Class: B.Tech 2nd yr ECE
Sub: Signal & System
Faculty: Mandeep Singh
Date - 12/4/20
Topic: Eignfunction

Eign function: In Mathematics an eignfunction of a linear operator "D" defined on some function space ~~of that~~ is any non zero function "f" in that space that when acted upon by "D", is only multiplied by some scaling factor called an "eign value". As an equation, this condition can be written as:

$$D f = \lambda f$$

for some scalar eignvalue λ.

The solutions to this equation may also be subject to boundary condition that limit the allowable eignvalue and eignfunctions

An eignfunction is a type of eignvector. Because of the Boundary conditions the possible value of λ are generally limited for exemple to a discrete set λ_1, λ_2 --- or to a continuous set over some range. The set of all possible eignvalue of D is sometimes called its spectrum which may be discrete continuous or combination of both.

Derivative Example:

For example consider the derivative operater $\frac{d}{dt}$ with

eignvalue equation:

$$\frac{d}{dt} f(t) = \lambda f(t)$$

This differential equation can be solved by multiplying both sides by $\frac{dt}{f(t)}$ and

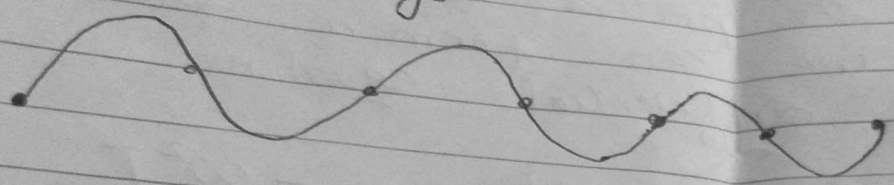
integrating its solutions the exponential function

$$f(t) = f_0 e^{\lambda t}.$$

is the eignfunction of the derivate operator where f_0 is a parameter that depends on the boundary condition

Applications of Eign function:

Vibrating. strings



The shape of a standing wave in a string fixed at its boundaries is an example of an eign function of a differential operator. The admissible eignvalues are governed by the length of the string and detеimine the frequcncy of oscillation

Let $h(x,t)$ denote the transverse displacement of a stressed elsatic chord, such as the vibrating strings of a string instrument, as a function of the position x along the string and of time t. Applying the laws of meachanics to infinitesimal portions of the string, the function h satisfies the partial defferential equation:-

$$\frac{\partial^2 h}{\partial t^2} = c^2 \frac{\partial^2 h}{\partial x^2}$$

which is called the (one-dimensional) wave equation. Here "c" is a constant speed that depends on the tension and mass of the string

This problem is amenable to the method of separation of variables. If we assume that $h(x,t)$ can be written as the product of the form $X(x)T(t)$, we can form a pair of ordinary differential equations.

$$\frac{d^2}{dx^2}X = -\frac{\omega^2}{c^2}X, \qquad \frac{d^2}{dt^2}T = -\omega^2 T$$

Each of these is an eignvalue equation with eignvalue $-\frac{\omega^2}{c^2}$ and $-\omega^2$ respectively.

For any value of ω and c the equations are satisfied by functions-

$$X(x) = \sin\left(\frac{\omega x}{c} + \phi\right); \quad T(t) = \sin$$

where phase angles ϕ and ψ are arbitrary real constants