SCRIET

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Subject :- Signal and system

Submitted To :- Submitted By :-

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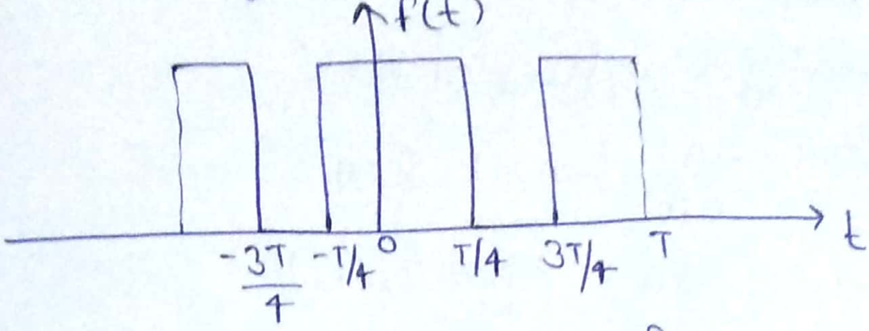
ASSIGNMENT-5

ques.) find the fourier series expansion of the periodic rectangular wave form shown in fig.



$$f(t) = A, \quad 0<t<T/4$$
$$= 0, \quad T/4<t<3T/4$$
$$= A, \quad 3T/4<t<T$$

Sol. $a_n = \frac{2}{T}\left[\int_0^{T/4} A\cdot\cos n\omega t\, dt + \int_{T/4}^{3T/4} 0\cdot\cos n\omega t\, dt + \int_{3T/4}^{T} A\cdot \cos n\omega t\, dt\right]$

$$a_n = \frac{2}{T}\left[A\int_0^{T/4}\cos n\omega t\, dt + 0 + A\int_{3T/4}^{T}\cos n\omega t\, dt\right]$$

$$a_n = \frac{2A}{T}\left[\int_0^{T/4}\cos n\omega t\, dt + \int_{3T/4}^{T}\cos n\omega t\, dt\right]$$

$$a_n = \frac{2A}{T}\left[\left\{\frac{\sin n\omega t}{n\omega}\right\}_0^{T/4} + \left\{\frac{\sin n\omega t}{n\omega}\right\}_{3T/4}^{T}\right]$$

$$a_n = \frac{2A}{n\omega T}\left[\sin\frac{n\omega T}{4} + 0 + \left\{\sin n\omega T - \sin\frac{3n\omega T}{4}\right\}\right]$$

$$a_n = \frac{2A}{n\omega T}\left[\sin\frac{n\omega T}{4} + \sin 2\pi n - \sin\frac{3n\omega T}{4}\right]$$

$$\sin 2\pi n = 0$$

$$\boxed{a_n = \frac{2A}{n\omega T}\left[\sin\frac{n\omega T}{4} - \sin\frac{3n\omega T}{4}\right]}$$

$$\omega T = 2\pi$$

$$a_n = \frac{2A}{n2\pi}\left(\sin\frac{2\pi n}{4} - \sin 3\times\frac{2\pi}{4}\times n\right)$$

$$\boxed{a_n = \frac{2A}{n\pi}}$$

for all integer values of n

$$a_n = \frac{2A}{n\pi} \; ; n = 1, 5, 9$$

$$a_n = -\frac{2A}{n\pi} \; ; n = 3, 7, 11$$

$$a_n = 0, \quad n = \text{even}$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t\, dt$$

$$b_n = \frac{2}{T}\left[\int_0^{T/4} A\cdot\sin n\omega t\, dt + \int_{T/4}^{3T/4} 0\cdot\sin n\omega t\, dt + \int_{3T/4}^{T} A\cdot\sin n\omega t\, dt\right]$$

$$b_n = \frac{2A}{n\omega T}\left[-\cos\frac{n\omega T}{4} + 1 - \cos n\omega T + \cos\frac{3n\omega T}{4}\right]$$

$$\omega T = 2\pi$$

$$b_n = \frac{2A}{n2\pi}\left[-\cos\frac{2\pi}{4}n + 1 - \cos 2\pi n + \cos 3\times\frac{2n\pi}{4}\right]$$

