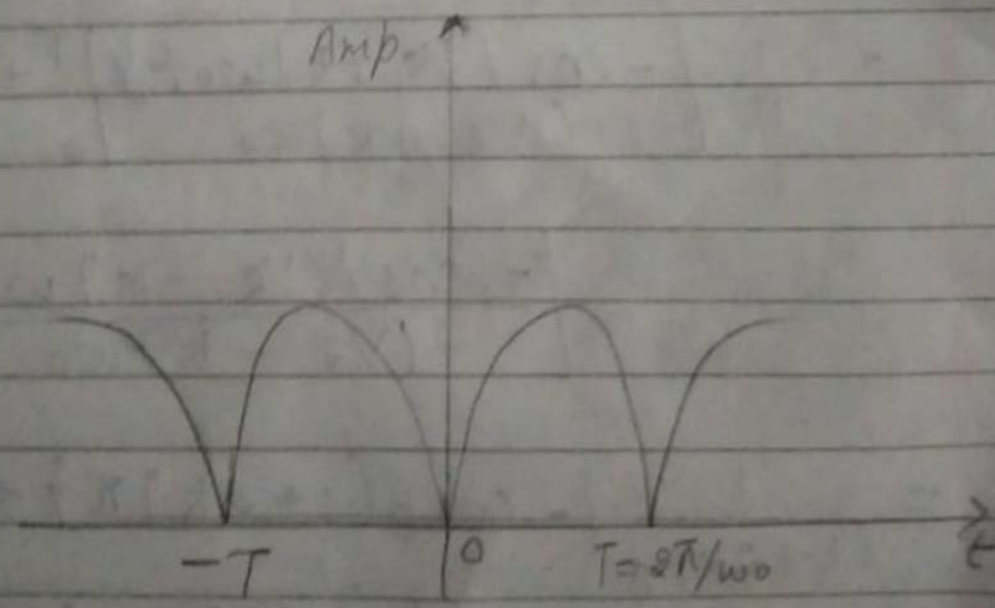
Q1 Consider the signal $f(t)$ defined as

$$f(t) = \sin\left[\frac{\omega_0 t}{2}\right] ; \; 0 \le t \le T = \frac{2\pi}{\omega_0}$$

The signal is periodic with period T. Find its Fourier series. To begin with sketch the signal.

Sol. From the expression of the periodic signal the signal is sketched in Fig. It can be seen that it is a full-wave rectified sinusoidal wave



In order to find the fourier series of the signal given.

we first evaluate the cofficients a_0, a_k and b_k in the following manner.

$$a_0 = \frac{1}{T}\int_0^T \sin(\omega_0 t/2)$$

$$= \frac{1}{T}\left[-\frac{\cos(\omega_0 t/2)}{[\omega_0 t/2]}\right] = \frac{2}{\pi} \quad (1)$$

$$a_k = \frac{2}{T}\int_0^T \sin(\omega_0 t/2)\cos k\omega_0 t \, dt$$

$$= \frac{1}{T}\int_0^T [\sin\{(\omega_0 t/2) + k\omega_0 t\} + \sin\{(\omega_0 t/2) - k\omega_0 t\}] \, dt$$

$$= \frac{1}{T}\left[\frac{-\cos\{(1/2 + k)\omega_0 t\}}{(1/2 + k)\omega_0}\right]_0^T + \frac{1}{T}\left[\frac{-\cos\{(1/2 - k)\omega_0 t}{(1/2 - k)\omega_0}\right]_0^T$$

$$= \frac{1}{(1+2k)\pi}[-\cos\{(1+2k)\pi\} + 1] + \frac{1}{[1-2k]\pi}[-\cos\{(1-2k)\pi\} + 1] \quad (2)$$

in eq^n (ii)

$$\cos\{(1+2k)\pi\} = -1 = \cos\{(1-2k)\pi\}$$
for all values of k

Thus

$$a_k = \frac{2}{(1+2k)\pi} + \frac{2}{(1-2k)\pi} = \frac{4}{(1+4k^2)\pi} \quad \text{--- (3)}$$

Evaluating b_k by eq^n, it is found that $b_k = 0$: for all values of k.

