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# Lecture-7

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Microwave Engg.

EC-6th Sem (3rd yr.)

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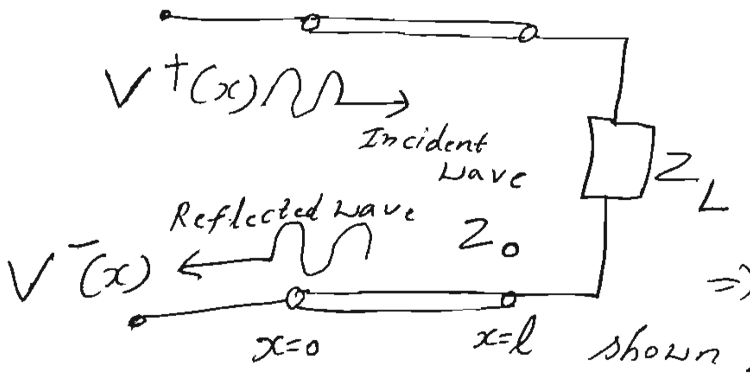
## Relation b/w S-Parameters & ABCD Parameters

**Step-1** Define waves in terms of voltages & currents

=) Let's take a simple transmission line, which has been terminated with a load impedance $Z_L$.

↳ will not go into details of transmission lines as you have already studied that in EMFT (2nd year).



=) Incident wave is shown by sign (+). $V^+(x)$.

=) Reflected wave is shown by sign (−) $V^-(x)$.

=) $$V^+(x) = C_1 e^{-j\beta x}$$

↑ Incident wave

↑ Constant

↖ This term comes here because $x=0$ here.

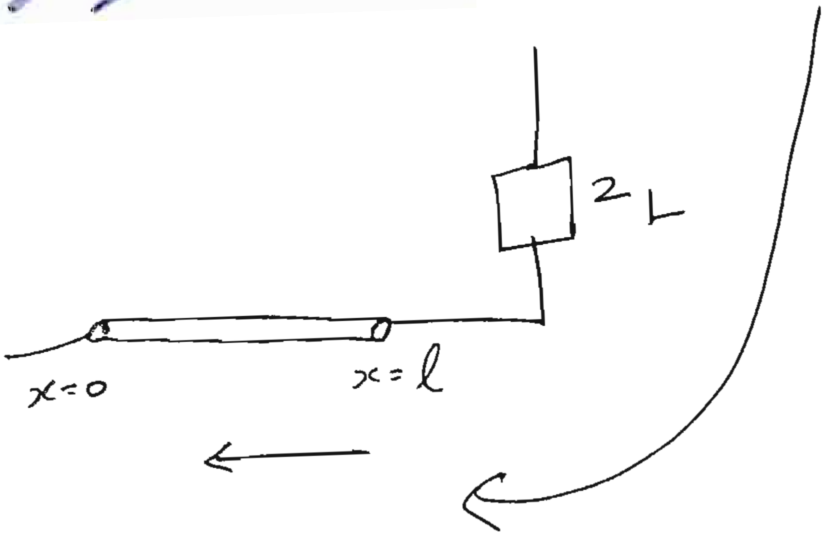
=) Similarly for reflected wave, $V^-(x) = C_2 e^{+j\beta x}$ ↙ (+)ve sign because it is

Note:-

~~$e^{-j\beta x}$~~

$$V^-(x) = C_2 e^{j\beta x}$$

The (+)ve sign here doesnot mean that it is going to be (+)ve. Look at the fig. of transmission line. 'x' is going to be (—)ve when ~~you~~ we are going from $x = l$ to $x = 0$.



⇒ Total voltage,

$$\left[ \underset{\substack{\uparrow \\ \text{Total} \\ \text{voltage}}}{V(x)} = \underset{\substack{\uparrow \\ \text{Incident} \\ \text{voltage}}}{V^+(x)} + \underset{\substack{\uparrow \\ \text{Reflected} \\ \text{voltage}}}{V^-(x)} \right]$$

↳ Note:- This is same as we have talked earlier about $V_1$ for Port-1.

⇒ Similarly,

Total current, $I(x) = I^+(x) + I^-(x)$

$$= \frac{V^+(x)}{Z_0} - \frac{V^-(x)}{Z_0}$$

⇒ At any port of the network,

$$a = \frac{V^+}{\sqrt{Z_0}}, \quad b = \frac{V^-}{\sqrt{Z_0}}$$

⇒ If you forgot what is 'a' & 'b' then check previous lectures.

↳ This is generalized one

ex:- If it is port '1' then we will write

$$a_1 = \frac{V_1^+}{\sqrt{Z_0}}, \quad b_1 = \frac{V_1^-}{\sqrt{Z_0}}$$

Similarly for port-'2' it will be.

$$a_2 = \frac{V_2^+}{\sqrt{Z_0}}, \quad b_2 = \frac{V_2^-}{\sqrt{Z_0}}$$

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⇒ Reflection coefficient $(\Gamma)$ = $\frac{\text{Reflected } \sout{\text{Power}}}{\text{Incident}}$

↳ $S_{11}, S_{22}, S_{33}$ etc
↓
Depending upon which port we are looking at.

$$\therefore \left[ \Gamma = \frac{V^-}{V^+} = \frac{b\sqrt{Z_0}}{a\sqrt{Z_0}} = \frac{b}{a} \right]$$

⇒ So, reflection coefficient at Port-1 will be

$$\Gamma_1 = \frac{b_1}{a_1}$$

Similarly ~~P~~ at port 2 $\Gamma_2 = \frac{b_2}{a_2}$, and so on.

