Q.2) $F^{-1}\left[\frac{1}{1+j\omega} * \frac{2}{j\omega}\right]$

Sol^n We can use Multiplication property

$$F^{-1}\left[\frac{1}{1+j\omega}\right] = e^{-t}u(t) \quad -(1)$$

$$F^{-1}\left[\frac{2}{j\omega}\right] = \cancel{sig}\ sgn(t) \quad -(2)$$

$$F^{-1}\left[\frac{1}{1+j\omega}\right] * F^{-1}\left[\frac{2}{j\omega}\right] = F^{-1}\left[\frac{1}{1+j\omega} * \frac{2}{j\omega}\right]$$

$$F^{-1}\left[\frac{1}{1+j\omega} * \frac{2}{j\omega}\right] = e^{-t}u(t) \cdot sgn(t)$$

Q.3) Find Fourier transform of $\left[\frac{6}{t^2+9}\right]$

Sol^n we know that $e^{-a|t|} = \frac{2a}{a^2+\omega^2}$

So $\frac{6}{\omega^2+9} = F.T\left[e^{-3|t|}\right]$

If $e^{-3|t|} \xleftrightarrow{F.T} \frac{6}{\omega^2+9}$

$$\frac{6}{t^2+9} \xleftrightarrow{F.T} 2\pi\, e^{-3|-\omega|} \text{ (Duality)}$$

$$= 2\pi\, e^{-3|\omega|}$$

Q) $F.T\left(\frac{1}{jt}\right)$

$$F.T\,[\,\mathrm{sgn}(t)\,] = \frac{2}{jw}$$

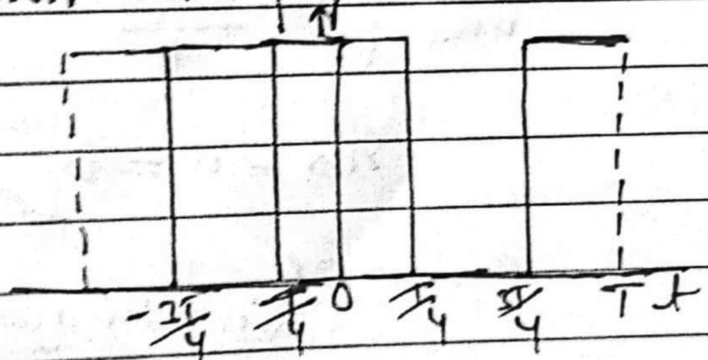
$$\mathrm{sgn}(t) \xleftrightarrow{F.T} \frac{2}{jw}$$

$$\frac{2}{jt} \xleftrightarrow{F} 2\pi\,\mathrm{sgn}(-w)$$

(Duality property)

FOURIER SERIES NUMERICAL

Q Find the fourier series expansion of the periodic rectangular wave from shown in fig.



$$f(t) = 4 \quad 0 < t < \frac{T}{4}$$
$$= 0 \quad \frac{T}{4} < t < \frac{3T}{4}$$
$$= 4 \quad \frac{3T}{4} < t < T$$

$$a_n = \frac{2}{T}\left[\int_0^{T/4} 4\cdot\cos n\omega t\,dt + \int_{T/4}^{3T/4} 0\cdot\cos n\omega t\,dt + \int_{3T/4}^{T} 4\cdot\cos n\omega t\,dt\right]$$

Date / /
Page No.

$$a_n = \frac{2A}{T}\left[A\int_0^{T/4}\cos n\omega t\,dt + 0 + A\int_{3T/4}^{T}\cos n\omega t\,dt\right]$$

$$a_n = \frac{2A}{T}\left[\left\{\frac{\sin n\omega t}{n\omega}\right\}_0^{T/4} + \left\{\frac{\sin n\omega t}{n\omega}\right\}_{3T/4}^{T}\right]$$

$$a_n = \frac{2A}{n\omega t}\left[\sin\frac{n\omega t}{4} + \sin 2\pi n - \sin\frac{3n\omega T}{4}\right]$$

$$\sin 2\pi n = 0$$

$$a_n = \frac{2A}{n\omega t}\left[\sin\frac{n\omega t}{4} - \sin\frac{3n\omega T}{4}\right]$$

$$\sin 2\pi n = 0$$

$$a_n = \frac{2A}{n\omega t}\left[\sin\frac{n\omega T}{4} + \sin 2\pi n - \sin\frac{3n\omega T}{4}\right]$$

$$\sin 2\pi n = 0$$

$$a_n = \frac{2A}{n\omega t}\left[\sin\frac{n\omega t}{4} - \sin\frac{3n\omega T}{4}\right]$$

$$a_n = \frac{2A}{n\omega t}\left[\sin\frac{n\omega t}{4} + \sin 2\pi n - \sin\frac{3n\omega t}{4}\right]$$