The fourier series of the given periodic signal is, finally, obtained in the following form.

$$f(t) = \frac{2}{\pi} - \frac{34}{3\pi}\cos w_0 t - \frac{4}{15\pi}\cos 2w_0 t$$

$$- \frac{4}{35\pi}\cos 3w_0 t - \frac{4}{63\pi}\cos 4w_0 t$$

$$- \frac{4}{99\pi}\cos 5w_0 t$$

Ans.

Q2. Consider the signal

$$x(t) = 1 + \sin \omega_0 t + 2\cos \omega_0 t + \cos\left[2\omega_0 t + \frac{\pi}{4}\right]$$

Determine its fourier series coefficients in the form of exponential series.

Sol: we will proceed directly rather than use the general relationship writing sinsoidal fn in exponential form,

we have.

$$\sin \omega_0 t = \frac{1}{2j}\left[e^{j\omega_0 t} - e^{-j\omega_0 t}\right] \text{ ——— (i)}$$

$$2\cos \omega_0 t = \left[e^{j\omega_0 t} + e^{-j\omega_0 t}\right] \text{ ——— (ii)}$$

$$\cos\left[2\omega_0 t + \frac{\pi}{4}\right] = \frac{1}{2}\left[e^{j2\omega_0 t} e^{j\pi/4} + e^{-j2\omega_0 t} e^{-j\pi/4}\right]$$

$$= \frac{\sqrt{2}}{4}(1+j1)e^{-j2\omega_0 t} + \frac{\sqrt{2}}{4}[1-j1]e^{-j2\omega_0 t}$$

——— (iii)

(i), (ii) and (iii) we can write

$$x(t) = 1 + \left[1 + \frac{1}{2j}\right] e^{j\omega_0 t} + \left[1 - \frac{1}{2j}\right] e^{-j\omega_0 t}$$

$$+ \frac{\sqrt{2}}{4}(1+j) e^{j2\omega_0 t} + \frac{\sqrt{2}}{4}(1-j) e^{-j2\omega_0 t} \quad \text{(iv)}$$

By inspection of eq^n (iv) we write down the four series coefficients as

$$F_0 = 1$$

$$F_1 = 1 + \frac{1}{2j}, \quad F_{-1} = 1 - \frac{1}{2j}$$

$$F_2 = \frac{\sqrt{2}}{4}(1+j), \quad F_{-2} = \frac{\sqrt{2}}{4}(1-j)$$

Q.3. The non-zero fourier series coefficients in exponential form of a continuous-time signal $f(t)$ with fundamental time period $T = 8$ are.

$$F_1 = F_{-1} = 2, \quad F_3 = F_{-3} = 4j$$

Determine $f(t)$ in sinusoidal form

Sol $\omega_0 = \frac{2\pi}{8} = \frac{\pi}{4}$: fundamental frequency

Only fundamental frequency and third harmonic are presents in the signal.

$$f(t) = \left[2e^{j\frac{\pi}{4}t} + 2e^{-j\frac{\pi}{4}t}\right] +$$

$$\left[4je^{j\frac{3\pi}{4}t} - 4je^{-j\frac{3\pi}{4}t}\right]$$

$$= 4\cos\frac{\pi}{4}t - 8\sin\frac{3\pi}{4}t$$

$$= 4\cos\frac{\pi}{4}t + 8\cos\left[\frac{3\pi}{4}t + \frac{\pi}{2}\right]$$

Q.4 A continuous-time periodic signal has

$$x(t) = \begin{cases} -2 & -1 \le t \le 0 \\ +2 & 0 \le t \le 1 \end{cases}$$

Its fundamental frequency is $\omega_0 = \pi$ calculating the fourier series coefficients f_k.

