces in the members of a perfect frame:

1. All the members are pin-jointed. **(UPTU 2009-2010)**
2. The frame is loaded only at the joints.
3. The frame is a perfect one.
4. The weight of the members, unless stated otherwise, is regarded as negligible in comparison with the other external forces or loads acting on the truss.

The forces in the members of a perfect frame may be found out either by analytical method or graphical method. But in this chapter, we shall discuss the analytical method only.

11.11. Analytical Methods for the Forces

The following two analytical methods for finding out the forces, in the members of a perfect frame, are important from the subject point of view :

1. Method of joints. 2. Method of sections.

11.12. Method of Joints

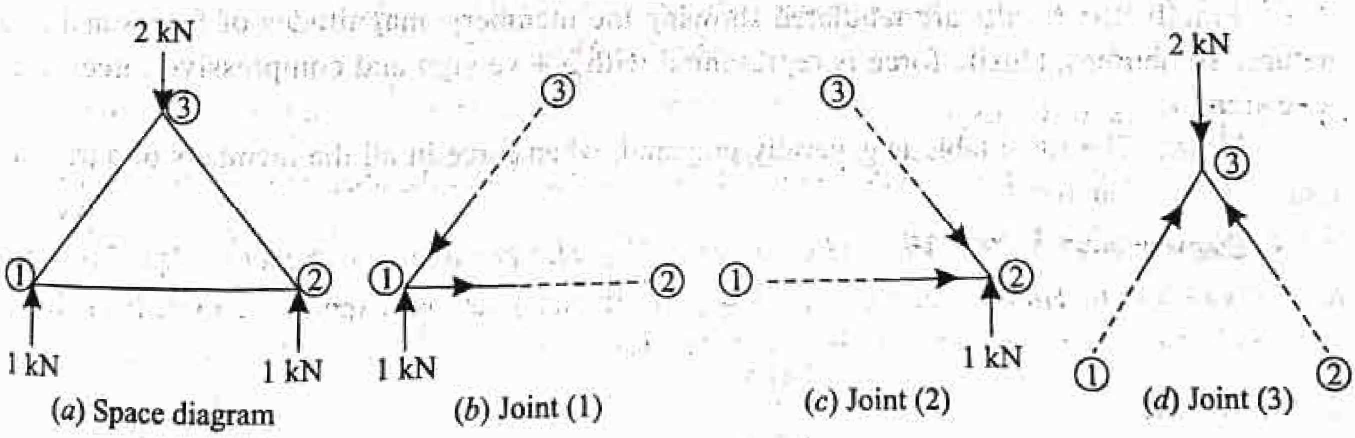


Fig. 11.3.

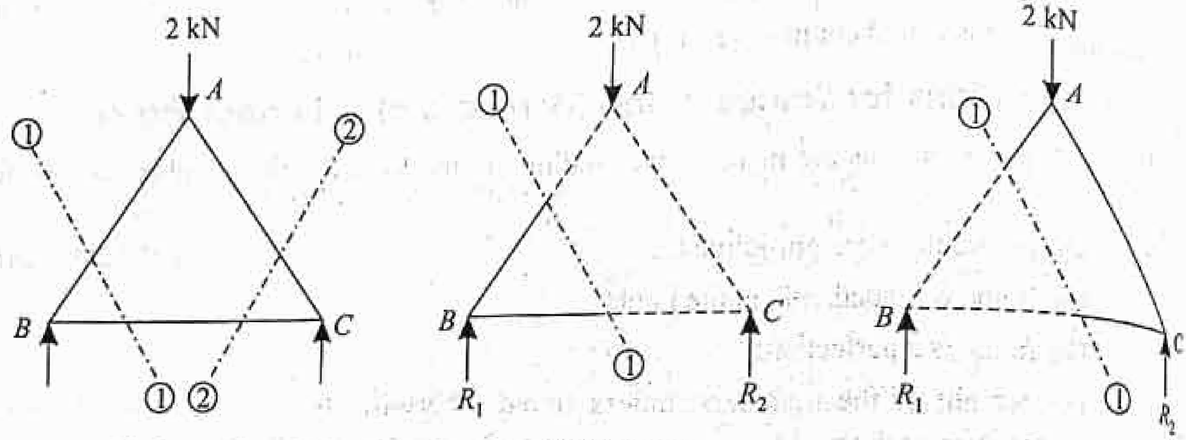
In this method, each and every joint is treated as a free body in equilibrium as shown in Fig. 11.3 (*a*), (*b*), (*c*) and (*d*). The unknown forces are then determined by equilibrium equations viz., $\Sigma V = 0$ and $\Sigma H = 0$. *i.e.*, Sum of all the vertical forces and horizontal forces is equated to zero.

Notes: 1. The members of the frame may be named either by Bow's methods or by the joints at their ends.

2. While selecting the joint, for calculation work, care should be taken that at any instant, the joint should not contain more than two members, in which the forces are unknown.

11.13. Method of Sections (or Method of Moments)

This method is particularly convenient, when the forces in a few members of a frame are required to be found out. In this method, a section line is passed through the member or members, in which the forces are required to be found out as shown in Fig. 11.4 (*a*). A part of the structure, on any one side of the section line, is then treated as a free body in equilibrium under the action of external forces as shown in Fig. 11.4 (*b*) and (*c*).



(*a*) Space diagram (*b*) Left part (*c*) Right part

Fig. 11.4.

The unknown forces are then found out by the application of equilibrium or the principles of statics *i.e.*, $\Sigma M = 0$.

Notes: 1. To start with, we have shown section line 1-1 cutting the members *AB* and *BC*. Now in order to find out the forces in the member *AC*, section line 2-2 may be drawn.

2. While drawing a section line, care should always be taken not to cut more than three members, in which the forces are unknown.

11.14. Force Table

Finally, the results are tabulated showing the members, magnitudes of forces and their nature. Sometimes, tensile force is represented with a + ve sign and compressive force with a – ve sign.

Note: The force table is generally prepared, when force in all the members of a truss are required to be found out.

Example 11.1. *The truss ABC shown in Fig. 11.5 has a span of 5 metres. It is carrying a load of 10 kN at its apex.*

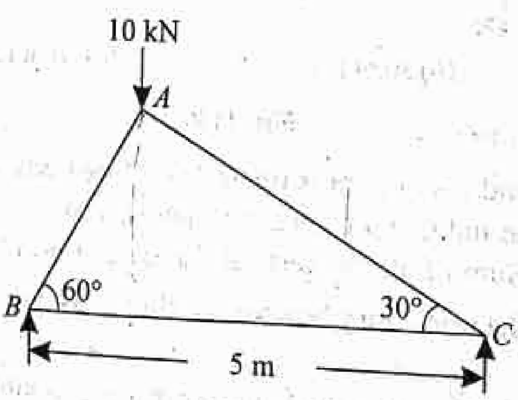


Fig. 11.5.

Find the forces in the members AB, AC and BC.