$$\sin 2\pi n = 0$$

Date / /
Page No.
Shivalal

$$a_n = \frac{2A}{n\omega T}\left[\sin\frac{n\omega T}{4} - \sin\frac{3n\omega T}{4}\right]$$

$$\omega T = 2\pi$$

$$a_n = \frac{2A}{n \times 2\pi}\left(\sin\frac{2\pi n}{4} - \sin 3 \times \frac{2\pi}{4} \times n\right)$$

$$a_n = \frac{2A}{n\pi}$$

for all integer values of n :-

$a_n = \frac{2A}{n\pi}$; n = 1, 5, 9

$a_n = \frac{-2A}{n\pi}$; n = 3, 7, 11

$a_n = 0$; n even

$$b_n = \frac{2A}{n\omega T}\left[-\cos\frac{n\omega T}{4} + 1 - \cos n\omega T + \cos\frac{3n\omega T}{4}\right]$$

$$\omega T = 2\pi$$

$$b_n = \frac{2A}{n \times 2\pi}\left[-\cos\frac{2\pi}{4}n + 1 - \cos 2\pi n + \cos 3 \times \frac{2\pi n}{4}\right]$$

$$b_n = \frac{2A}{n \times 2\pi}\left[1 + (-1)\right]$$

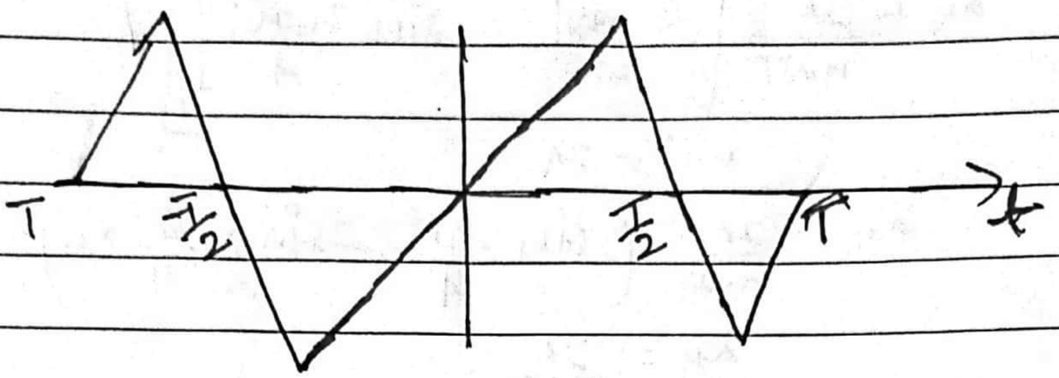
$$b_n = \frac{2A}{2\pi n}[0]$$

$$b_n = 0$$

The fourier series is obtained as

$$f(t) = \frac{A}{\pi}\left[2\cos\omega t - \frac{2}{3}\cos 3\omega t + \frac{2}{5}\cos 5\omega t + \frac{2}{7}\cos 7\omega t + \dots\right]$$

Q)



The function being odd, $a_0 = 0$ & $a_n = 0$ the sine coff is given by

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\,\sin(n\omega_0 t)\,dt$$

$$= \frac{4}{T}\int_0^{T/2} A\cdot\sin(n\omega_0 t)\,dt$$

$$= \frac{4A}{T}\int_0^{T/2} \sin(n\omega_0 t)\,dt$$

$$= \frac{4A}{n\omega_0 T}\Big[\cos(n\omega_0 t)\Big]_0^{T/2}$$

$$= \frac{4A}{n\omega_0 T}\left[\cos\frac{n\omega_0 T}{2} - 1\right]$$

$$= \frac{4A}{n\cdot\omega_0 T}\left[\cos\frac{n\omega_0 T}{2} - 1\right]$$

$$\omega T = 2\pi$$

$$b_n = \frac{4A}{n\cdot 2\pi}\left[\cos n\frac{2\pi}{2} - 1\right]$$

Scanned with CamScanner

Date / /
Page No.