Sol. Time period

$$T = \frac{2\pi}{\omega_0} = 2$$

Thus one period is from -1 to 1.

$$F_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t) e^{-jk\omega_0 t}\, dt$$

$$F_k = \frac{1}{2}\int_{-1}^{0} (-2) e^{-jk\pi t}\, dt + \frac{1}{2}\int_{0}^{1} 2 e^{-jk\pi t}\, dt$$

$$F_k = \frac{1}{jk\pi}\{(1 - e^{-jk\pi}) - (e^{-jk\pi} - 1)\}$$

$$= \frac{2}{jk\pi}(1 - e^{jk\pi}) = \frac{2}{jk\pi} e^{jk(\pi/2)}$$

$$(e^{jk(\pi/2)} - e^{-jk(\pi/2)})$$

$$F_k = \frac{4}{k\pi} e^{jk(\pi/2)} \sin(k\pi/2) = -j\frac{4}{k\pi}$$

$$F_0 = 0$$

F_2 (odd) = Pure imaginary

F_k (even) = 0

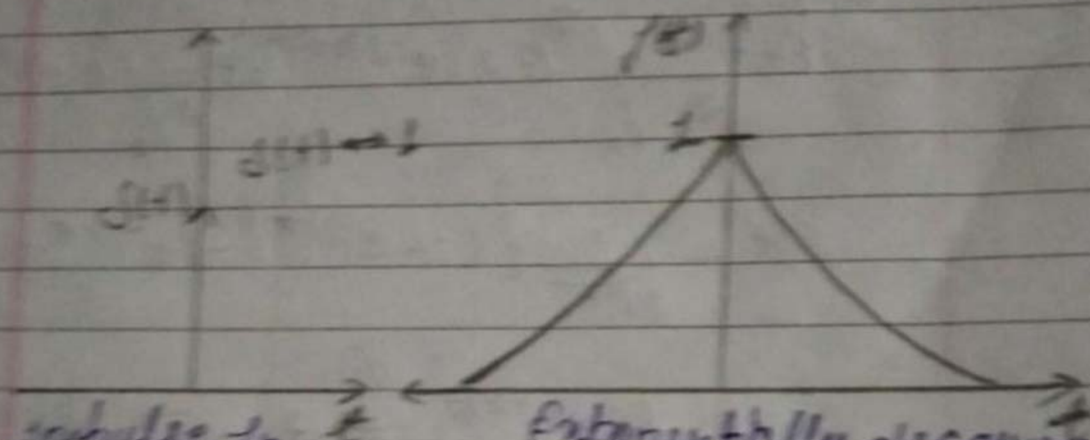
Observation

$x(t)$ real and odd $\Rightarrow$ F_k pure imaginary and odd.

Fourier Transform

(3) obtain the fourier transform of the following.

(a) Impulse function
(b) Exponentially decaying fn.



Some basic pulse fn.

Sol.

(a) $F(\omega) = \mathcal{F}\{\delta(t)\} = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt = 1$

Hence the pair

(b) Exponentially decaying function

It can be mathematically expressed

$f(t) = e^{-|t|/\tau}$ ———— (i)

Its fourier transform is

$$F(\omega) = \int_{-\infty}^{\infty} e^{-|t|/\tau} e^{-j\omega t}\, dt \quad \text{——— (i)}$$