$$b_n = \frac{2A}{n\times 2\pi}\left[1 + (-1)\right]$$

$$b_n = \frac{2A}{2\pi n}[0]$$

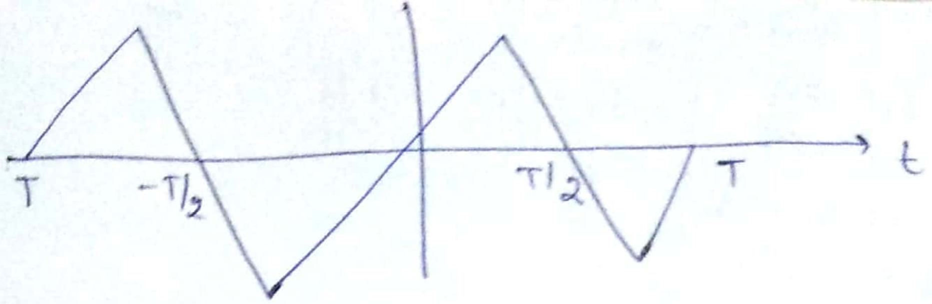
$$b_n = 0$$

The fourier series is the obtained as → $f(t) = \frac{A}{\pi}$

$$\left[2\cos\omega t - \frac{2}{3}\cos 3\omega t + \frac{2}{5}\cos 5\omega t - \frac{2}{7}\cos 7\omega t + \dots\right]$$

$$f(t) = \frac{2A}{\pi}\left[\cos\omega t - \frac{1}{3}\cos 3\omega t + \frac{1}{5}\cos 5\omega t - \frac{1}{7}\cos 7\omega t + \dots\right]$$

ans.)



Sol. The function being odd, $a_0 = 0$ & $a_n = 0$ The sine coefficent is given by

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\, \sin(n\omega_0 t)\, dt$$

$$= \frac{4}{T}\int_0^{T/2} A \cdot \sin(n\omega_0 t)\, dt$$

$$= \frac{4A}{T}\int_0^{T/2} \sin(n\omega_0 t)\, dt$$

$$= \frac{4A}{n\omega_0 T}\left[\cos(n\omega_0 t)\right]_0^{T/2}$$

$$= \frac{4A}{n\omega_0 T}\left[\cos\frac{n\omega_0 T}{2} - 1\right]$$

$$= \frac{4A}{n\omega_0 T}\left[\right. \qquad \omega T = 2\pi$$

$$b_n = \frac{4A}{n \times 2\pi}\left[\cos n\frac{2\pi}{2} - 1\right]$$

$$b_n = \frac{2A}{n\pi}\left[\cos n\pi - 1\right]$$

$$b_n = \frac{2A}{n\pi}\left[\cos n\pi - 1\right]$$

$$b_n = \frac{2A}{n\pi}\left[(-1)^n - 1\right]$$

for $n = 1$

$$b_1 = \frac{2A}{\pi}\left[-1-1\right] = -\frac{4A}{\pi}$$

for $n = 2$

$$b_2 = \frac{2A}{\pi}\left[4-1\right] = 0$$

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for $n=3$

$$b_3 = \frac{2A}{3\pi}[-1-1] = \frac{-4A}{3\pi}$$

for $n=4$

$$b_4 = 0$$

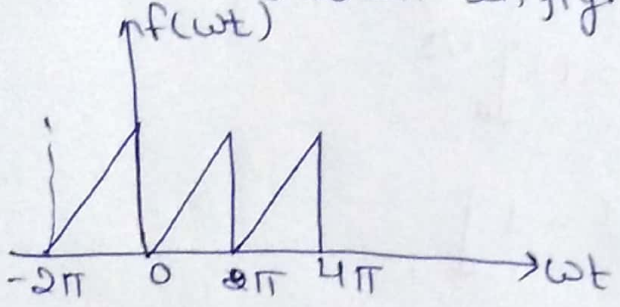
for $n=5$

$$b_5 = -\frac{4A}{5\pi}$$

fourier series is given by

$$f_t = \frac{4A}{\pi}\left[-\sin\omega_0 t - \frac{1}{3}\sin 3\omega_0 t - \frac{1}{5}\sin 5\omega_0 t + \cdots\right]$$

Ques.) Determine the fourier series for the saw-tooth wave shown in fig.