Solution. From the geometry of the truss, we find that the load of 10 kN is acting at a distance 1.25 m from the left hand support *i.e.*, *B* and 3.75 m from *C*. Taking moments about *B* and equating the same,

$$R_C \times 5 = 10 \times 1.25 = 12.5$$

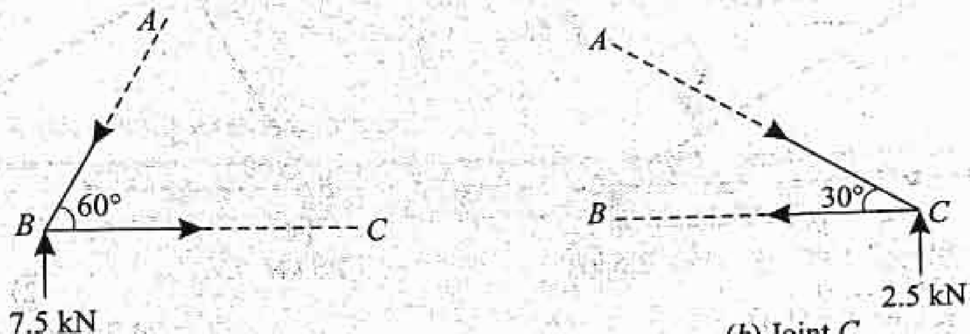
$$\therefore \quad R_C = \frac{12.5}{5} = 2.5 \text{ kN}$$

$$\text{and} \quad R_B = 10 - 2.5 = 7.5 \text{ kN}$$

The example may be solved by the method of joints or by the method of sections. But we shall solve it by both the methods.

Methods of Joints

First of all consider joint *B*. Let the *directions of the forces P_{AB} and P_{BC} (or P_{BA} and P_{CB}) be assumed as shown in Fig 11.6 (*a*).



(*a*) Joint *B* (*b*) Joint *C*

Fig. 11.6.

Resolving the forces vertically and equating the same,

$$P_{AB} \sin 60° = 7.5$$

$$\text{or} \quad P_{AB} = \frac{7.5}{\sin 60°} = \frac{7.5}{0.866} = 8.66 \text{ kN (Compression)}$$

and now resolving the forces *horizontally* and equating the same,

$$P_{BC} = P_{AB} \cos 60° = 8.66 \times 0.5 = 4.33 \text{ kN (Tension)}$$

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* The idea, of assuming the direction of the force P_{AB} to be downwards, is that the vertical component of the force P_{BC} is zero. Therefore in order to bring the joint *B* in equilibrium, the direction of the force P_{AB} must be downwards, or in other words, the direction of the force P_{AB} should be *opposite* to that of the reaction R_B. If, however the direction of the force P_{AB} is assumed to be upwards, then resolving the forces vertically and equating the same,

$$P_{AB} \sin 60° = -7.5 \text{ (Minus sign due to same direction of } R_B \text{ and } P_{AB}.)$$

$$\therefore \quad P_{AB} = \frac{-7.5}{\sin 60°} = \frac{-7.5}{0.866} = -8.66 \text{ kN}$$

Minus sign means that the direction assumed is wrong. It should have been downwards instead of upwards. Similarly, the idea of assuming the direction of the force P_{BC} to be towards right is that the horizontal component of the reaction R_B is zero. Therefore in order to bring the joint *B* in equilibrium, the direction of the force P_{AB} must be towards right (because the direction of the horizontal component of the force P_{AB} is towards left).

Now consider the joint C. Let the *directions of the forces P_{AC} and P_{BC} (or P_{CA} and P_{CB}) be assumed as shown in Fig. 11.6 (*b*). Resolving the forces vertically and equating the same,

$$P_{AC} \sin 30° = 2.5$$

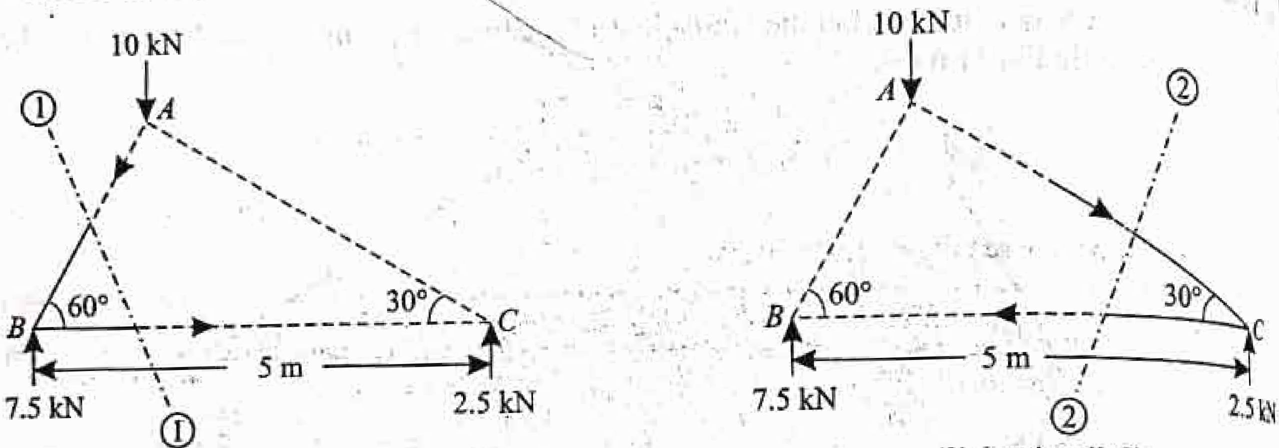
$$\therefore \quad P_{AC} = \frac{2.5}{\sin 30°} = \frac{2.5}{0.5} = 5.0 \text{ kN (Compression)}$$

and now resolving the forces horizontally and equating the same,

$$P_{BC} = P_{AC} \cos 30° = 5.0 \times 0.866 = 4.33 \text{ kN (Tension).}$$

...(As already obtained)

Method of Sections



(*a*) Section (1-1) (*b*) Section (2-2)

Fig. 11.7.

First of all, pass section (1-1) cutting the truss into two parts (one part shown by firm lines and the other by dotted lines) through the members AB and BC of the truss as shown in Fig 11.7 (*a*). Now consider equilibrium of the left part of the truss (because it is smaller than the right part). Let the directions of the forces P_{AB} and P_{AC} be assumed as shown in Fig 11.7 (*a*).

Taking** moments of the forces acting in the left part of the truss only about the joint C and equating the same,

$$P_{AB} \times 5 \sin 60° = 7.5 \times 5$$

$$\therefore \quad P_{AB} = \frac{7.5 \times 5}{5 \sin 60°} = \frac{7.5}{0.866} = 8.66 \text{ kN (Compression)}$$

and now taking moments of the forces acting in the left part of the truss only about the joint A and equating the same,

$$P_{BC} \times 1.25 \tan 60° = 7.5 \times 1.25$$

$$\therefore \quad P_{BC} = \frac{7.5 \times 1.25}{1.25 \tan 60°} = \frac{7.5}{1.732} = 4.33 \text{ kN (Tension)}$$

* For details, please refer to the foot note on last page.