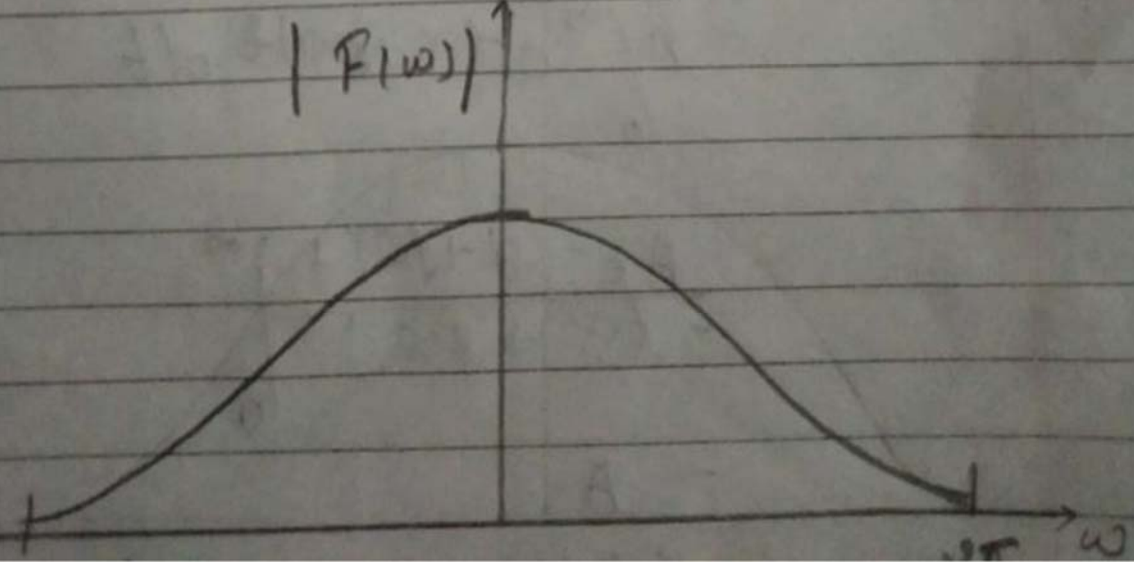
$$= \int_{-\infty}^{0} e^{(t/\tau)-j\omega t}\, dt + \int_{0}^{\infty} e^{(-t/\tau)-j\omega t} \quad \text{——— (iii)}$$

$$= \frac{e^{(t/\tau)-j\omega t}\Big|_{-\infty}^{0}}{(1/\tau)-j\omega} + \frac{e^{(-t/\tau)-j\omega t}\Big|_{0}^{\infty}}{(-1/\tau)-j\omega} \quad \text{——— (iv)}$$

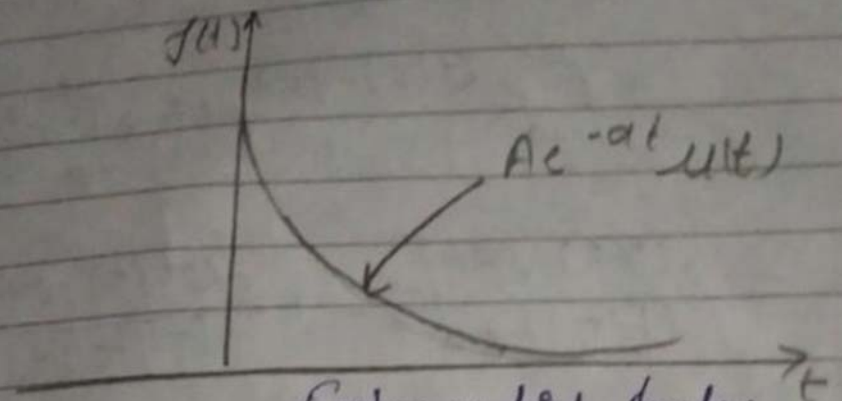
$$= \frac{1}{(1/\tau)-j\omega} + \frac{1}{(1/\tau)+j\omega} \quad \text{——— (v)}$$

$$F(\omega) = \frac{2\tau}{1+\omega^2\tau^2} \quad \text{——— (vi)}$$

The plot of $|F(\omega)|$ with ω is shown in fig



Q2 Find the fourier transform of a One-sided exponential pulse is sketched in Fig



Exponential pulse

Sol The fourier transform of this fn is

$$f(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} Ae^{-\alpha t} u(t) e^{-j\omega t}\, dt$$

$$= A\int_{0}^{\infty} e^{-(\alpha + j\omega)t}\, dt$$

$$= \left. \frac{Ae^{-(\alpha + j\omega)t}}{-(\alpha + j\omega)} \right|_{0}^{\infty}$$

