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Class - B.Tech 2nd yr (E.C.E)
Subj: Signal & System
Faculty: Mandeep Singh
Date: 2/4/2020

Topic: Numerical of DFT.

Q1 Calculate - 8 point DFT.

$x(n) = \{1, 2, 1, 2\}$

Sol: Firstly we will make length of given sequence "8" by doing zero padding.

$\therefore x(n) = \{1, 2, 1, 2, 0, 0, 0, 0\}$ ... (1)

We have $W_N = e^{-j\frac{2\pi}{N}}$

$\therefore W_8^{kn} = e^{-j\frac{2\pi}{8}} = e^{-j\frac{\pi}{4}kn}$ ... (2)

Here the range of k and n is from 0 to N-1 that means 0 to 7.

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Now the matrix W_8^{FT} is as follows

$n=0$	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$
W_8^0	W_8^0	W_8^0	W_8^0	W_8^0	W_8^0	W_8^0	W_8^0
W_8^0	W_8^1	W_8^2	W_8^3	W_8^4	W_8^5	W_8^6	W_8^7
W_8^0	W_8^2	W_8^4	W_8^6	W_8^8	W_8^{10}	W_8^{12}	W_8^{14}
W_8^0	W_8^3	W_8^6	W_8^9	W_8^{12}	W_8^{15}	W_8^{18}	W_8^{21}
W_8^0	W_8^4	W_8^8	W_8^{12}	W_8^{16}	W_8^{20}	W_8^{24}	W_8^{28}
W_8^0	W_8^5	W_8^{10}	W_8^{15}	W_8^{20}	W_8^{25}	W_8^{30}	W_8^{35}
W_8^0	W_8^6	W_8^{12}	W_8^{18}	W_8^{24}	W_8^{30}	W_8^{36}	W_8^{42}
W_8^0	W_8^7	W_8^{14}	W_8^{21}	W_8^{28}	W_8^{35}	W_8^{42}	W_8^{49}

We have already obtained different values of W_8^{kn}

$W_8^0 = W_8^8 = W_8^{16} = W_8^{24} = W_8^{32} = W_8^{40} = \cdots = 1$

$W_8^1 = W_8^9 = W_8^{17} = W_8^{25} = W_8^{33} = W_8^{41} = W_8^{49} = 0.707 - j0.707$

$W_8^2 = W_8^{10} = W_8^{18} = W_8^{26} = W_8^{34} = W_8^{42} = \cdots = -j$

$W_8^3 = W_8^{11} = W_8^{19} = W_8^{27} = W_8^{35} = W_8^{43} = \cdots = -0.707 - j0.707$

$W_8^4 = W_8^{12} = W_8^{20} = W_8^{28} = W_8^{36} = W_8^{44} = \cdots = -1$

$W_8^5 = W_8^{13} = W_8^{21} = W_8^{29} = W_8^{37} = W_8^{45} = \cdots = -0.707 + j0.707$

$W_8^6 = W_8^{14} = W_8^{22} = W_8^{30} = W_8^{38} = W_8^{46} = \cdots = j$

$W_8^7 = W_8^{15} = W_8^{23} = W_8^{31} = W_8^{39} = W_8^{47} = \cdots = 0.707 + j0.707$

Putting values of W_8^{kn} in Equation (3) and write x_n in matrix form we get,

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0.707-j0.707 & -j & -0.707-j0.707 & -1 & -0.707+j0.707 & j & 0.707+j0.707 \\ 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & -0.707-j0.707 & j & 0.707-j0.707 & -1 & 0.707+j0.707 & -j & -0.707+j0.707 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -0.707+j0.707 & -j & 0.707+j0.707 & -1 & 0.707-j0.707 & j & -0.707-j0.707 \\ 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & 0.707+j0.707 & j & -0.707+j0.707 & -1 & -0.707-j0.707 & -j & 0.707-j0.707 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore X_8 = \begin{bmatrix} 1+2+1+2+0+0+0+0 \\ 1+1.414-j\,1.414-j-1.414-j\,1.414+0+0+0+0 \\ 1-j\,2-1+j\,2+0+0+0+0 \\ 1-1.414-j\,1.414+j+1.414-j\,1.414+0+0+0+0 \\ 1-2+1-2+0+0+0+0 \\ 1-1.414+j\,1.414-j+1.414+j\,1.414+0+0+0+0 \\ 1+j\,2-1-j\,2+0+0+0+0 \\ 1+1.414+j\,1.414+j-1.414+j\,1.414+0+0+0+0 \end{bmatrix}$$

