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PDC

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oct 27/14

PDC

$x(t) \rightarrow \boxed{\text{Process}} \rightarrow y(t)$

Process Dynamics

If there is any change in input then there will be a corresponding in the output.

The study of these changes & the process output/response of the process) correspondig change in input is process dynamics.

DE preapare modle are created

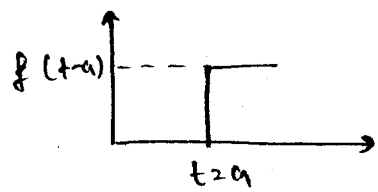
its eqn interms

we apply L.T to form eqn in algebraic form which is easily to solve.

* $L\{e^{at} f(t)\}$ first shifting

$f(t-a)\, u(t-a)$

$$\begin{cases} 0 & , t<a \\ f(t-a) & , t\geq a \end{cases}$$



$$L\{t^n f(t)\} = (-1)^n \frac{d^n \bar{f}(s)}{ds^n}$$

$$L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty \bar{f}(s)\,ds$$

<u>Transform of Derivatives</u>

$$L\left\{\frac{d f(t)}{dt}\right\} = s\bar{f}(s) - f(0)$$

$$L\left\{\frac{d^2 f(t)}{dt}\right\} = s^2\bar{f}(s) - sf(0) - f'(0)$$

<u>Transform of integrals</u>

$$L\left[\int_0^t f(t)\,dt\right] = \frac{\bar{f}(s)}{s}$$

Multiplication of | wid exponential = change of base

<u>Initial Value Theorem</u>

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sf(s)$$

<u>Final Value</u>

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sf(s)$$

Step function

$$L\{u(t-a)\} = \frac{e^{-as}}{s}$$

Impulse function

$$L\{\delta(t-a)\} = e^{-as}$$

Partial fractions

$$\frac{(ax+b)}{(cx+d)(px+q)} = \frac{A}{(cx+d)} + \frac{B}{(px+q)}$$

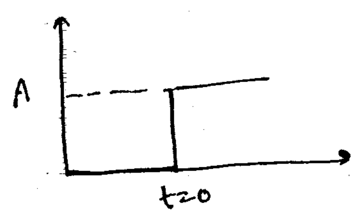
$$\frac{(ax+b)}{(cx+d)(px+q)^3} = \frac{A}{(cx+d)} + \frac{B}{(px+q)} + \frac{C}{(px+q)^2} + \frac{d}{(px+q)^3}$$

$$\frac{(ax+b)}{(cx+d)(px^2+q)} = \frac{A}{(cx+d)} + \frac{Bx+C}{(px^2+q)}$$

degree of numerator is less than degree of denominator

Input functions or forcing function

Step Input



t=0, means intially the process is going on at the steady state and that is represented by tym $t<0$, if at some pt. of tym there is a change in the process, then the kinetics of process is disturbed. The tym at which the change in introduced is designated at tym $t=0$.

$$L\{u(t-a)\} = \frac{e^{-as}}{s} \cdot A$$; from stepchange A.

$$\boxed{L\{u(t)\} = A/s}$$; in PDC $a=0$

Applying F.V.T

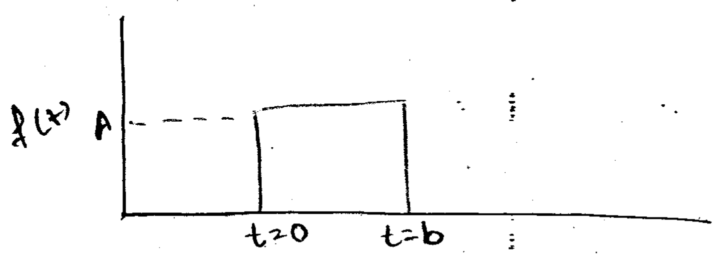
$$f(t) = A$$

$$\lim f(t) = A$$

$$\lim_{s\to 0} s f(s) = \lim_{s\to 0} A = A$$

* Step input is a bounded input.

Rectangular Pulse Input/function :-



$$f(t) = \begin{cases} 0 & , \ t<0 \\ A & , \ 0 \le t \le b \\ 0 & , \ t>0 \end{cases}$$

$$L\{f(t)\} = 0\,u(t-0) + A\,[u(t-0) - u(t-b)] + 0\,u(t-0)$$

$$L^{-1}\{\bar{f}(t)\} = A\{u(t-0) - u(t-b)\}$$

$$\boxed{f(s) = A\left[\frac{1}{s} - \frac{e^{-bs}}{s}\right]}$$

Apply final value theorem.

$$\lim_{s\to 0} s f(s) = \lim_{s\to 0} s \cdot \frac{A}{s}[1 - e^{-bs}]$$

$$= A$$

; we get defined result.

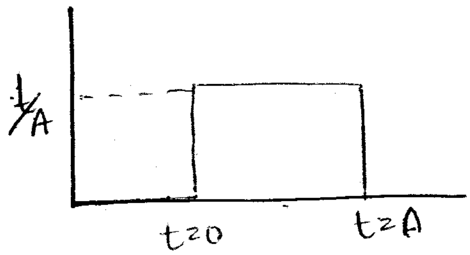
A

$\alpha\, A A$

* Rectangular pulse input is bounded input

Unit pulse Input :—

Special case of Rectangular Pulse



∵ such that area under curve is $\boxed{\frac{1}{A} \cdot A = 1}$

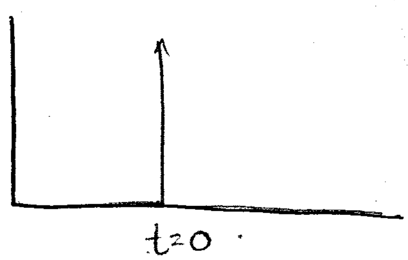
$$f(t) = \begin{cases} 0 & , t<0 \\ 1/A & , 0 \le t \le A \\ 0 & , t>0 \end{cases}$$