** The moment of the force P_{AB} about the joint C may be obtained in any one of the following two ways:

1. The vertical distance between the member AB and the joint C (*i.e.*, AC in this case) is equal to $5 \sin 60°$ m. Therefore moment about C is equal to $P_{AB} \times 5 \sin 60°$ kN-m.
2. Resolve the force P_{AB} vertically and horizontally at B. The moment of horizontal component about C will be zero. The moment of vertical component (which is equal to $P_{AB} \times \sin 60°$) is equal to $P_{AB} \times \sin 60° \times 5 = P_{AB} \times 5 \sin 60°$ kN-m.

Now pass section (2-2) cutting the truss into two parts through the members AC and BC. Now consider the equilibrium of the right part of the truss (because it is smaller than the left part). Let the †direction of the forces P_{AC} and P_{BC} be assumed as shown in Fig 11.7 (*b*).

Taking moments of the force acting in the right part of the truss only about the joint B and equating the same,

$$P_{AC} \times 5 \sin 30° = 2.5 \times 5$$

$$\therefore \quad P_{AC} = \frac{2.5}{\sin 30°} = \frac{2.5}{0.5} = 5 \text{ kN} \quad \text{(Compression)}$$

and now taking moments of the forces acting in the right part of the truss only about the joint A and equating the same,

$$P_{BC} \times 3.75 \tan 30° = 2.5 \times 3.75$$

$$\therefore \quad P_{BC} = \frac{2.5 \times 3.75}{3.75 \tan 30°} = \frac{2.5}{0.577} = 4.33 \text{ kN} \quad \text{(Tension)}$$

...(As already obtained)

Now tabulate the results as given below :

S.No.	*Member*	*Magnitude of force in kN*	*Nature of force*
1	*AB*	8.66	Compression
2	*BC*	4.33	Tension
3	*AC*	5.0	Compression

Example 11.2. *Fig 11.8 shows a Warren girder consisting of seven members each of 3 m length freely supported at its end points.*

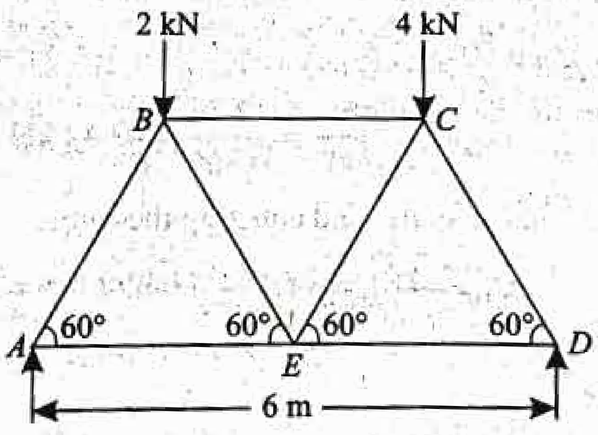


Fig. 11.8.

The girder is loaded at B and C as shown. Find the forces in all the members of the girder, indicating whether the force is compressive or tensile.

Solution. Taking moments about A and equating the same,

$$R_D \times 6 = (2 \times 1.5) + (4 \times 4.5) = 21$$

$$\therefore \quad R_D = \frac{21}{6} = 3.5 \text{ kN}$$

and

$$R_A = (2 + 4) - 3.5 = 2.5 \text{ kN}$$

Now tabulate the results as given below :

S.No.	Member	Magnitude of force in kN	Nature of force
1	AB	2.887	Compression
2	AE	1.444	Tension
3	CD	4.042	Compression
4	DE	2.021	Tension
5	BE	0.577	Tension
6	BC	1.732	Compression
7	CE	0.577	Compression

Example 11.3. *A plane is loaded and supported as shown in Fig 11.13.*

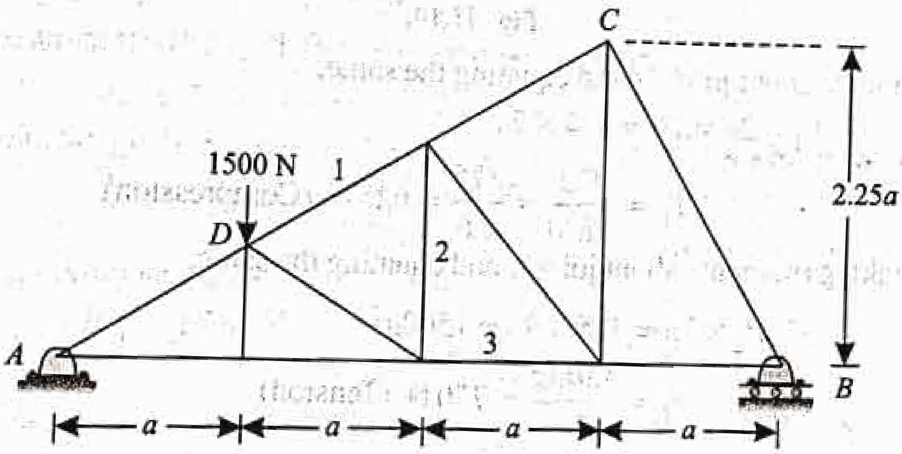


Fig. 11.13.

Determine the nature and magnitude of the forces in the members 1,2 and 3.

Solution. Taking moments about A and equating the same,

$$V_B \times 4a = 1500 \times a$$

$$\therefore \qquad V_B = \frac{1500}{4} = 375 \text{ N}$$

and

$$V_A = 1500 - 375 = 1125 \text{ N}$$

From the geometry of the figure, we find that

$$\tan\theta = \frac{2.25\,a}{3\,a} = 0.75$$

and

$$\sin\theta = \frac{3}{5} = 0.6 \text{ and } \cos\theta = \frac{4}{5} = 0.8$$

The example may be solved by any method. But we shall solve it by the method of sections, as one section line can cut the members 1, 2 and 3 in which the forces are required to be found out. Now let us pass section (1-1) cutting the truss into two parts as shown in Fig 11.14.

Now consider the equilibrium of the right part of the truss. Let the directions of P_1, P_2 and P_3 be assumed as shown in Fig. 11.14.

