2.5. Reconstruction Filter (Low Pass Filter)

The low pass filter is used to recover original signal from its samples. This is also called interpolation filter.

A low-pass filter is that type of filter which passes only low-frequencies upto a specified cut-off frequency and rejects all other frequenices above cut-off frequency. Figure 2.3 shows the frequency response of low-pass filter.

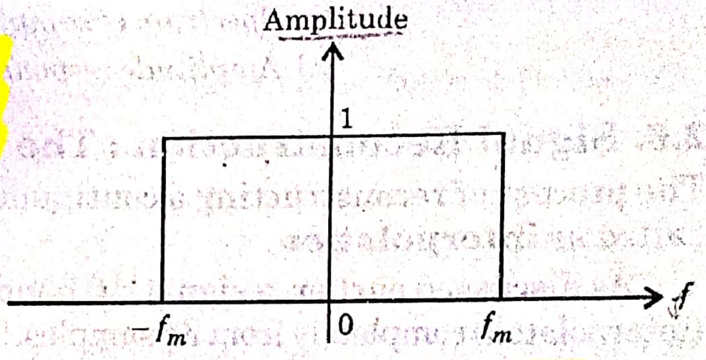

Fig. 2.3. Ideal low-pass filter

From figure 2.3 it may be observed that in case of low-pass filter, there is sharp-change in response at cut-off frequency, that is amplitude response becomes suddenly zero at cut-off frequency which is not possible practically. This means that an ideal low-pass filter is not physically realizable. In place of ideal-low pass filter, we use practical filter.

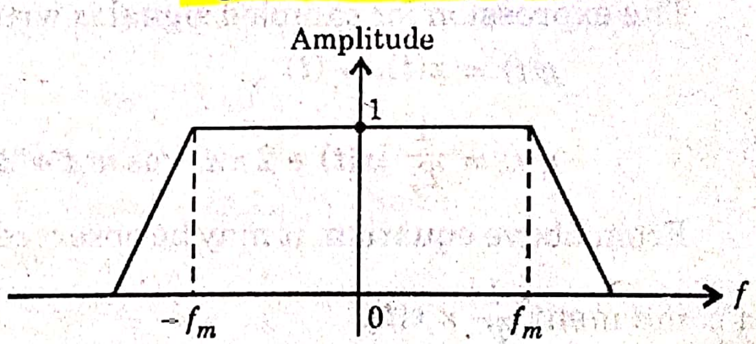

Fig. 2.4. Practical low-pass filter

Figure 2.4 shows the frequency response of practical low-pass filter. From figure 2.4, it may be observed that in case of practical filter, the amplitude response decreases slowly to become zero. This means that there is a transition band in case of practical filter. Figure 2.5 shows the use of practical low-pass filter in reconstruction of original signal from its sample.

2.7. Effect of Under Sampling : Aliasing

When a continuous-time band-limited signal is sampled at a rate lower than Nyquist rate $f_s < 2f_m$, then the successive cycles of the spectrum $G(w)$ of the sampled signal $g(t)$ overlap with each other as shown in figure 2.7.

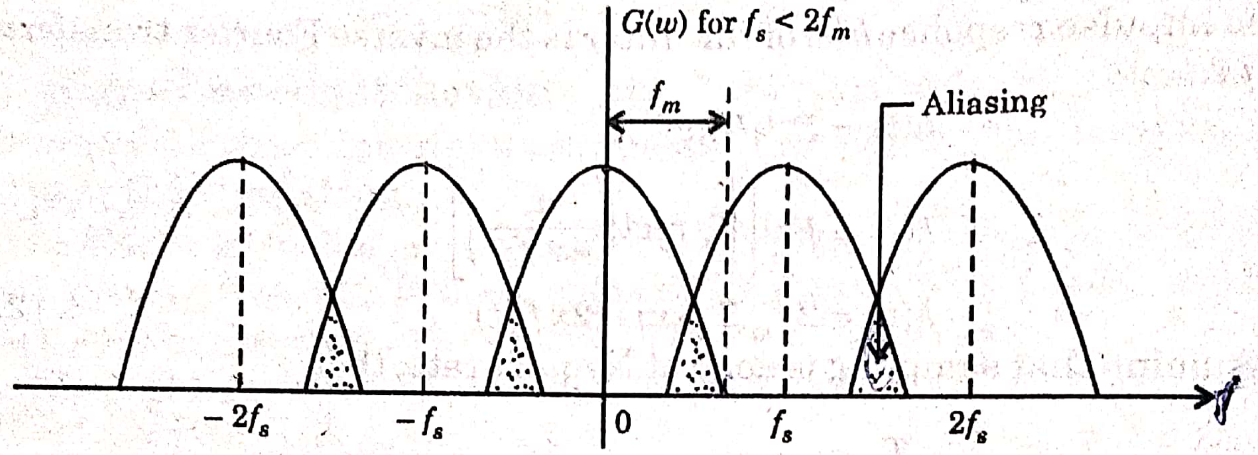

Fig. 2.7. *Spectruym of the sampled signal for the case $f_s < 2f_m$*

Hence the signal is under-slampled in this case ($f_s < 2f_m$) and some amount of aliasing is produced in this under-sampling process. In fact, aliasing is the phenomenon in which a high frequency component in the frequency-spectrum of the signal takes identity of a lower-frequency component in the spectrum of the sampled signal.