Sol. The wave form is periodic and continuous for $0<\omega t<2\pi$, The function is difined as

$$f(\omega t) = \frac{1}{2\pi}\omega t \quad , 0 < \omega t < 2\pi$$

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(\omega t)\, d(\omega t) = \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{\omega t}{2\pi}\right) d(\omega t)$$

$$= \frac{1}{2\pi}\left[\frac{(\omega t)^2}{2\times 2\pi}\right]_0^{2\pi} = \frac{1}{2\pi}\times\left[\frac{(2\pi)^2}{4\pi} - 0\right]$$

$$= \frac{1}{2\cancel{\pi}}\,\frac{4\cancel{\pi^2}}{4\cancel{\pi}} = \frac{1}{2}$$

$$= .5$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} \left(\frac{1}{2\pi}\omega t\right)\cos n\omega t \, d\omega t$$

$$= \frac{1}{2\pi^2}\left[\frac{\omega t}{n}\sin n\omega t + \frac{\cos n\omega t}{n^2}\right]_0^{2\pi}$$

Putting limits we, get

$$= \frac{1}{2\pi^2}\left[\frac{2\pi}{n}\overset{0}{\sin 2\pi n} + \frac{\overset{1}{\cos 2\pi n}}{n^2} - 0 - \frac{\overset{1}{\cos 0}}{n^2}\right]$$

$$= \frac{1}{2\pi}\left[\frac{1}{n^2} - \frac{1}{n^2}\right] = 0$$ i.e all cosine terms are absent

$$b_n = \frac{1}{\pi}\int_0^{2\pi} \frac{1}{2\pi}\omega t \sin \omega t \, d\omega t$$

$$= \frac{1}{2\pi^2}\left[-\frac{\omega t}{n}\cos n\omega t + \frac{1}{n^2}\sin n\omega t\right]_0^{2\pi}$$

$$= \frac{1}{2\pi^2}\left[-\frac{(2\pi)}{n}\cos 2\pi n + \frac{1}{n^2}\overset{0}{\sin 2\pi} + 0 - 0\right]$$

$$= \frac{-2\cancel{\pi}}{2\cancel{\pi}^2 n} = \frac{-1}{\pi n}$$

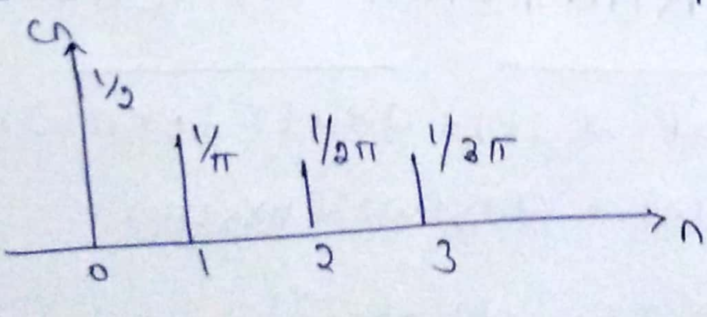
Thus eq (i) yields the fourier series as:

$$f(t) = .5 - \sum_{n=1}^{\infty} \frac{1}{n\pi}\sin n\omega t$$

$$\boxed{\begin{aligned} a_0 &= .5 \\ a_n &= 0 \\ b_n &= \frac{1}{-\pi n}\end{aligned}}$$

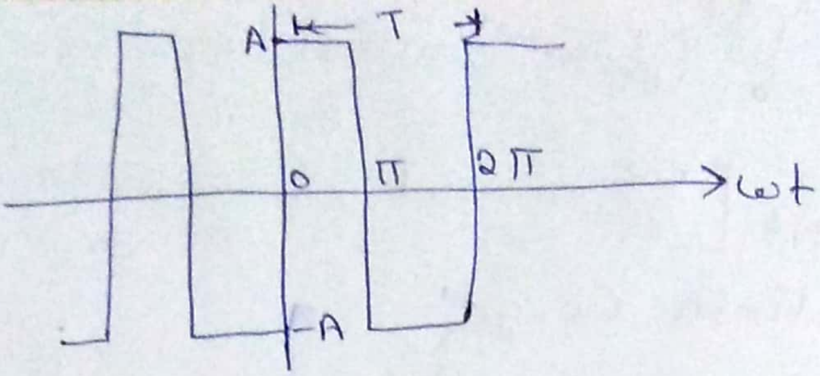
$$C_0 = a_0 = .5$$

as $a_n = 0$ hence $C_n = |b_n| = \frac{1}{n\pi}$



magnitude spectrum of sawtooth wave

Ques.)