$$L^{-1}\{\bar{f}(s)\} = 1/A \{u(t-0) - u(t-A)\}$$

$$\boxed{f(s) = 1/A \left[1/s - \frac{e^{-As}}{s} \right]}$$

Unit Impulse function

Special case of unit pulse



almost practically impossible as it is not possible to give input & remove it at the moment / or same tym

$$f(s) = \lim_{A \to 0} \frac{1}{A} \left[\frac{1}{s} - \frac{e^{-As}}{s} \right]$$

$$= \lim_{A \to 0} \left[\frac{1 - e^{-As}}{As} \right]$$

L- Hospital Rule -

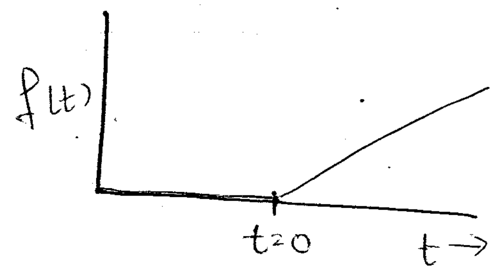
$$= \lim_{A \to 0} \left[\frac{0 - e^{-As}(-s)}{s} \right]$$

$$= \lim_{A \to 0} \left[e^{-As} \right] = 1$$

or

$$L\{\delta(t-\underset{0}{a})\} = e^{-as} = 1$$

RAMP Input :—



$f(t) = At$

$$\boxed{L\{f(t)\} = A/s^2}$$

Apply F.V.T

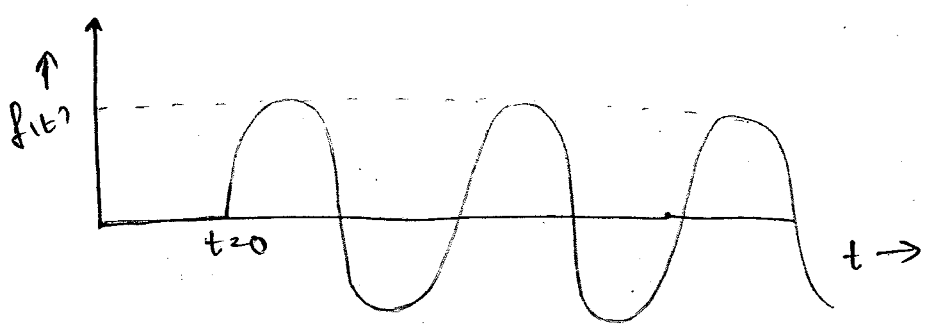
$f(t) = At$

$\lim f(t) = At$

$\lim_{s \to 0} s f(s) = \lim_{s \to 0} \cancel{s} \, A/s^{\cancel{2}} = \infty$

RAMP Input is unbounded input.

Sinusodial Input :-



$f(t) = A \sin \omega t$

$$\boxed{L\{f(t)\} = \frac{A\omega}{s^2+\omega^2}}$$

It comes automatically as order increases.

Apply f.v.T

$\lim_{s \to 0} s f(s) \Rightarrow \lim_{s \to 0} \frac{s \cdot A\omega}{s^2 + \omega^2} = \cancel{A\omega}\ 0 =$

Sinusodial Input is bounded.

Step
Sinusodial } bounded RAMP } unbounded , Impulse } impossible.

First order System

These are those which can be modeled by the help of a first order differential eqn

$$a_1 \frac{dy}{dt} + a_0 y = b\, f(t)$$

$\frac{dy}{dt} \rightarrow dp$; $f(t) \rightarrow$ input

$x(t) \rightarrow$ Process $\rightarrow y(t)$

Simulation $\quad x(t) \rightarrow$ Model $\rightarrow y(t)$ $\quad$ set of mathematical eqn that defines same process.

Model

Set of Mathematical eqns that gives the same result if the same ~~It satisfy~~ i/p is given

If $a_0 \neq 0$

$$\frac{a_1}{a_0} \frac{dy}{dt} + y = \frac{b}{a_0} f(t)$$

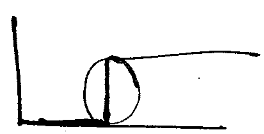
$\frac{a_1}{a_0} \rightarrow \tau$; $\frac{b}{a_0} \rightarrow k_p$; $f(t) \rightarrow$ input

$$\boxed{\tau \frac{dy}{dt} + y = K_p f(t)} \quad \text{—— (i)}$$

Deviation Variables

They are defined as the value of the function at any tym t minus the value of the function at steady state.

$$\boxed{F(t) = f(t) - f(0)}$$



t=0

By using deviation variable, we ~~are~~ resistricted ourselves to the study only the changes in i/p not the final steady state.

» The use of deviation variable allow us to study the behaviour of the o/p corresponding to the change in the i/p given

At steady state

$$\tau \frac{dy_s}{dt} + y_s = k_p f_0(t) \quad \text{—— (ii)}$$

(i) − (ii)

$$\tau \frac{d}{dt}(y-y_s) + (y-y_s) = k_p \left[f(t) - f(0)\right]$$

$$\boxed{\tau \frac{dY}{dt} + Y = k_p F(t)}$$

$$Y = y - y_s$$

$$F(t) = f(t) - f(0)$$

by applying the L.T

$$\tau \left[s\,Y(s) - Y(0) \right] + Y(s) = k_p F(s)$$

$$Y(0) = y(0) - y_s$$
$$\downarrow$$
$$y(0)$$
$$= 0$$

$$Y(s)\left[1 + \tau s\right] = k_p F(s)$$

L T of o/p

$$\boxed{\frac{Y(s)}{F(s)} = \frac{k_p}{1+\tau s}}$$

LT of i/p

» If we work wid deviation variable, the final eq^n is free from initial values.