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Now the DFT is given by.

$$X_g = [W_g] x_N$$

putting the value of W_g^k in eq. (3)

$$\therefore x_g \begin{bmatrix} 6 \\ 1 - j\,2.4k \\ 0 \\ 1 - j\,1.828 \\ -2 \\ 1 + j\,1.828 \\ 0 \\ 1 + j\,3.828 \end{bmatrix}$$

This is required DFT

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Q. 2

The first five points of the 8-point DFT of a real valued sequences are

$\{0.25, 0.125 - j0.3018, 0, 0.125 - j0.0518, 0\}$

Determine the remaining three points.

Solution :- Given DFT points are:

$X(0) = 0.25$

$X(1) = 0.125 - j\,0.3018$

$X(2) = 0.$

$X(3) = 0.125 - j\,0.0518$

$X(4) = 0$

Given sequence is a real valued sequence. According to the symmetry property

$$X^*(k) = X(N-k)$$

or $$X(k) = X^*(N-k) \quad \text{— (1)}$$

This is 8-point DFT. Thus $N = 8$.

$$\therefore X(k) = X^*(8-k) \quad \text{— (2)}$$

Now we want remaining three samples $X(5)$ $X(6)$ and $X(7)$. Putting $k = 5$ in eq. (2)

$$X(5) = X^*(8-5) = X^*(3)$$

$$X(3) = 0.125 - j\,0.0518.$$

$$\therefore X^*(3) = 0.125 + j\,0.0518.$$

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$\therefore X(5) = 0.125 + j\ 0.518$

$k = 6$ in eq (2)

$X(6) = X^*(8-6) = X^*(2)$

$X(2) = 0$, thus $X^*(2) = 0$

$\therefore X(6) = 0$

Similarly in putting $k = 7$ in eq. (2) we get

$X(7) = X^*(8-7) = X^*(1)$

We have

$X(1) = 0.125 - j\ 0.301 8$

$\therefore X(7) = 0.125 + j\ 0.301 8$

Ex. 4.4.4 : Compute 8 point DFT of the sequence x (n) = {0, 1, 2, 3}. Sketch the magnitude and pha plot also.

Solution : Given sequence is, x (n) = {0, 1, 2, 3}

It is asked to calculate 8 point DFT to we will add four zeros in sequence x (n), to make length equal to 8; we will add four zeros.

$\therefore$ x (n) = {0, 1, 2, 3, 0, 0, 0, 0}Now $X(K) = [W_8]\, x_N$

$$\therefore X(K) = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$X_6 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0.707-j0.707 & -j & -0.707-j0.707 & -1 & -0.707+j0.707 & j & 0.707+j0.707 \\ 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & -0.707-j0.707 & j & 0.707-j0.707 & -1 & 0.707+j0.707 & -j & -0.707+j0.707 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -0.707+j0.707 & -j & 0.707+j0.707 & -1 & 0.707-j0.707 & j & -0.707-j0.707 \\ 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & 0.707+j0.707 & j & -0.707+j0.707 & -1 & -0.707-j0.707 & -j & 0.707-j0.707 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore X(K) = \{6, -\sqrt{2} - j\,4.82\,j, -2 + j2, \sqrt{2} - j\,0.82, -2, \sqrt{2} + j\,0.82, -2 - j2, -\sqrt{2} + j4 - 82\}$$

Now magnitude response $|X(K)| = \sqrt{[X(R)\,(\text{Real})]^2 + [X(K)\,(\text{Imaginary})]^2}$

$$\therefore |X(k)| = \{6, 5.02, 2.82; 1.63, 2, 1.63, 2.82, 5.02\}$$

The magnitude plot is shown in Fig. P. 4.4.4.