From figure 2.7 it is clear that because of the overlap due to aliasing phenomenon, it is not possible to recover original signal $x(t)$ from sampled signal $g(t)$ by low-pass filtering since the spectral components in the overlap regions add and hence the signal is distorted.

Since any information signal contains a large number of frequencies, so, to decide a sampling frequency is always a problem. Therefore, a signal is first passed through a low-pass filter. This low-pass filter blocks all the frequencies which are above f_m *Hz*. This process is known as band limiting of the original signal $x(t)$. This low-pass filter is called **prelias filter** because it is used to prevent aliasing effect. After band-limiting, it becomes easy to decide sampling frequency since the maximum frequency is fixed at f_m Hz.

In short, to avoid aliasing :

(*i*) Preaias filter must be used to limit band of frequencies of the signal to f_m Hz.

(*ii*) Sampling frequency 'f_s' must be selected such that

$$f_s > 2f_m$$

2.8. Sampling of BandPass Signals

In previous sections, we discussed sampling theorem for low-pass signals. However, when the given signal is a bandpass signal, then a different criteria must be used to sample the signal. Therefore, the sampling theorem for bandpass signals may be expressed as under :

The bandpass signal $x(t)$ whose maximum bandwidth is $2f_m$ can be completely represented into and recovered from its samples if it is sampled at the minimum rate of twice the bandwidth. Here, f_m is the maximum frequency component present in the signal.

Hence if the bandwidth is $2f_m$, then the minimum sampling rate for bandpass signal must be $4f_m$ samples per second. Figure 2.8 shows the spectrum of an arbitary bandpass signal.

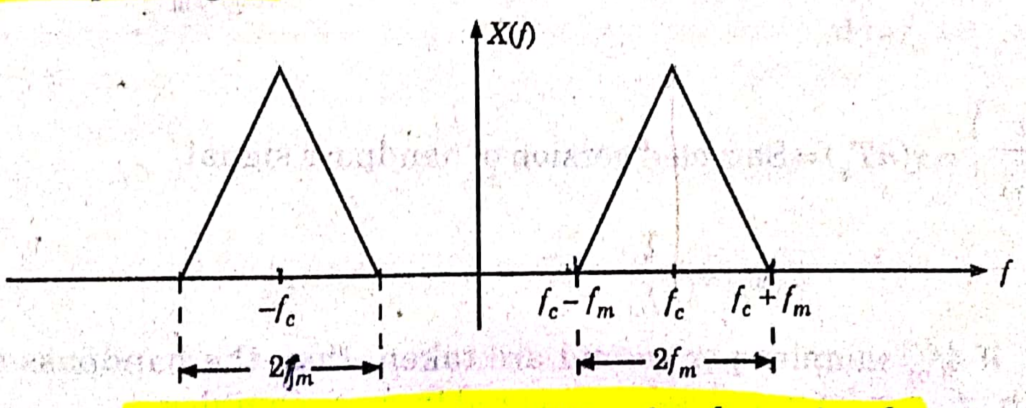

Fig. 2.8. *Spectrum of an orbitary bandpass signal.*

The spectrum in figure 2.8 is centred around frequency f_c. The bandwidth is $2f_m$. Thus, the frequencies present in the bandpass signal are from $f_c - f_m$ to $f_c + f_m$. This means that the highest frequency present in the bandpass signal is $f_c + f_m$. Generally the centre frequency $f_c > f_m$.

This bandpass signal is first represented in terms of its inphase and quadrature

Example 2.5. *Show that a bandlimited signal of finite energy which has no frequency components higher than f_m Hz is completely described by specifying values of the signals at instants of time separated by $1/2f_m$ seconds and also show that if the instantaneous values of the signal are separated by intervals larger than $1/2f_m$ seconds, they fail to describe the signal. A bandpass signal has spectral range that extends from 20 to 82 KHz.*

Find the acceptable range of sampling frequency f_s.

Sampling Frequency for Bandpass Signal :

Since the spectral range of the bandpass signal is 20 kHz to 82kHz

Therefore

$$\text{Bandwidth} = 2f_m$$
$$= 82 \text{ kHz} - 20 \text{ kHz} = 62 \text{ kHz}$$

Hence, Minimum Sampling rate = 2 × bandwidth

$$= 2 \times 62$$
$$= 124 \text{ kHz}$$

Generally, the range of minimum sampling frequencies is specified for band-pass signals.

It lies between $4f_m$ to $8f_m$ samples per second.

Therefore,

Range of minimum sampling frequencies

$$= (2 \times \text{bandwidth}) \text{ to } (4 \times \text{bandwidth})$$
$$= 2 \times 62 \text{ kHz to } 4 \times 62 \text{ kHz}$$
$$= 124 \text{ kHz to } 248 \text{ KHz}$$ **Ans.**