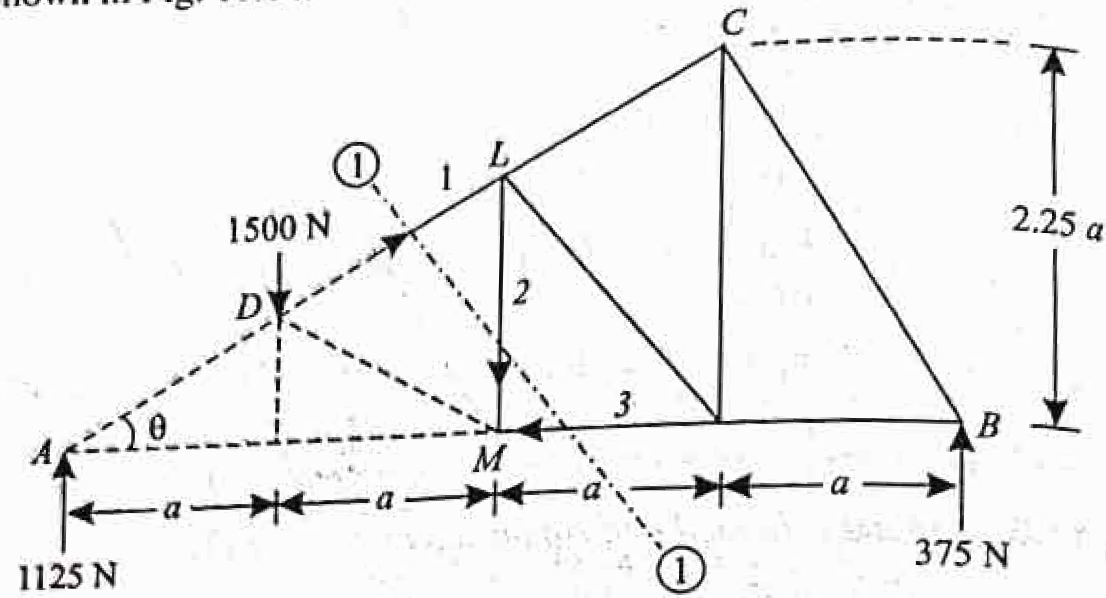


Fig. 11.14.

Taking moments about joint M and equating the same,

$$P_1 \times 2a \sin\theta = 375 \times 2a$$

$$\therefore \qquad P_1 = \frac{375}{\sin\theta} = \frac{375}{0.6} = 625 \text{ N (Compression)}$$

Similarly, taking moments about joint A and equating the same,

$$P_2 \times 2a = 375 \times 4a = 1500a$$

$$\therefore \qquad P_2 = \frac{1500a}{2a} = 750 \text{ N (Tension)}$$

and now taking moments about the joint L, and equating the same,

$$P_3 \times \frac{3a}{2} = 375 \times 2a = 750a$$

$$\therefore \qquad P_3 = \frac{750}{1.5} = 500 \text{ N (Tension)}$$

5. The roof truss shown in Fig. 11.23 is supported at A and B and carries vertical loads at each of the upper chord points.

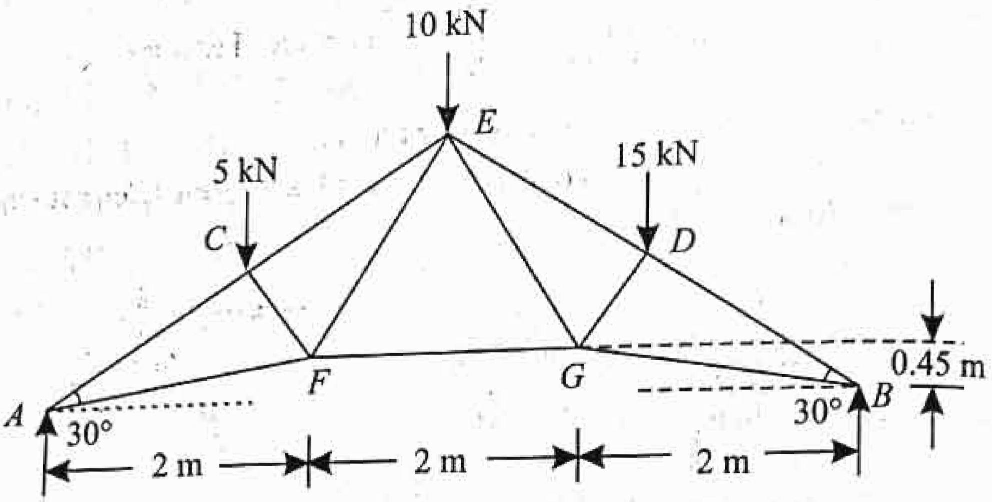


Fig. 11.23.

Using the method of sections, determine the forces in the members CE and FG of truss, stating whether they are in tension or compression.

[**Ans.** 38.5 kN (Compression); 24.2 kN (Tension)]

11.15. Cantilever Trusses

A truss, which is connected to a wall or a column at one end, and free at the other is known as a cantilever truss. In the previous examples, the determination of support reactions was absolutely essential to start the work. But in the case of cantilever trusses, determination of support reaction is not essential, as we can start the calculation work from the free end of the cantilever.

Example 11.6. *A cantilever truss of 3 m span is loaded as shown in Fig 11.24.*

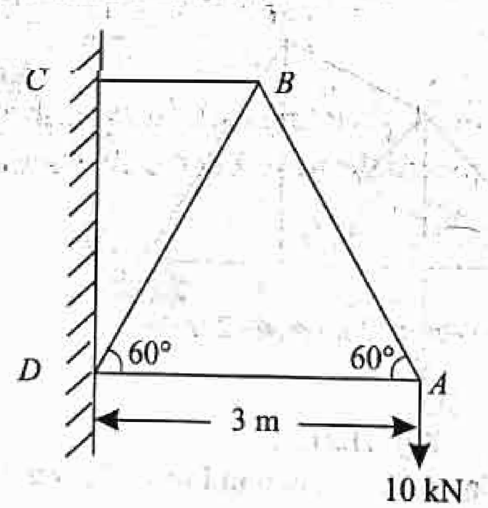


Fig. 11.24.

Find the forces in the various members of the framed truss, and tabulate the results.

Solution. The example may be solved either by the method of joints or method of sections. But we shall solve it by both the methods one by one.

Method of joints

First of all, consider the joint A, Let the directions of the forces P_{AB} and P_{AD} be assumed as shown Fig 11.25 (*a*).

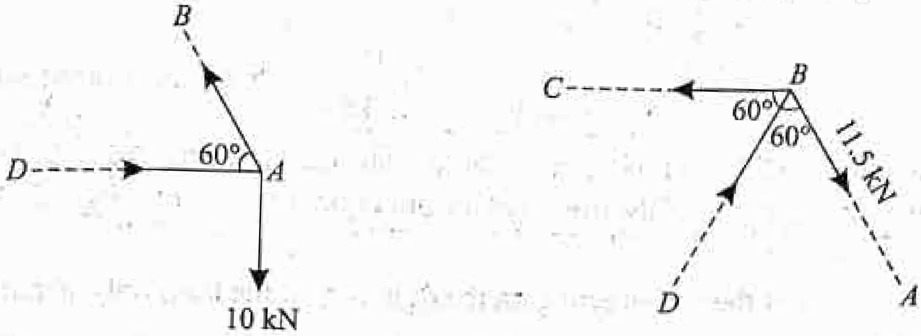
Resolving the forces vertically and equating the same,

$$P_{AB} \sin 60° = 10$$

$$\therefore \quad P_{AB} = \frac{10}{\sin 60°} = \frac{10}{0.866} = 11.5 \text{ kN (Tension)}$$

and now resolving the forces horizontally and equating the same,

$$P_{AD} = P_{AB} \cos 60° = 11.5 \times 0.5 = 5.75 \text{ kN (Compression)}$$



(*a*) Joint *A* (*b*) Joint *B*

Fig. 11.25.