$$\boxed{G(s) = \frac{Y(s)}{F(s)} = \frac{k_p}{1+\tau s}}$$

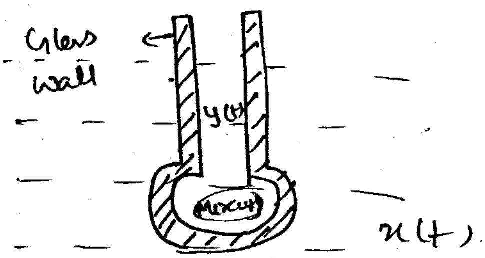
G(s) is defined as the ratio of L.T of o/p to the L.T of i/p

$F(s) \longrightarrow \boxed{G(s)} \longrightarrow Y(s)$

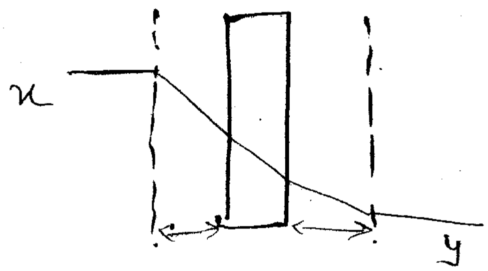
K_p = Steady state gain of the process
τ = Tym constant of the process.

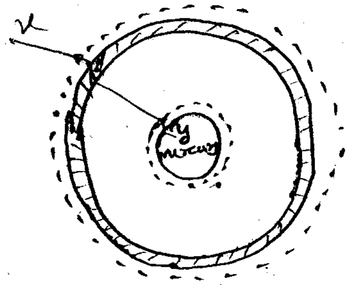
Mercury Thermometer :— (lumped parameter System)

The thermometer is placed in a moving liq stream temp (x)



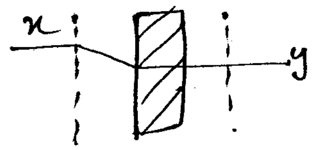
The thermometer reads the value of the liq temp (y)





Assumptions

⇒ All the resistance to h.t lies widin film only (film b/w the glass wall and the liquid)



⇒ The Glass wall containing the mercury doesn't expand or contract during the change. i.e all the thermal capacity lies widin Mercury only.

Applying the energy balance

Input − O/p = Acc.

$$hA(x-y) - 0 = mC_p \, dy/dt \quad \text{—(i)}$$

h = film coeff. of H.T

A = Surface Area of the bulb for heat transfer

C_p = heat capacity of the Mercury.

m = mass of the mercury in the bulb.

x = fluid temp.

y = Mercury thermometer Reading.

At steady state

$$hA(x_s - y_s) = m C_p \frac{dy_s}{dt} \quad \text{---(ii)}$$

(i) −(ii)

$$hA\left((x - x_s) - (y - y_s)\right) = m C_p \frac{d(y - y_s)}{dt}$$

$$hA(X - Y) = m C_p \frac{dY}{dt}$$

Applying L.T.

$$hA(X - Y) = m C_p \, dY/dt$$

$$X - Y = \underbrace{\frac{m C_p}{hA}}_{\tau} \, dY/dt$$

$a_0 \neq 0$

$$X - Y = \tau \frac{dY}{dt}$$

Taking Laplace

$$X(s) - Y(s) = \tau\left[sY(s) - Y(0)\right] \qquad | \; Y(0) = 0$$

$$Y(s)(1 + \tau s) = X(s)$$

$$\boxed{\frac{Y(s)}{X(s)} = \frac{1 = k_p}{1 + \tau s}}$$

example of first order system.

$$\tau = (mC_p) \times \left(\frac{1}{hA}\right)$$ — Resistan

Capacitance

$$\boxed{\tau = \text{Capacitance} \times \text{Resistance}}$$

Q.) A Mercury thermometer having a tym constant of 0.1 min is at s.s temp. of 90°F at tym t=0 thes thermometer is placed in a temp bath maintained at 100°F. Determine the tym needed for the thermometer to read 98°F

$$G(s) = \frac{Y(s)}{X(s)} = \frac{k_p}{1+\tau s}$$

$y_s = 90°F$

$A = 10°F$

$k_p = 1$

$$y(t) = A k_p (1 - e^{-t/\tau})$$

$y(t) - y_s$

$$98 - 90 = 10(1 - e^{-\frac{t}{0.1}})$$

$t = 0.16$

Response ~~for~~ of the first order system towards diff i/p functions

i) Step Input

$$\frac{Y(s)}{X(s)} = \frac{K_p}{1+\tau s}$$

$$X(s) = A/s$$

$$Y(s) = \frac{K_p}{1+\tau s} \cdot A/s$$

$$Y(t) = \frac{K_p A}{s(1+\tau s)}$$

$$\frac{P}{s} + \frac{Q}{(1+\tau s)}$$

$P = k_p A$

$P\tau + Q = 0$

$P\tau = -Q$

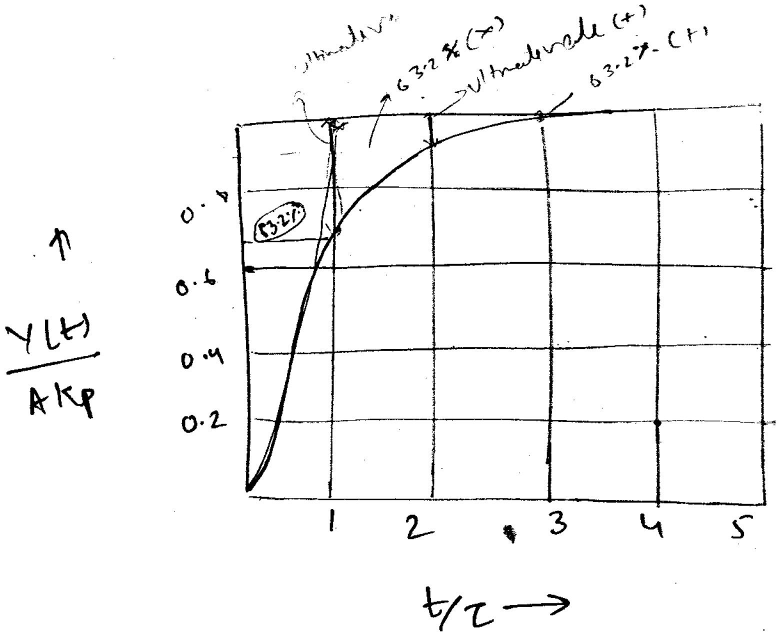
$$= \frac{k_p A}{\tau\left[s + 1/\tau\right] s}$$