$$= \frac{A}{\alpha + j\omega} \quad \text{—— (i)}$$

Its amplitude and phase spectrum are given as

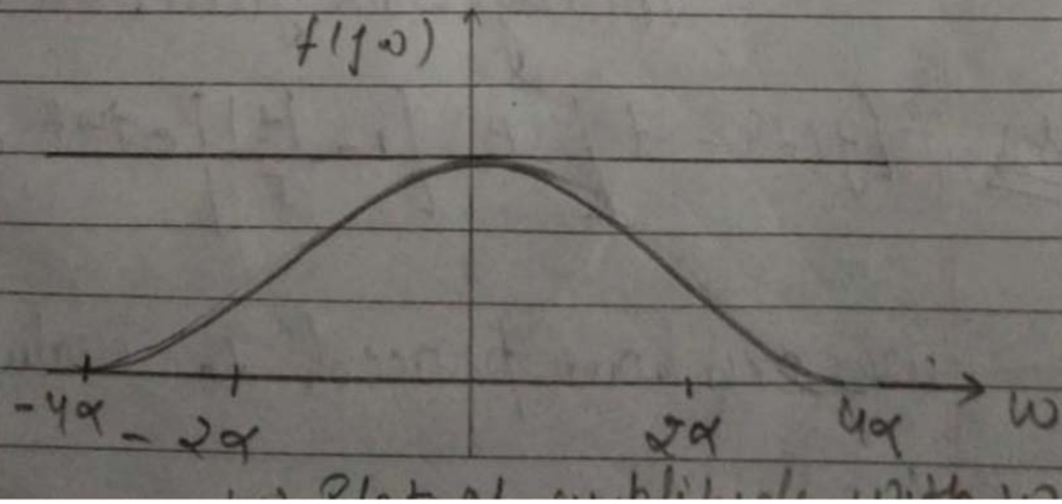
$$|F(\omega)| = \frac{A}{(a^2+\omega^2)^{1/2}} \quad \text{—— (ii)}$$

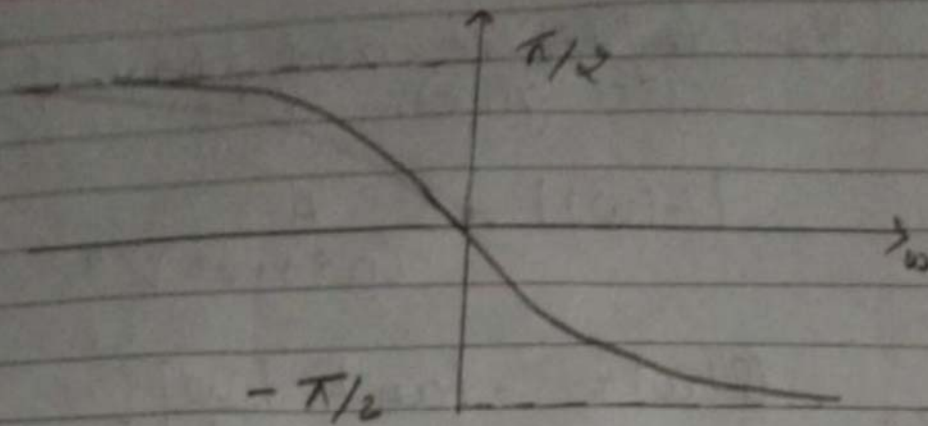
$$\Phi(\omega) = -\tan^{-1}\left[\frac{\omega}{\alpha}\right] \quad \text{—— (iii)}$$

These spectra are plotted in fig observe that $|F(\omega)|$ is an even function and $\Phi(\omega)$ is an function.

Q.3 Obtain fourier transform of the triangular function shown Mathematically described as