Now consider the joint *B*. Let the directions of P_{BD} and P_{BC} be assumed as shown in Fig 11.25 (*b*). We have already found out that the force in member *AB* is 11.5 kN (Tension) as shown in the figure 11.25 (*b*). Resolving the forces vertically and equating the same,

$$P_{BD} \sin 60° = P_{AB} \sin 60° = 11.5 \sin 60°$$

$$\therefore \quad P_{BD} = P_{AB} = 11.5 \text{ kN (Compression)}$$

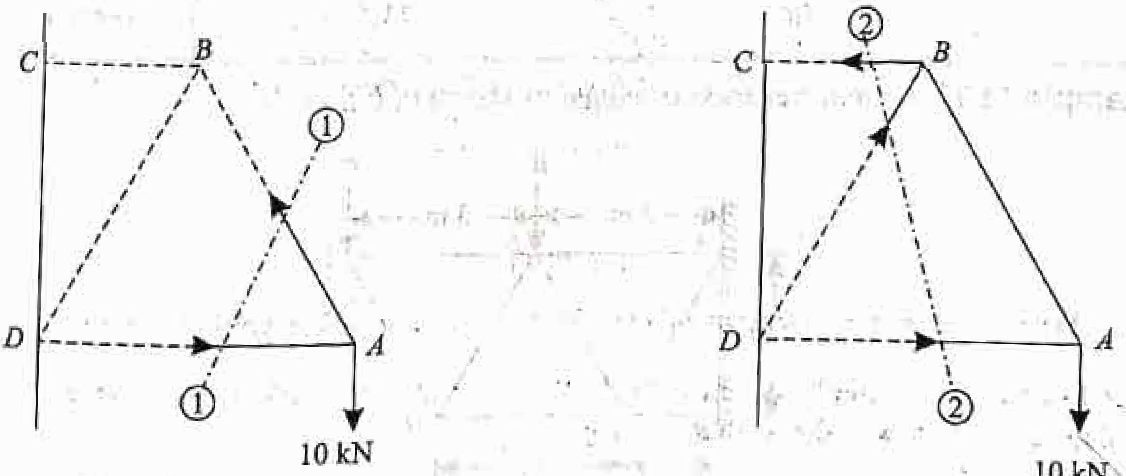
and now resolving the forces horizontally and equating the same,

$$P_{BC} = P_{AB} \cos 60° + P'_{BD} \cos 60°$$

$$= (11.5 \times 0.5) + (11.5 \times 0.5) = 11.5 \text{ kN (Tension)}$$

Method of sections

First of all, pass section (1-1) cutting the truss through the members *AB* and *AD*. Now consider the equilibrium of the right part of the truss. Let the directions of the forces P_{AB} and P_{AD} be assumed as shown in Fig 11.26 (*a*).



(*a*) Section (1-1) (*b*) Section (2-2)

Fig. 11.26.

Taking moments of the forces acting on right part of the truss only, about the joint D and equating the same,

$$P_{AB} \times 3 \sin 60° = 10 \times 3$$

$$\therefore \quad P_{AB} = \frac{10}{\sin 60°} = \frac{10}{0.866} = 11.5 \text{ kN (Tension)}$$

and now taking moments of the forces in the right part of the truss only about the joint B and equating the same,

$$P_{AD} \times 3 \sin 60° = 10 \times 1.5 = 15$$

$$\therefore \quad P_{AD} = \frac{15}{3 \sin 60°} = \frac{15}{3 \times 0.866} = 5.75 \text{ kN (Compression)}$$

Now pass section (2-2) cutting the truss through the members BC, BD and AD. Now consider the equilibrium of the right part of the truss. Let the directions of the forces P_{BC} and P_{BD} be assumed as shown in Fig. 11.26 (b)

Taking moments of the forces acting on the right part of the truss only, about the joint D and equating the same,

$$P_{BC} \times 3 \sin 60° = 10 \times 3$$

$$\therefore \quad P_{BC} = \frac{10}{\sin 60°} = \frac{10}{0.866} = 11.5 \text{ kN (Tension)}$$

and now taking moments of the forces in the right part of the truss only, about the joint C and equating the same,

$$P_{BD} \times 1.5 \sin 60° = (10 \times 3) - P_{AD} \times 3 \sin 60° = 30 - (5.75 \times 3 \times 0.866) = 15$$

$$P_{BD} = \frac{15}{1.5 \sin 60°} = \frac{15}{1.5 \times 0.866} = 11.5 \text{ kN (Compression)}$$

Now tabulate the results as given below :

S.No.	Members	Magnitude of force in kN	Nature of force
1	AB	11.5	Tension
2	AD	5.75	Compression
3	BD	11.5	Compression
4	BC	11.5	Tension

2. A cantilever truss of 4 m span is carrying two point loads of 1.5 kN each as shown in Fig. 11.38. Find the stresses in the members *BC* and *BD* of the truss.

Ans. 2.52 kN (Tension) ; zero

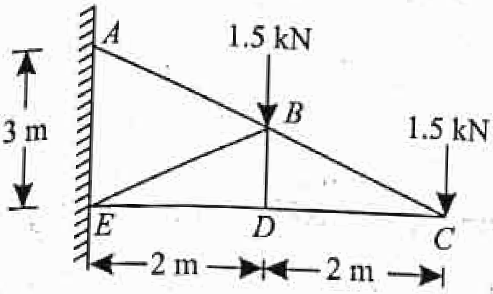


Fig. 11.38.

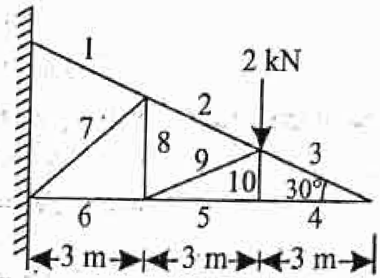


Fig. 11.39.

3. A cantilever truss carries two vertical load as shown in the Fig. 11.39. Find the magnitude and nature of strees in the members 2, 9, 5 and 10 of the truss.

Ans. $P_2 = 6.0$ kN (Tension)
$P_9 = 2.9$ kN (Compression)
$P_5 = 3.46$ kN (Compression)
$P_{10} = 0$

4. A cantilever truss is subjected to two point loads of 3 kN each at *B* and *C* as shown in Fig 11.40. Find by any method the forces in the members *AB*. *BE* and *ED* of the truss.