$$= A k_p \left[\cancel{\frac{\tau}{1+\tau(0)}} \quad \frac{\tau}{(1+\tau s)(-1)} + \frac{1}{s(1)} \right]$$

$$Y(s) = A k_p \left[\frac{1}{s} - \frac{\tau}{1+\tau s} \right]$$

$$\boxed{Y(t) = A k_p \left[1 - e^{-t/\tau}\right]}$$

Response of~~for~~ the first order system for the step change of A.



<u>Observations</u>

1) At $t/\tau = 1$, $t = \tau$, tym constant of the process is defined as the tym taken by the process to reach 63.2%.. of its ultimate value.

2) $$\frac{d\left(Y(t)/A k_p\right)}{d(t/\tau)} = 0 - e^{-t/\tau}(-1)$$
$$= e^{-t/\tau}$$

$$\text{"} \quad \Big|_{t=0} = 1$$

3) if the initial rate of change of the response is maintained, when the tym taken by the process to reach its ultimate value is known as the constant of the value.

⇒ Apply f.v.t in response.

$$\lim_{t \to \infty} Y(t) = AK_p$$

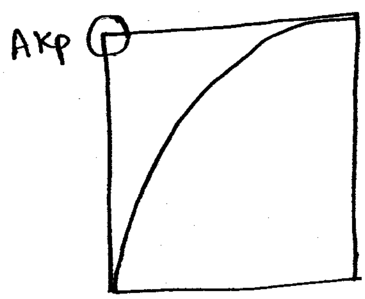
<u>first order systems</u> are <u>self regulating system</u> i.e doesn't require any controller.

$$\Delta(\text{output}) = K_p \, \Delta(\text{input})$$

Oct 28,14

<u>K_p = Steady state gain.</u>

Ultimate value of response is noticed. is (AK_p) Thus we can say that ~~the if~~ the steady state gain is the steady state value that the system attains after being disturbed by a step input.



for any step change in the input, the resulting change in the o/p is

$$\boxed{\Delta(\text{output}) = K_p \, (\Delta \text{input})}$$

This eqn tells us by how much we should change the value of the i/p in order to achieve the desired change in the o/p, for a process wid given gain K_p.

Thus to affect the same change in o/p, we need
⇒ a small change in the i/p, if K_p is large
⇒ a large change in the i/p, if K_p is small

If K_p is large the system is known as very sensitive system very sensitive system.

If k_p is small, the system is known as less sensitive system.

- generally value k_p should be low, but sometimes it should be high. and τ is as less as possible thus response is fast

less sensitive system wid low τ

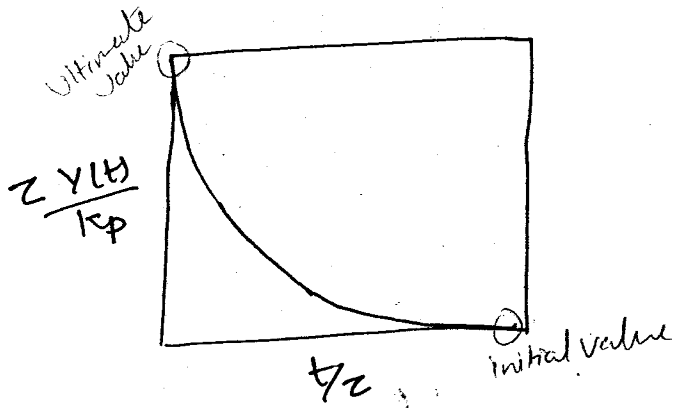
ii.) Impulse Input :-

$$X(s) = 1$$

$$Y(s) = \frac{k_p}{\tau(s+\frac{1}{\tau})} \cdot 1$$

$$\boxed{Y(t) = \frac{k_p}{\tau} e^{-t/\tau}}$$

Response of impulse i/p for first order system



initial steady state $\boxed{y(\infty) = y_s}$

ii.) RAMP Input :-

$$X(s) = A/s^2.$$

$$Y(s) = \frac{k_p}{\tau(s+1/\tau)} \cdot \frac{A}{s^2}$$

$$Y(s) = \frac{A k_p}{\tau s^2 (s+1/\tau)}$$

$$\frac{1}{s^2(s+1/\tau)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1/\tau}$$

$A(s(s+1/\tau)) + B(s+1/\tau)$
$+ Cs^2 = 1$
$As^2 + \frac{As}{\tau} + Bs + \frac{B}{\tau} + Cs^2 = 1$

$$A + C = 0$$

$$\frac{A}{\tau} + B = 0$$

$$\frac{B}{\tau} = 1$$

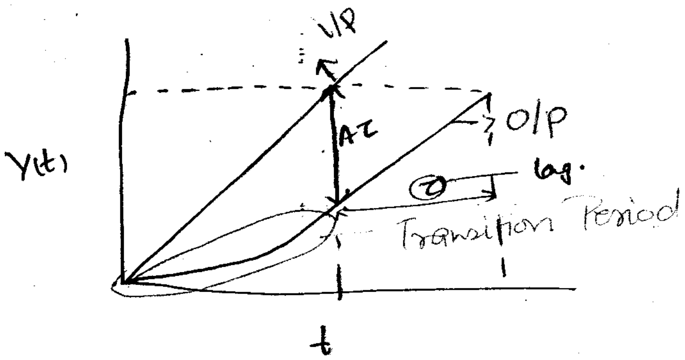
$$\boxed{\begin{aligned} B &= \tau \\ A &= -\tau^2 \\ C &= \tau^2 \end{aligned}}$$

$$\boxed{Y(t) = AK_p t - AK_p \tau \left(1 - e^{-t/\tau}\right)}$$ Response of Ramp i/p for 1st order System

if $K_p = 1$

$$Y(t) = At - A\tau\left(1 - e^{-t/\tau}\right)$$

$$X(t) = At$$



after sometime when eponential term vanishes.