ABCD to S-Parameter ⇐ No need of derivation. (refer Pozar book if you are interested)

| S | ABCD |
|---|---|
| $S_{11}$ | $\frac{A + (B/Z_0) - CZ_0 - D}{A + (B/Z_0) + CZ_0 + D}$ |
| $S_{12}$ | $\frac{2(AD - BC)}{A + (B/Z_0) + CZ_0 + D}$ |
| $S_{21}$ | $\frac{2}{A + (B/Z_0) + CZ_0 + D}$ |
| $S_{22}$ | $\frac{-A + B/Z_0 - CZ_0 + D}{A + (B/Z_0) + CZ_0 + D}$ |

For symmetrical Network
$S_{11} = S_{22}$
$(A = D)$
For reciprocal network
$S_{21} = S_{12}$
$(AD - BC = 1)$

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Note:- ① We are ~~doing~~ doing this conversion because we know how a large network can be divided into smaller network for which we use ABCD Parameters (Previous lecture).

↳ We multipled ABCD matrix of individual networks, for finding overall ABCD matrix.

② Our objective is not ABCD when we are talking about μWave, In microwave we talk about incident wave & reflected wave. So, we are more interested in finding S-Parameters.

③ So, the ABCD Parameters that we discussed in previous lectures is an intermediate step to reach the final goal of S-Parameters.

Note:- Now,

$$S_{11} = \frac{A + (B/Z_0) - CZ_0 - D}{A + (B/Z_0) + CZ_0 + D}$$

What is unit of 'A' → It is dimensionless

" 'D' → Dimensionless

" 'B' → unit of impedance

so, $B/Z_0$ → Dimensionless.

Unit of 'C' = mho

so, $CZ_0$ → Dimensionless

⇒ So,

$$\frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D}$$

can be written in ~~no~~ the form of normalized values.

~~so, $S_{11} = a + b - c -$~~

Let normalized value be,

$$A = a, \quad \frac{B}{Z_0} = b, \quad CZ_0 = c, \quad D = d$$

$$\therefore \; S_{11} = \frac{a + b \mp c - d}{a + b + c + d}$$

Note:- In all parameters (~~S~~ $S_{11}$, $S_{12}$, $S_{21}$, $S_{22}$) mentioned in table denominators are same, i.e. $A + B/Z_0 + CZ_0 + D$.

Now look at numerator,

$$S_{11} = A + B/Z_0 - CZ_0 - D \quad \leftarrow \text{Numerator}$$

$$S_{22} = -A + B/Z_0 - CZ_0 + D \quad \leftarrow \text{Numerator}$$

i.e. in Normalized form numerator will be,

$$S_{11} = a + b - c - d$$

$$S_{22} = -a + b - c + d$$

In these two equation 'a' has become '-a' & -d has become d.

Otherwise $S_{11}$ & $S_{22}$ are exactly same.

⇒ Using symmetrical Property,

if $\left[A = D\right]$

then,

$$S_{11} = \frac{\cancel{A} + B/Z_0 - CZ_0 - \cancel{A}}{\cancel{A} + B/Z_0 + CZ_0 + A} = \frac{B/Z_0 - CZ_0}{B/Z_0 + CZ_0 + A}$$

$$S_{22} = \frac{-\cancel{A} + B/Z_0 - CZ_0 + \cancel{A}}{A + B/Z_0 + CZ_0 + A} = \frac{B/Z_0 - CZ_0}{B/Z_0 + CZ_0 + A}$$

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$\therefore \left[ S_{11} = S_{22} \right]$

ii) So, we can easily prove from $S_{11} = S_{22}$ or $A = D$ that it is symmetrical network.

Note:- Now let's look at $S_{12}$ & $S_{21}$.

Numerator is,

$S_{12} = 2(AD - BC)$

$S_{21} = 2$

For reciprocal network, $AD - BC = 1$ $\Rightarrow$ $\therefore S_{12} = \dfrac{2}{A + B/Z_0 + CZ_0 + D}$

& $S_{21} = \dfrac{2}{A + \frac{B}{Z_0} + CZ_0 + D}$

$\therefore \left[ S_{12} = S_{21} \right]$

Note:- In some books they have shown how to convert from S-Parameter to ABCD. But in MWave we don't need this.

We need only ABCD to S-Parameter conversion.

Note:- We know,

$$I(x) = I^{+}(x) + I^{-}(x)$$

$$\Rightarrow \quad = \frac{V^{+}(x)}{Z_0} - \frac{V^{-}(x)}{Z_0}$$

$$\Rightarrow \quad a = \frac{V^{+}}{\sqrt{Z_0}} \quad \text{i.e.} \quad V^{+} = a\sqrt{Z_0}$$

$$b = \frac{V^{-}}{\sqrt{Z_0}} \quad \text{i.e.} \quad V^{-} = b\sqrt{Z_0}$$

$$\therefore \quad I(x) = I^{+}(x) + I^{-}(x)$$

both are same.

$$I = I^{+} + I^{-}$$

→ If you don't want to write 'x'.

$$= \frac{a\sqrt{Z_0}}{Z_0} - \frac{b\sqrt{Z_0}}{Z_0}$$