Ans. *AB* = 8.6 kN (Tension)
BE = 2.0 kN (Tension)
ED = 2.0 kN (Compression)

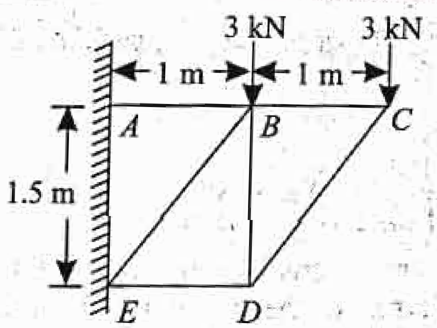


Fig. 11.40.

11.16. Structures with One End Hinged (or Pin-jointed) and The Other Freely Supported on Rollers and Carrying Horizontal Loads

Sometimes, a structure is hinged or pin-jointed at one end, and freely supported on rollers at the other end. If such a truss carries vertical loads only, it does not present any special features. Such a structure may be solved just as a simply supported structure.

But, if such a structure carries horizontal loads (with or without vertical loads) the support reaction at the roller supported end will be normal to the support; where the support reaction at the hinged end will consist of :

1. Vertical reaction, which may be found out, by substracting the vertical support reaction at the roller supported end from the total vertical load.
2. Horizontal reaction, which may be found out, by algebraically adding all the horizontal loads.

After finding out the reactions, the forces in members of the frame may be found out as usual.

Example. 11.11. *Figure 11.41 shows a framed of 4 m span and 1.5 m height subjected to two point loads at B and D.* **(UPTU 2009-2010)**

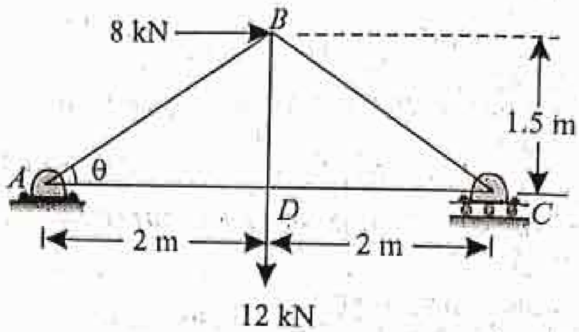


Fig. 11.41.

Find graphically, or otherwise, the forces in all the members of the structure.

Solution. Since the structure is supported on rollers at the right hand support (*C*), therefore the reaction at this support will be vertical (because of horizontal support). The reaction at the left hand support (*A*) will be the resultant of vertical and horizontal forces and inclined with the vertical.

Taking moments about *A* and equating the same,

$$V_C \times 4 = (8 \times 1.5) + (12 \times 2) = 36$$

$$V_C = \frac{36}{4} = 9 \text{ kN} (\uparrow)$$

$$V_A = 12 - 9 = 3 \text{ kN} (\uparrow) \quad \text{and} \quad H_A = 8 \text{ kN} (\leftarrow)$$

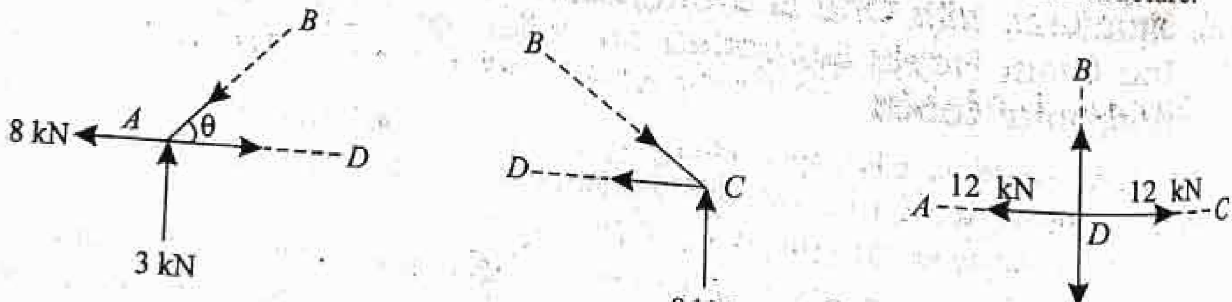
From the geometry of the figure, we find that

$$\tan \theta = \frac{1.5}{2} = 0.75 \quad \text{or} \quad \theta = 36.9°$$

Similarly

$$\sin \theta = \sin 36.9° = 0.6 \quad \text{and} \quad \cos \theta = \cos 36.9° = 0.8$$

The example may be solved either by the method of joints or method of sections. But we shall solve it by the method of joints as we have to find forces in all the members of the structure.



(*a*) Joint *A* (*b*) Joint *C* (*c*) Joint *D*

Fig. 11.42.

First of all, consider joint *A*. Let directions of the forces P_{AB} and P_{AD} be assumed as shown in Fig. 11.42 (*a*). We have already found that a horizontal force of 8 kN is acting at *A* as shown in Fig. 11.42 (*a*).

Resolving the forces vertically and equating the same,

$$P_{AB} \sin 36.9° = 3$$

$$\therefore \quad P_{AB} = \frac{3}{\sin 36.9°} = \frac{3}{0.6} = 5.0 \text{ kN (Compression)}$$

and now resolving the forces horizontally and equating the same,

$$P_{AD} = 8 + P_{AB} \cos 36.9° = 8 + (5 \times 0.8) = 12.0 \text{ kN (Tension)}$$

Now consider the joint *C*. Let the directions of the forces P_{BC} and P_{CD} be assumed as shown in Fig. 11.42 (*b*).

Resolving the forces vertically and equating the same,

$$P_{BC} \sin 36.9° = 9$$

$$P_{BC} = \frac{9}{\sin 36.9°} = \frac{9}{0.6} = 15 \text{ kN (Compression)}$$

and now resolving the forces horizontally and equating the same,

$$P_{CD} = P_{BC} \cos 36.9° = 15 \times 0.8 = 12.0 \text{ kN (Tension)}$$

Now consider the joint *D*. A little consideration will show that the value of the force P_{BD} will be equal to the load 12 kN (Tension) as shown in Fig 11.42. (*c*). This will happen as the vertical components of the forces P_{AD} and P_{CD} will be zero.

Now tabulate the results as given below :

S.No.	*Member*	*Magnitude of force in kN*	*Nature of force*
1	*AB*	5.0	Compression
2	*AD*	12.0	Tension
3	*BC*	15.0	Compression
4	*CD*	12.0	Tension
5	*BD*	12.0	Tension

Example 11.12. *2 A truss of 8 metres span, is loaded as shown in Fig. 11.43.*

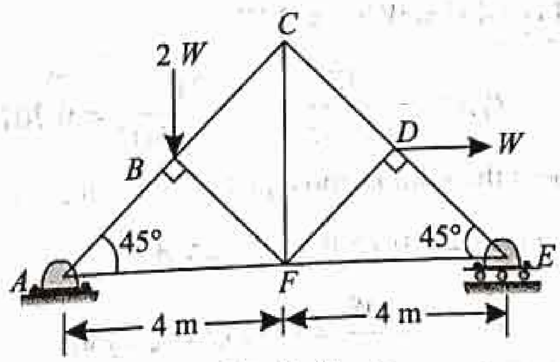


Fig. 11.43.