$$Y(t) = At - A\tau = A(t - \tau)$$

$$X(t) = A(t - 0)$$

<u>Sinusodial Input</u>

$$X(s) = \frac{A\omega}{s^2 + \omega^2}$$

$$Y(s) = \frac{K_p}{s(1 + 1/\tau)} \cdot \frac{A\omega}{s^2 + \omega^2}$$

check all these

$$Y(t) = \frac{A\omega\tau e^{-t/\tau}}{1 + \tau^2\omega^2} - \frac{A\omega\tau}{1 + \tau^2\omega^2} \cos\omega t$$

$$Y(t) = \frac{A\omega\tau e^{-t/\tau}}{1+\tau^2\omega^2} - \frac{A\omega\tau}{1+\tau^2\omega^2}\cos\omega t + \frac{A}{1+\tau^2\omega^2}\sin\omega t$$

value: const.!

$A \quad \frac{\tau}{1+\omega^2\tau^2} \qquad \frac{A}{s+1/\tau} \qquad \frac{s}{s^2+\omega^2} \quad B \to -\frac{\tau^2}{1+\omega^2\tau^2} \qquad \frac{1}{s^2+\omega^2} \quad C = \frac{\tau}{1+\omega^2\tau^2}$

$$\boxed{Y(t) = \frac{A\omega\tau e^{-t/\tau}}{1+\tau^2\omega^2} - \frac{A\omega\tau}{1+\tau^2\omega^2}\cos\omega t + \frac{A}{1+\tau^2\omega^2}\sin\omega t}$$

$a\sin\theta + b\cos\theta = r\sin(\theta+\phi)$

$r\left(\sqrt{a^2+b^2}\right)\left(\frac{a}{\sqrt{a^2+b^2}}\sin\theta + \frac{b}{\sqrt{a^2+b^2}}\cos\theta\right)$

$\phi = \tan^{-1}(b/a)$

$\left(\frac{\sin\theta}{\sqrt{2}} + \frac{\cos\theta}{\sqrt{2}}\right)\sqrt{2}$

$\sqrt{2}\,(\cos 45^\circ \sin\theta + \sin 45^\circ \cos\theta)$

$r\,(\sin(\theta + 45^\circ))$

$\downarrow$

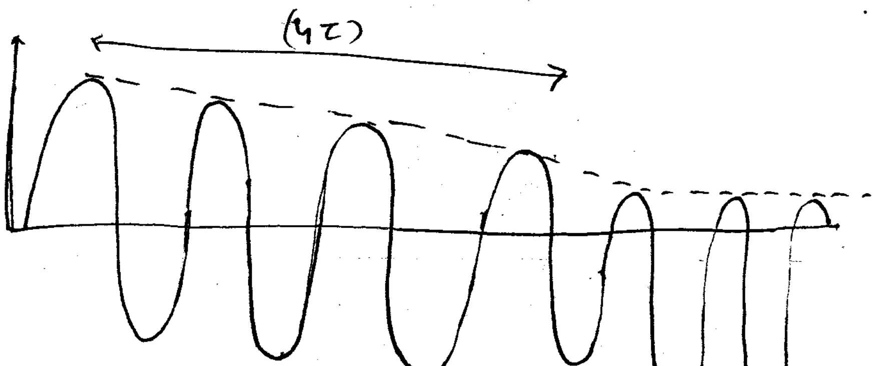
$\sqrt{2}$

$\frac{A^2\omega^2\tau^2 + A^2}{(1+\tau^2\omega^2)^2}$

$\frac{A^2(1+\omega^2\tau^2)}{(1+\tau^2\omega^2)^2} \qquad \frac{A}{(1+\tau\omega)}$

$\phi = \tan^{-1}(-\omega\tau)$

$$\boxed{Y(t) = \frac{A\omega\tau e^{-t/\tau}}{1+\tau^2\omega^2} + \frac{A}{\sqrt{1+\tau^2\omega^2}}\sin(\omega t + \phi)}$$

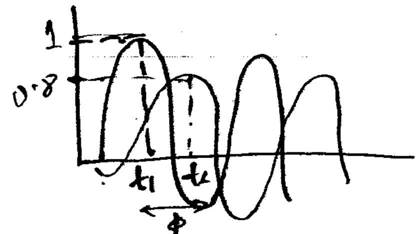


Ultimate value of Response

$$\boxed{Y(t)\Big|_{t\to\infty} = \frac{A}{\sqrt{1+\tau^2\omega^2}}\sin(\omega t+\phi)}$$ Ultimate solution

Observation :-

⇒ I/p amplitude is equals to A & O/p amplitude is equals to $\frac{A}{\sqrt{1+\tau^2\omega^2}}$



Amplitude ratio is defined as O/p amplitude to the I/p amplitude

$AR = \frac{1}{\sqrt{1+\tau^2\omega^2}}$, always less than 1.

~~and~~ an amplitude ratio is a factor which the gives the information abt the op amplitude w.r.t to i/p amplitude.

⇒ The i/p phase angle is ωt & o/p angle $(\omega t+\phi)$

The O/p lags behind the i/p by an angle ϕ.

ϕ has range btween (0–90) ; As $\tan 0 = 0$ & $\tan 90 = \infty$

⇒ O/p amplitude is lesser than the i/p amplitude and this the process known as Attenuation

Attenuation :- It is the gradual loss ~~of~~ intensity of any kind of flux through a medium.

eg→ Sunglasses.