Sol. $C_n = \frac{1}{2\pi}\left[\int_0^{\pi} A e^{-jk(\omega t)} d(\omega t) + \int_{\pi}^{2\pi} (-A) e^{-jk(\omega t)} d(\omega t)\right]$

$$C_n = \frac{1}{2\pi}\left[\frac{A}{-jk} e^{-jk(\omega t)}\Big]_0^{\pi} + \frac{-A}{-jk} e^{-jk(\omega t)}\Big|_{\pi}^{2\pi}\right]$$

$$C_n = \frac{1}{2\pi}\left[\frac{A}{-jk}\left(e^{-jk\pi} - 1\right) + \frac{A}{jk}\left(e^{-j2k\pi} - e^{-jk\pi}\right)\right]$$

$$C_n = \frac{A}{2\pi jk}\left[-1 - e^{-jk\pi} + e^{-j2k\pi} - e^{-jk\pi}\right]$$

$$C_n = \frac{A}{2j\pi k}\left[e^{-j2k\pi} - 2e^{-jk\pi} - 1\right] = \frac{A}{2j\pi k}\left(e^{-jk\pi} - 1\right)^2$$

for $n =$ odd; $e^{-jk\pi} = -1$

So

$$C_{n=odd} = \frac{A}{2j\pi k}\left(e^{-jk\pi} - 1\right)^2 = \frac{A}{2j\pi k}(-1-1)^2$$

$$C_{n=odd} = \frac{2A}{j\pi k} \quad \text{Ans}$$

FOURIER TRANSFORM NUMERICALS

Ques) show that if $x_3(t) = a x_1(t) + b x_2(t)$, then
$$X_3(\omega) = a X_2(\omega) + b X_2(\omega)$$

Sol. $x_3(t) = a x_1(t) + b x_2(t)$ — (i)

Taking fourier transform

$$X_3(\omega) = F[a x_1(t)] + F[b x_2(t)]$$

$$= \int_{-\infty}^{\infty} a x_1(t) e^{-j\omega t} dt + \int_{-\infty}^{\infty} b x_2(t) e^{-j\omega t} dt$$

$$= a\int_{-\infty}^{\infty} x_1(t) e^{-j\omega t} dt + b\int_{-\infty}^{\infty} x_2(t) e^{-j\omega t} dt$$

Thus $X_3(\omega) = a X_1(\omega) + b X_2(\omega)$

Hence Proved

Ques.) Find the fourier transform of $\left(\frac{1}{jt}\right)$
(Duality property)

Sol. $F.T.[sgn(t)] = \frac{2}{j\omega}$

$$Sgn \xleftrightarrow{FT} \frac{2}{j\omega}$$

$$\frac{2}{jt} \longleftrightarrow 2\pi\, sgn(-\omega)$$

Ans

Ques.) $FT[e^{-at}u(t) * e^{-bt}u(t)]$

Sol. $F[e^{-at}u(t)] = \frac{1}{a+j\omega}$ — i)

$F[e^{-bt}u(t)] = \frac{1}{b+j\omega}$ — (ii)

by using convolution property

$$F.T.[(i) * (ii)] = \frac{1}{a+j\omega} \cdot \frac{1}{b+j\omega}$$ Ans

Ques.) $F^{-1}\left[\frac{1}{a+j\omega} * \frac{2}{j\omega}\right]$

Sol. $F^{-1}\left[\frac{1}{a+j\omega}\right] = e^{-at}u(t)$ — i)

$$F^{-1}\left[\frac{2}{J\omega}\right] = sgn(t)$$

$$F^{-1}\left[\frac{1}{a+J\omega}\right] * F^{-1}\left[\frac{2}{J\omega}\right] = e^{-at}\, u(t) \cdot sgn(t)$$

Ans

ans.) $x(t) = e^{-3t}\, u(t)$

Sol. By using fourier transform:-

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-J\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} e^{-3t}\, e^{-J\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-(3+J\omega)t}\, dt$$

$$= \left[\frac{e^{-(3+J\omega)t}}{-(3+J\omega)}\right]_0^{\infty}$$

$$= -\frac{1}{(3+J\omega)}\,[0-1]$$

$$= \frac{1}{3+J\omega} \quad \text{Ans}$$