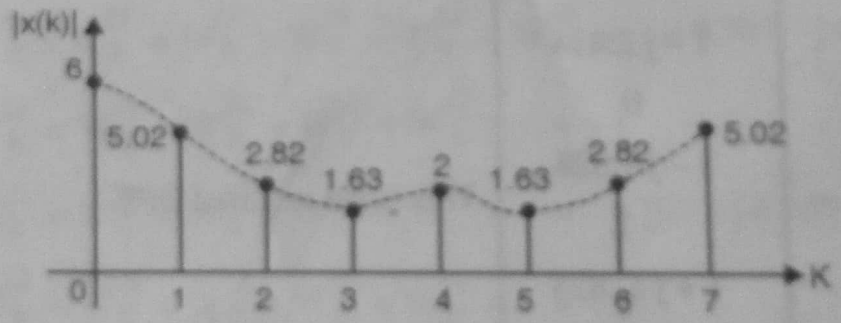

Fig. P. 4.4.4 : Magnitude response of X (K)

Phase response : $\angle X(k) = \tan^{-1}\left\{\dfrac{X(K)\,(\text{Imaginary})}{[X(K)\,(\text{Real})]}\right\}$

$$\angle X(k) = \{0°, 73.6°, -45°, -30°, 0, 30°, 45°, 73.6°\}$$

The phase response is shown in Fig. P. 4.4.4.

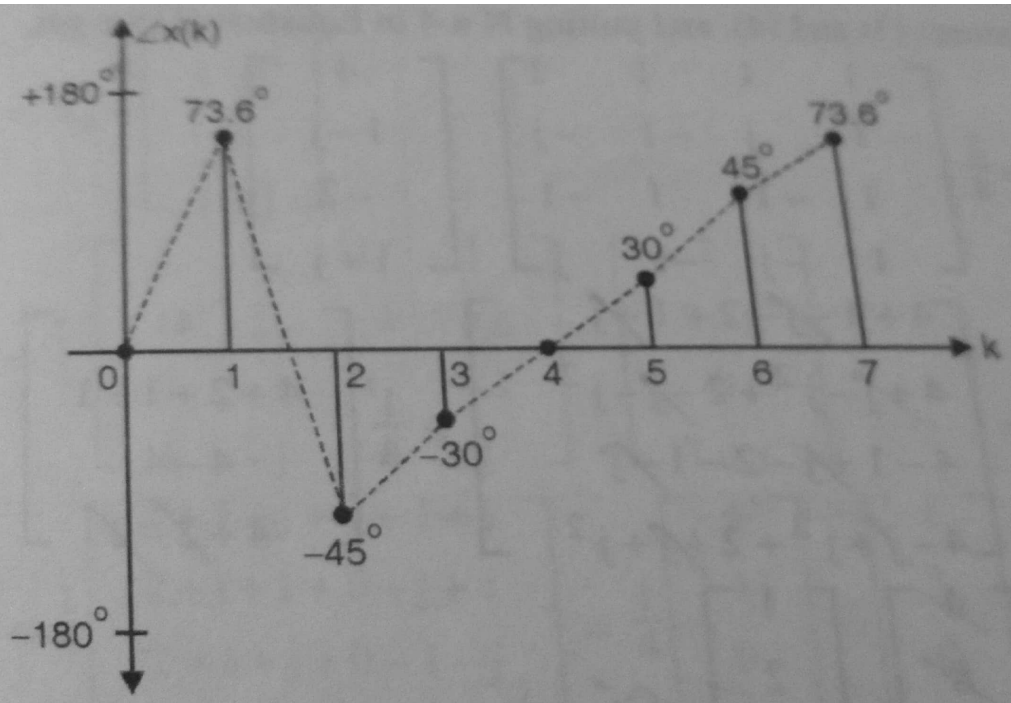

Fig. P. 4.4.4(a) : Phase response of X (k)