Q) A Mercury thermometer having a tym const of 0.1 min is placed in a temp bath at 100°F and allow to come to equilibrium wid the bath. At tym t=0, the temp of the bath begins to varry sinusodialy abt its avg. temp of 100°F wid an amplitude of 2°F. If the frequency of the oscillation is $10/\pi$ cycles/min. Determine the phase lag, amplitude ratio, response of the system and the lag tym.

$$\underset{\text{rad per min}}{\omega} = 2\pi \underset{\text{cycles/min}}{f}$$

$$\tau = 0.1 \text{ min}$$

$$\omega = 2\pi \times 10/\pi$$

$$= 20 \text{ radian/min}$$

$$\phi = \tan^{-1}(-\omega\tau) = \tan^{-1}(-20 \times 0.1)$$

$$\phi = \cancel{1.107}\ 63.5^\circ = 1.11 \text{ rad}$$

$$AR = \frac{2}{\sqrt{1+\tau^2\omega^2}} = \cancel{0.15617}\ 0.896$$

$$= 0.4$$

Response $\boxed{Y(t) = 0.896 \sin(20t - 1.11)}$

$$360^\circ = 1 \text{ cycle tym}$$

$$63.5^\circ = \frac{\pi}{10} \times 63.5 \text{ min}$$

$$= 0.055 \text{ min}.$$

Q) The unit impulse response of a first order process is given by $2e^{-0.5t}$, Calculate the gain & tym constant of the process.

$$2e^{-0.5t} = \frac{K_P}{\tau} e^{-t/\tau}$$

$$\frac{K_P}{\tau} = 2 \quad , \quad 1/\tau = 0.5$$

$\tau = 2$

$k_p = 4$

Q.) A unit step i/p is given to a process that is represented by the transfer funct^n $\frac{s+2}{s+5}$. Calculate the initial value of the response towards the step i/p.

$$G(s) = \frac{s+2}{s+5} = \frac{Y(s)}{X(s)}$$

$$X(s) = 1/s$$

$$Y(s) = \frac{s+2}{s(s+5)}$$

$$\lim_{t\to 0} Y(t) = \lim_{s\to\infty} sY(s) = \frac{s+2}{s(s+5)} = \frac{\cancel{s}(1+2/s)}{\cancel{s}(1+5/s)}$$

$$= 1 \quad \text{Ans}$$

Q.) For the following first order system find the value of tym constant and steady state gain. $2/s+1/3$

$$\frac{k_p}{1+\tau s} = \frac{2}{s+1/3} = \frac{6}{3s+1}$$

$k_p = 6$

$\tau = 3$

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for the case of step i/p, the response reaches the ultimate value and stays there. but for the case of sinusoidal i/p the response oscillate abt its ultimate value wid an amplitude of $\frac{A}{\sqrt{1+\tau^2\omega^2}}$ and never give a constant result. bt it remains under the limit of $\pm \frac{A}{\sqrt{1+\tau^2\omega^2}}$ and therefore the step response and sinusoidal response are bounded.

Purely Capacitance System :—

$$a_1 \frac{dy}{dt} + a_0 y = b f(t)$$

$$a_1 \frac{dy}{dt} = b f(t) \quad \text{—(i)} \qquad ; \; a_0 = 0$$

At ss,
$$a_1 \frac{dy_s}{dt} = b f(0) \quad \text{—(ii)}$$

(i) − (ii)

$$\boxed{a_1 \frac{dY}{dt} = bF(s)}$$

$$a_1 [s Y(s) - Y(0)] = b F(s)$$

$$Y(0) = 0$$

$$sY(s) = b/a_1 \; F(s)$$

$$\boxed{\frac{Y(s)}{F(s)} = \frac{K_p}{s}}$$

Ultimate Value of Response

$$\lim_{t\to\infty} Y(t) = \lim_{s\to 0} sY(s)$$

$$= \lim_{s\to 0} s\left(\frac{K_p}{s^2}\right) = \infty$$

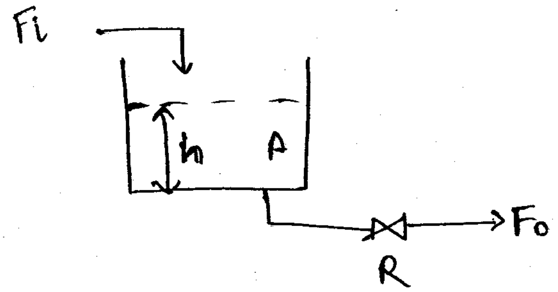
Ultimate value of response is infinite for a step change. If any change is introduced in the system, then the system can't sustain that change and ultimately the process stops. These kind of system that can't regulate themselves are known as Non-self regulating System. and we need a controller to control them.

These systems are also known as purely i integrated system bcz it behave as if there were an integrator present btw its i/p & o/p.

$$X(s) \rightarrow \boxed{G(s)} \rightarrow Y(s)$$

Integrator.

First order System wid capacity of mass storage :-



H = holp up of a liq in a tank / liq level

A = Area of cross-section

Fi/Fo = i/p / o/p flow rate.

R (valve) = act as a linear resistance wid a resistance R.

Consider a tank shown in fig, the output flow rate is related linearly to the hydrostatic pressure and resistance of the valve

$$\boxed{F_o = h/R}$$

1st assumption
F_o is linearly changes.