Find the forces in the members CD, FD and FE of the truss.

Now consider the joint H. We have already found out that $P_{HE} = 2.5$ kN (Tension). It will be interesting to know that the force P_{DH} will be zero, as there is no other member at joint H to balance the component of this forces (if any) at right angle to the member GHE.

11.17. Structures with One End Hinged (Or Pin-jointed) and the other Freely Supported on Rollers and Carrying Inclined Loads

We have already discussed in the last article that if a structure is hinged at one end, freely supported on rollers at the other, and carries horizontal loads (with or without vertical loads), the support reaction at the roller- supported end will be normal to the support. The same principle is used for structures carrying inclined loads also. In such a case, the support reaction at the hinged end will be the resultant of :

1. Vertical reaction, which may be found out by subtracting the vertical component of the support reaction at the roller supported end from the total vertical loads.
2. Horizontal reaction, which may be found out algebraically by adding all the horizontal loads.

Example 11.14. *Figure 11.47 represents a north-light roof truss with wind loads acting on it.*

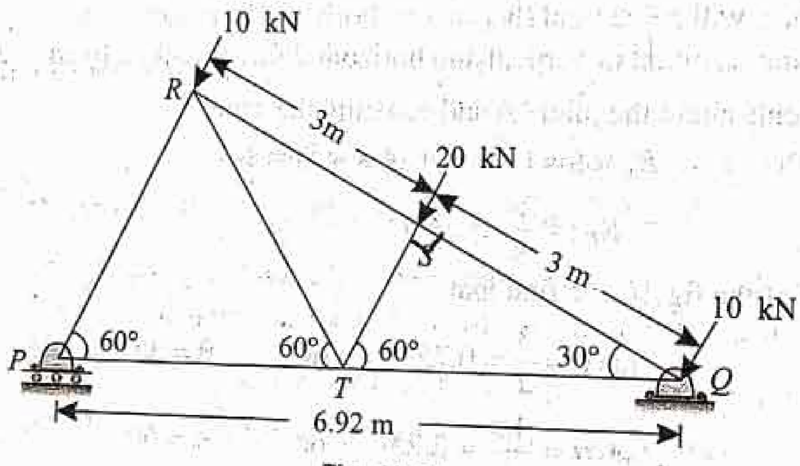


Fig. 11.47.

Find graphically, or otherwise, the forces in all the members of the truss Give your results in a tabulated form.

Solution. Since the truss is supported on rollers at P, therefore the reaction at this end will be vertical (because of horizontal support). Moreover, it is hinged at Q, therefore the reaction at this end will be the resultant of horizontal and vertical forces and inclined with the vertical.

Taking moments about Q and equating the same,

$$V_P \times 6.92 = (20 \times 3) + (10 \times 6) = 120$$

$$\therefore \quad V_P = \frac{120}{6.92} = 17.3\text{kN}$$

We know that total wind loads on the truss

$$= 10 + 20 + 10 = 40 \text{ kN}$$

$\therefore$ Horizontal component of wind load,

$$H_Q = 40 \cos 60° = 40 \times 0.5 = 20 \text{ kN} (\rightarrow)$$

and vertical component of the wind load

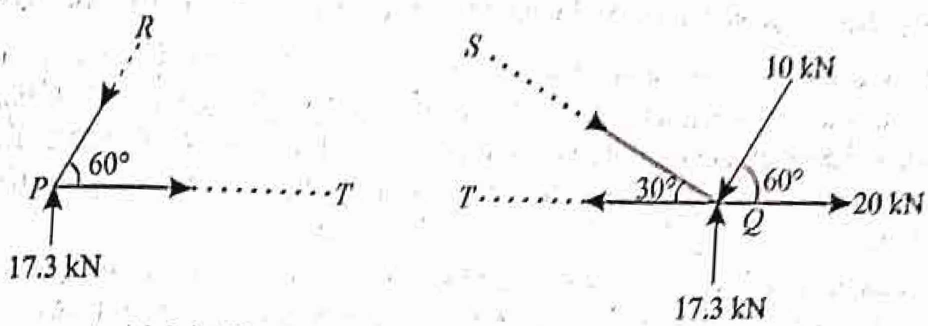
$$= 40 \sin 60° = 40 \times 0.866 = 34.6 \text{ kN} (\downarrow)$$

$\therefore$ Vertical reaction at Q,

$$V_Q = 34.6 - 17.3 = 17.3 \text{ kN} (\uparrow)$$

The example may be solved either by the method of joints or method of sections. But we shall solve it by the method of joints, as we have to find out the forces in all the members of the truss.

First of all, consider the joint P. Let the directions of the forces P_{PR} and P_{PT} be assumed as shown in Fig 11.48(a). We know that a horizontal force of 20 kN is acting at Q as shown in Fig. 11.48 (b).



(a) Joint P (a) Joint Q

Fig. 11.48.

Resolving the forces vertically and equating the same,

$$P_{PR} \sin 60° = 17.3$$

$$\therefore \quad P_{PR} = \frac{17.3}{\sin 60°} = \frac{17.3}{0.866} = 20 \text{ kN (Compression)}$$

and now resolving the forces horizontally and equating the same,

$$P_{PT} = P_{PR} \cos 60° = 20 \times 0.5 = 10 \text{ kN (Tension)}$$

Now consider the joint Q. Let the directions of the forces P_{SQ} and P_{QT} be assumed as shown in Fig. 11.48 (b). We know that a horizontal force of 20 kN is acting at Q as shown in Fig 11.48 (b).

Resolving the forces vertically and equating the same,

$$P_{SQ} \sin 30° = 17.3 - 10 \cos 30° = 17.3 - (10 \times 0.866) = 8.64$$

$$\therefore \quad P_{SQ} = \frac{8.64}{\sin 30°} = \frac{8.64}{0.5} = 17.3 \text{ kN (Compression)}$$

and now resolving the forces horizontally and equating the same,

$$P_{QT} = P_{SQ} \cos 30° + 20 - 10 \sin 30°$$

$$= (17.3 \times 0.866) + 20 - (10 \times 0.5) = 30 \text{ kN (Tension)}$$

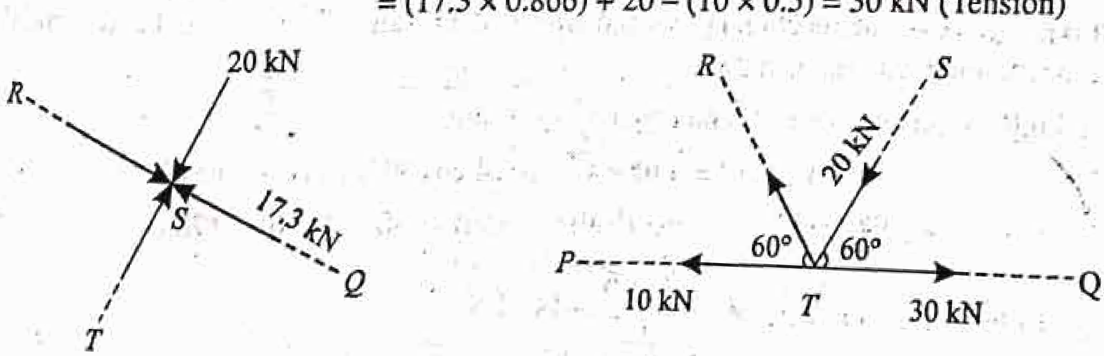


Fig. 11.49.