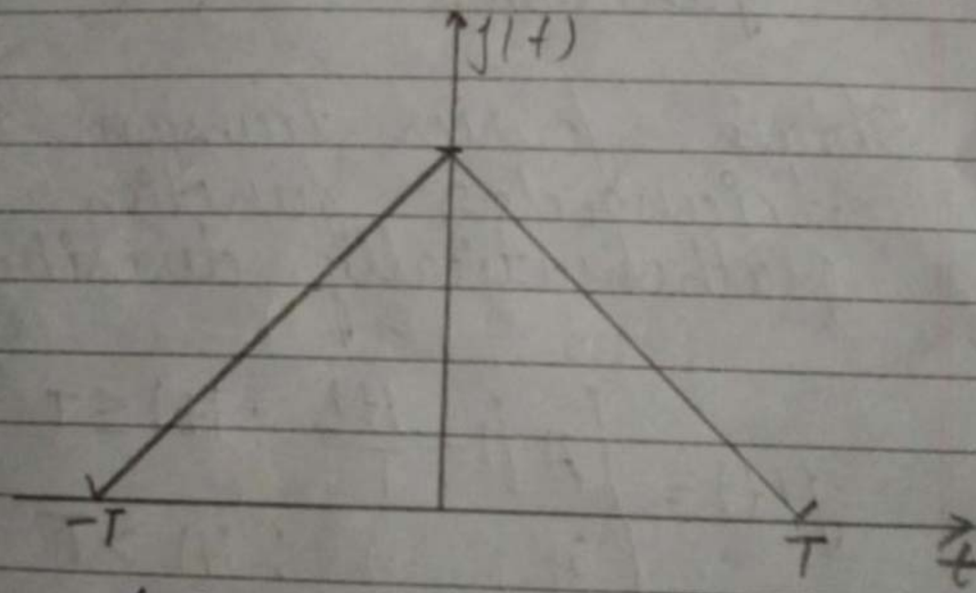
$$f(t) = \begin{cases} A\left[1-\frac{|t|}{T}\right] & ; |t| \le T \\ 0 & ; |t| > T \end{cases}$$





(b) plot of phase angle with ω

~~Ap~~ Amplitude and phase spectra of exponential pulse



Triangular function.

Sol. $F[f(t)] = \int_{-\infty}^{\infty} A\left[1 - \frac{|t|}{T}\right] e^{-j\omega t}\, dt ; |t| < T$

we can now proceed to evaluate right

side of above expression.

$$\int_{-T}^{0} A\left[1+\frac{t}{T}\right]e^{-j\omega t}+\int_{0}^{T} A\left[1-\frac{t}{T}\right]e^{-j\omega t}\,dt$$

or

$$\int_{-T}^{0} Ae^{-j\omega t}dt+\int_{-T}^{0}\frac{A}{T}te^{-j\omega t}dt+\int_{0}^{T}Ae^{-j\omega t}dt$$

$$-\int_{0}^{T}\frac{A}{T}te^{-j\omega t}dt$$

or

$$A\left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-T}^{0}+\frac{A}{T}\left[\frac{e^{-j\omega t}}{(1-j\omega)^2}(-j\omega t-1)\right]_{-T}^{0}+A\left[\frac{e^{-j\omega t}}{-j\omega}\right]_{0}$$

$$-\frac{A}{T}\left[\frac{e^{-j\omega t}}{(-j\omega)^2}(-j\omega t-1)\right]_{0}^{T}$$

or

$$\frac{-A}{j\omega}+\frac{Ae^{j\omega T}}{j\omega}+\left[\frac{-1}{(j\omega)^2}-\left\{\frac{Te^{j\omega T}}{j\omega}-\frac{e^{j\omega T}}{(j\omega)^2}\right\}\right]$$

$$-\frac{Ae^{-j\omega T}}{j\omega}+\frac{A}{j\omega}-\frac{A}{T}\left[\left\{\frac{Te^{-j\omega T}}{-j\omega}-\frac{e^{-j\omega T}}{(j\omega)^2}\right\}+\frac{1}{(j\omega)^2}\right]$$

combinning expponential terms into sine and cosine.

$$\frac{2A}{\omega^2 T}(1-\cos\omega T)=A\frac{\sin^2\omega T/2}{(\omega T/2)^2}=AT\,\mathrm{sinc}^2(\omega T/2)$$

Q.1 Find the fourier transform of the signal $\delta(t)$

Sol. $x(t) = \delta(t)$

By definition

$$x(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \delta(t) - j\omega t \, dt$$

$$= e^{-j\omega t} \Big|_{t=0} = 1$$

$$\delta(t) \longleftrightarrow 1$$

the unit impulse has a fourier transform consit of equal contribution of all frequency.