Applying the mass balance

$$\rho F_i - \rho F_o = \rho A \frac{dh}{dt}$$

IInd assumption
ρ = constant

$$F_i - F_o = A\, dh/dt$$

$$F_i - h/R = A\, dh/dt \quad \text{—(i)}$$

At steady state

$$F_{i_s} - \frac{h_s}{R} = A\frac{dh_s}{dt} \quad \text{—(ii)}$$

(i)-(ii)

$$(F_i - F_{i,s}) - \frac{(h-h_s)}{R} = A\frac{d(h-h_s)}{dt}$$

$$\bar{F_i} - \frac{\bar{H}}{R} = A\frac{d\bar{H}}{dt}$$

taking the laplace transform

but first make the variable unity

$$R\bar{F_i} - \bar{H} = AR\frac{d\bar{H}}{dt}$$

$$R\bar{F_i}(s) - \bar{H}(s) = AR\,(s\bar{H}(s) - \underset{0}{\underset{||}{\bar{H}(0)}})$$

$$R\bar{F_i}(s) = \bar{H}(s)(1 + ARs)$$

$$\boxed{\frac{\bar{H}(s)}{\bar{F_i}(s)} = \frac{R}{1+\tau s}} \quad \text{——(A)}$$

Mass storage systems can be modelled as first order systems

Steady state gain = R, where R is resistance of the valve

and tym constant $\tau = AR$.

Here also τ can be defined as .

The multiplication of storage capacitance (A) & the resistance (R)
It remains bounded.

Resistance

$$F_0 = h/R$$

$$F_{0,s} = h_s/R$$

$$(F_0 - F_{0,s}) = (h - h_s)/R$$

$$\bar{F}_0(s) = \bar{H}/R$$

taking laplace

$$\boxed{\frac{\bar{F}_0(s)}{\bar{H}(s)} = \frac{1}{R}} \quad \text{—(B)}$$

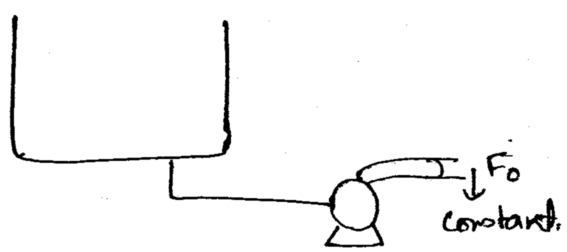
$$\frac{F_0(s)}{\bar{H}(s)} \cdot \frac{\bar{H}(s)}{\bar{F}_i(s)} = \frac{1}{R} \cdot \frac{R}{1+\tau s}$$

A×B

$$\boxed{\frac{\bar{F}_0(s)}{\bar{F}_i(s)} = \frac{1}{1+\tau s}}$$

System is bounded with the ultimate value of 1

if the o/p flow rate is constant by the help of some constant displacement pump



Apply mass balance.

$$\rho F_i - \rho F_0 = \rho A \frac{dH}{dt}$$

$$F_i - F_0 = A\frac{dH}{dt} \quad \text{—(1)}$$

ρ = const

$$F_{i,s} - F_o = A\frac{dh_s}{dt} \quad \text{—(ii)}$$

$F_o = $ const at s.s.

i—(ii)

$$\overline{F_i} = A\frac{d\overline{H}}{dt}$$

taking laplace.

$$\overline{F_i}(s) = A s \overline{H}(s)$$

$$\boxed{\frac{H(s)}{F_i(s)} = \frac{1}{As} = \frac{1/A}{s} = \frac{k_p}{s}}$$

example of purely capacitive or integrator system.

For a tank system wid a variable flow rate, if the inlet flow rate increases by the unit step. The liquid level goes up. As the liquid level goes up, the hydrostatic pressure increases which in turn increases the flow rate. as $\boxed{F_o = h/R}$. This action work towards the ~~resoto~~ restoration of an equilibrium stage at which the liquid level is constant for the i/p & o/p flow rate. Therefore this system is self regulating system.

For the tank system, wid a constant outlet flow rate, any small change in the inlet flow rate will make the tank flood or empty. These kinds of systems therefore behave in an unbounded manner for the changes in the input and therefore these are knownas Non self Regulating Systems. To keep the process bounded, we can adjust manually, the speed of the constant displacement pump so as to balance the flow coming in & out, to keep the level constant or need a controller for the same procedure.

Q.) Consider the two tanks wid different cross-sectional area A_1 & A_2 such that $A_1 > A_2$ & both the tanks have same resistance R. Draw the response curve of both tank towards step change.

$\tau = AR$

$\tau_1 = A_1 R$

$\tau_2 = A_2 R$

$A_1 > A_2$

$\tau_1 > \tau_2$

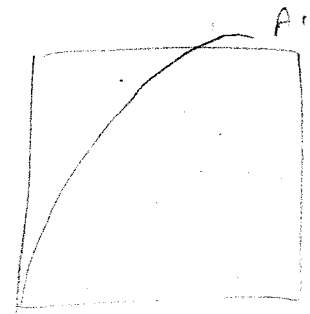
$A_1 > A_2$

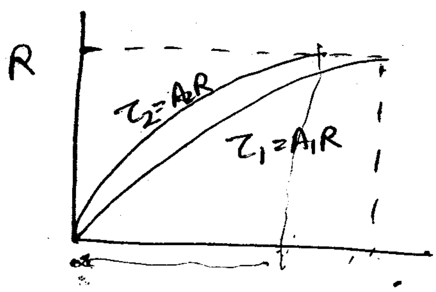
$\tau = A_1 R$

$\tau = A_2 R$

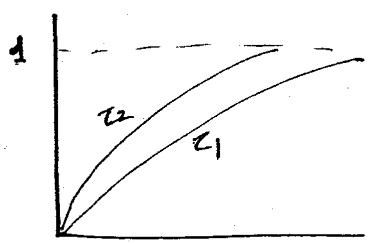
$K_p = R$

$R_1 = R_2 = R$

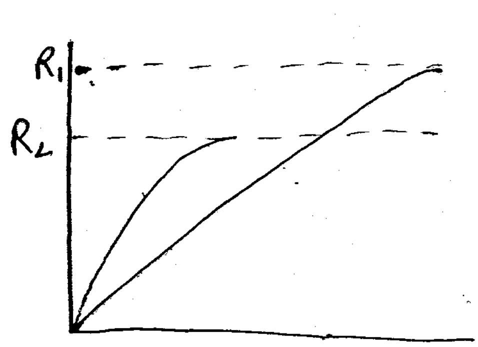


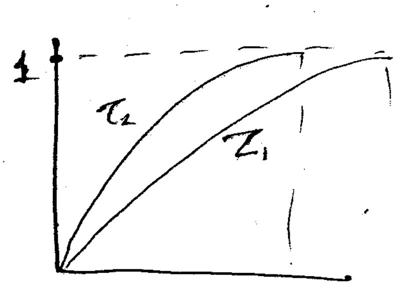


Q.) plot curve b/w $H(s)/F_i(s)$, $F_o(s)/F_i(s)$



Q.) if $R_1 > R_2$, and rest are same as above $H(s)/F_i(s)$ & $\frac{F_o(s)}{F_i(s)}$