Now consider the joint S. We have already found out that P_{SQ} = 17.3 kN (Compression). A ittle consideration will show that the value of the force P_{TS} will be equal to the force 20 kN Compression). Similarly, the value of the force P_{RS} will be equal to P_{SQ} *i.e.*, 17.3 kN (Compression) s shown in Fig. 11.49 (a).

Now consider the joint T. Let the directions of the force P_{RT} be assumed as shown in Fig. 11.49 (b). We have already found out that P_{ST} = 20 kN (Compression).

Resolving the forces vertically and equating the same,

$$P_{RT} \sin 60° = P_{ST} \sin 60° = 20 \sin 60°$$

or $$P_{RT} = 20 \text{ kN (Tension)}$$

Now tabulate the results as given below:

S.No.	*Member*	*Magnitude of force in kN*	*Nature of force*
1	*PR*	20.0	Compression
2	*PT*	10.0	Tension
3	*SQ*	17.3	Compression
4	*QT*	30.0	Tension
5	*ST*	20.0	Compression
6	*RS*	17.3	Compression
7	*RT*	20.0	Tension

Example 11.15. *A truss of 12 m span is loaded as shown in Fig 11.50.*

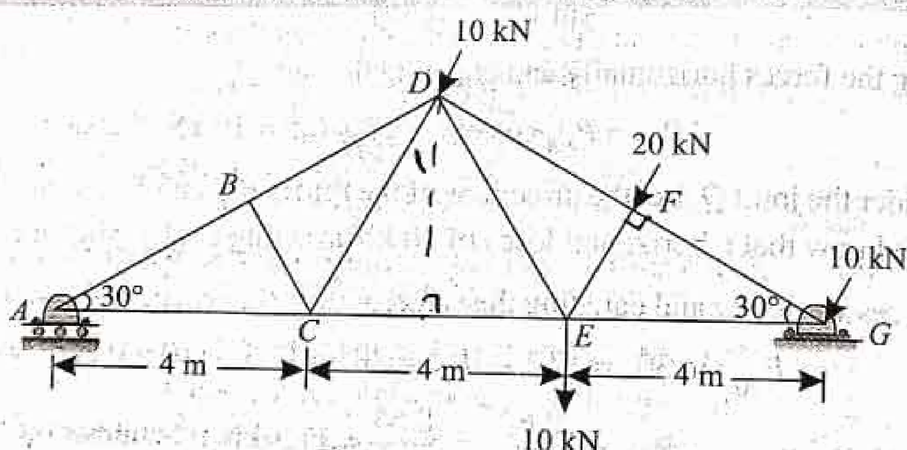


Fig. 11.50.

Determine the force in the members BD, CE and CD of the truss.

Solution. Since the truss is supported on rollers on the left end (A), therefore the reaction at this end will be vertical (because of horizontal support). Moreover, it is hinged at the right hand support (G), therefore the reaction at this end will be the resultant of horizontal and vertical forces and will be inclined with the vertical.

Taking * moments about G and equating the same,

$$V_A \times 12 = (10 \times 4) + (20 \times 4 \cos 30°) + (10 \times 8 \cos 30°)$$
$$= 40 + (80 \times 0.866) + (80 \times 0.866) = 178.6$$

$$\therefore \quad V_A = \frac{178.6}{12} = 14.9 \text{ kN}$$

The example may be solved either by the method of joints or method of sections. But we shall solve it by the method of sections, as one section line can cut the members BD, CE and CD in which forces are required to be found out.

* There is no need of finding out the vertical and horizontal reaction at G, as we are not considering this part of the truss.

Now let us pass section (1-1) cutting the truss into two parts as shown in Fig 11.51.

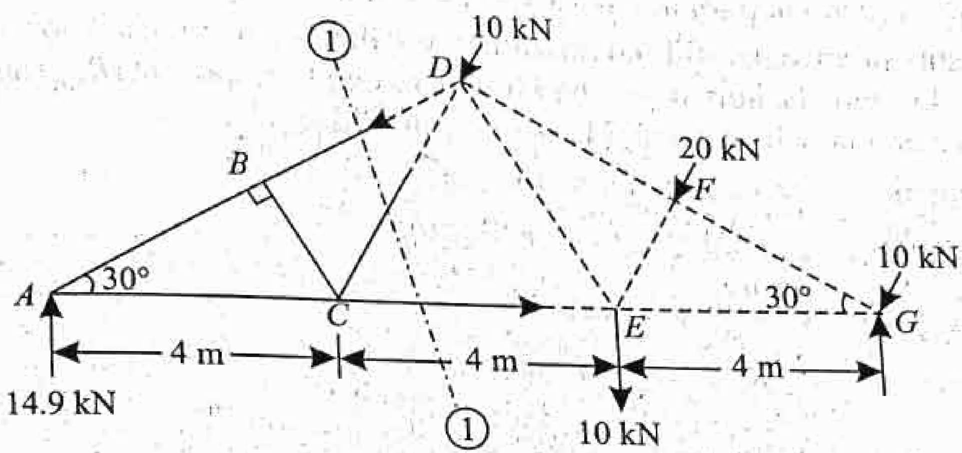


Fig. 11.51.

Now consider equilibrium of the left part of the truss. Let the directions of the forces P_{BD}, P_{CE} and P_{CD} be assumed as shown in Fig 11.51. Taking moments about the joint C and equating the same,

$$P_{BD} \times 2 = 14.9 \times 4 = 59.6$$

$$\therefore \qquad P_{BD} = \frac{59.6}{2} = 29.8 \text{ kN (Compression)}$$

Similarly taking moments about the joint D and equating the same,

$$P_{CE} \times 6 \tan 30° = 14.9 \times 6 = 89.4$$

$$\therefore \qquad P_{CE} = \frac{89.4}{6 \tan 30°} = \frac{89.4}{6 \times 0.5774} = 25.8 \text{ kN} \quad \text{(Tension)}$$

Now for finding out P_{CD}, we shall take moments about the A (where the other two members meet). Since there is no force in the lift of the truss (other than the reaction V_A, which will have zero moment about A), therefore the value of P_{CD} will be zero.

Note: The force P_{CD} may also be found out as discussed below :

At joint B, the force in member BC is zero, as there is no other member to balance the force (if any) in the member BC. Now at joint C, since the force in member BC is zero, therefore the force in member CD is also equal to zero.