$$b_n = \frac{2A}{n\pi}\left[\cos n\pi - 1\right]$$

$$b_n = \frac{2A}{n\pi}\left[\cos n\pi - 1\right]$$

$$b_n = \frac{2A}{n\pi}\left[(-1)^n - 1\right]$$

for $n = 1$

$$b_1 = \frac{2A}{\pi}\left[-1-1\right] = -\frac{4A}{\pi}$$

for $n=2$

$$b_2 = \frac{2A}{2\pi}\left[1 - 1\right] = 0$$

for $n=3$

$$b_3 = \frac{2A}{3\pi}\left[-1-1\right] = -\frac{4A}{3\pi}$$

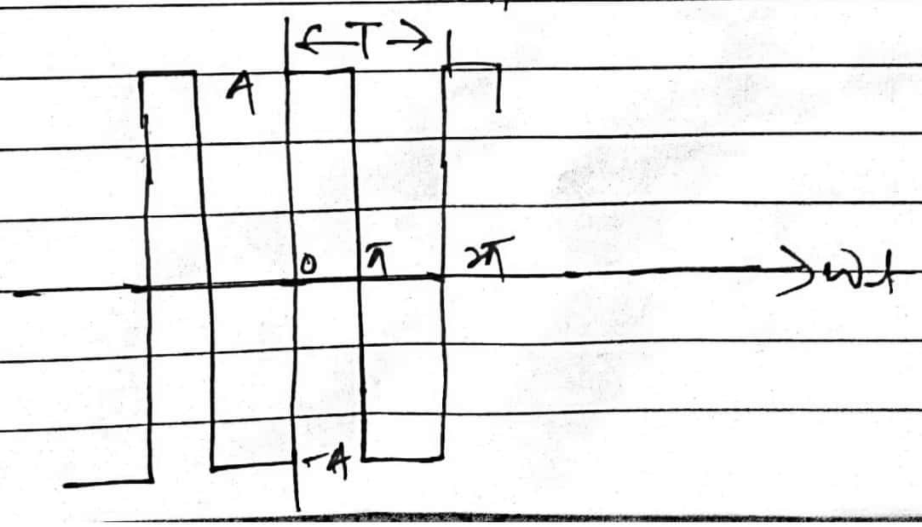
for $n=4$

$$b_4 = 0$$

for $n=5$

$$b_5 = -\frac{4A}{5\pi}$$

Q



Date / /
Page No.

Sol^n

$$c_n = \frac{1}{2\pi}\left[\int_0^{\pi} A\, e^{-jk(\omega t)}\, d(\omega t) + \int_{\pi}^{2\pi} (-A)\, e^{-jk(\omega t)}\, d(\omega t)\right]$$

$$c_n = \frac{1}{2\pi}\left[\frac{A}{-jk} e^{-jk(\omega t)}\right]_0^{\pi} + \frac{-A}{-jk} e^{-jk(\omega t)}\Big|_{\pi}^{2\pi}$$

$$c_n = \frac{1}{2\pi}\left[\frac{A}{-jk}\left(e^{-jk\pi} - 1\right) + \frac{A}{jk}\left(e^{-j2k\pi} - e^{-jk\pi}\right)\right]$$

$$c_n = \frac{A}{2\pi jk}\left[1 - e^{-jk\pi} + e^{-j2k\pi} - e^{-jk\pi}\right]$$

$$= \frac{A}{2j\pi k}\left(e^{-j2k\pi} - 2e^{-jk\pi} - 1\right) = \frac{A}{2j\pi k}\left(e^{-jk\pi} - 1\right)^2$$

for n odd; $e^{-jk\pi} = -1$

$$c_{n\,(n=odd)} = \frac{A}{2j\pi k}\left(e^{-jk\pi} - 1\right)^2$$

$$= \frac{A}{2j\pi k}(-1-1)^2$$

$$= \frac{2A}{j\pi k} \quad \text{Ans}$$

Scanned with CamScanner

Scanned with CamScanner

Fourier Transform
Numericals

Date / /
Page No.

1) Find the fourier transform of
$$x(t) = e^{-5t}u(t)$$ & also draw the magnitude spectrum.

Sol^n By definition of fourier transform —

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$X(\omega) = \int_{-\infty}^{\infty} e^{-5t} u(t)\, e^{-j\omega t}\, dt$$

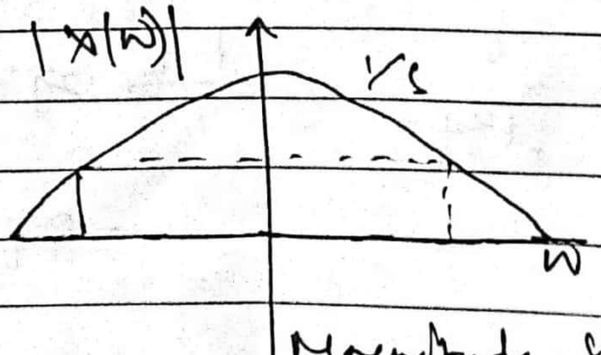
$$X(\omega) = \int_{0}^{\infty} e^{-5t - j\omega t}\, dt$$

$$X(\omega) = \int_{0}^{\infty} e^{-(5+j\omega)t}\, dt$$

$$X(\omega) = \left[\frac{e^{-(5+j\omega)t}}{-(5+j\omega)}\right]_0^{\infty}$$

$$X(\omega) = \frac{-1}{(5+j\omega)}(0-1)$$

$$X(\omega) = \frac{1}{5+j\omega}$$



Magnitude Spectrum