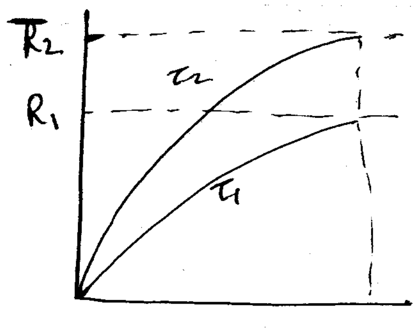


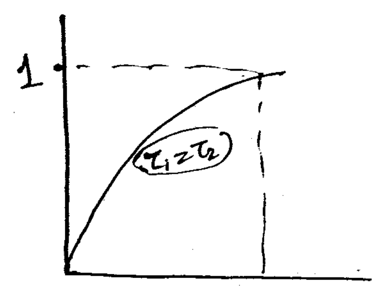


Q.) $A_1 > A_2$, $R_2 > R_1$ ⇒, $\boxed{A_1R_1 = A_2R_2}$

$\tau_1 = \tau_2$

$H_o/F(s)$, $\frac{F_o(s)}{F_i(s)}$





Oct 3914

Q.) A thermometer is initially at 100°C is dipped in the flowing bath. At the tym t=0, the oil bath is maintained at 150°C. If the recorded temp is 130°C after 3mins, then the tym constant of the thermometer is?

$Y(s) = 150°$

$t = 3min$

$$Y(t) = A(1 - e^{-t/\tau})$$

$t = 3$ min

$A = 150 - 100 = 50$ unit.

$$y(t) - y(s) = A(1 - e^{-t/\tau})$$

$130 - 100$

$$30 = 50(1 - e^{-3/\tau})$$

$$\tau = 3.27$$

Q.) A tank of volume 0.25m³ and height 1m has water flowing in at 0.05m³/min. The outlet flow rate is given by the relation $F_o = 0.1\sqrt{h}$, H is the height of water in the tank in m & F_o is outlet flow rate. The inlet flow rate suddenly changes ~~through it~~ from its nominal value of 0.05m³/min to

0.15 m³/min and remains there. The tym in min at which the tank will begin overflow is given by.

$V = 0.25 m^3$

$h = 1 m$

$F_i = 0.05\ m^3/min$

$F_o = 0.1 h$

$\bar{F_o} = h/R$

$F_o = 0.1 h$ m

$\frac{1}{R} = 0.1\ m^3/min$

$R = 10$

$F_o = h/R$

$F_o = 0.1\ h$

$R = 10$

$A = V/h = \frac{0.25 m^3}{1 m} = 0.25 m^2$

$\tau = AR$

$= 0.25 m^2 \times 10$

$= 2.5.$

Transfer function

$$\frac{\bar{h}(s)}{\bar{F_i}(s)} = \frac{R}{1+\tau s}$$

$$= \frac{10}{1+2.5s}$$

unit step change gives

if we substitute $0.15 - 0.05 = 0.1$ which ~~unit~~ step.

$$F_I = \frac{0.05\ m^3/min}{0.15\ m^3/min}$$

$\} \ 0.1\ m^3/min$

$$\bar{F_i}(s) = \frac{0.1}{s}$$ as it is unit step ch

$t^a = \frac{h}{sh+1}$

$$\bar{h}(s) = \frac{10}{1+2.5s} \times \frac{1}{s} = \frac{10}{s(1+2.5s)}$$

$$\bar{h}(s) = \frac{10}{s} + \frac{25}{(1+2.5s)}$$

$\frac{A}{s} + \frac{B}{(1+2.5s)}$

$A + 2.5As + Bs$

$A = 10$

$2.5A + B = 0$

$25 = B$

from my formula

$$h(t) - h(0) = 1 - e^{-t/2.5}$$

tank is overflow
$h(t) = 1$

$$e^{-t/2.5} = 0.5$$

$$F_{i/s} - F_{o/s} = \frac{A\,dh_s}{dt} \text{ const.}$$

Mass balance over tank.

$$F_{i/s} = F_{o/s}$$

$$0.05 = 0.1\, h_s$$

$$h_s = 0.5$$

$$t = 1.73 \text{ min}$$

Q) for a tank of cross-sectional area $100\,m^2$ & inlet flow rate F_i (cm^3/sec) Outlet flow rate is F_o in cm^3/sec. is related to the liquid height as $F_o = 3H$. The step change of magnitude ~~F_o~~ is $2\,cm^3/sec$ is given in the i/p flowrate Calculate the final height of the liquid level after 2 min ——. Given initial flow rate is $18\,cm^3/sec$.

$A = 100\,cm^2$

$F_i =$

$F_o =$

$F_o = 3H$

$F_i(s) = 2\,cm^3/sec$

$h_s = 9$

$t = 2\,min$

$F_o = 3H$

$\frac{1}{R} = 3$

$R = 0.333 \simeq \frac{1}{3}$

$$\tau = \frac{100}{3} = 33.33$$

$$\frac{h(s)}{F_i(s)} = \frac{0.333}{1 + 33.33\,s}$$

$$h(s) = \frac{0.333}{1+33.33s} \cdot \frac{2}{s}$$

$$=$$

$$h(t) = h(s) = 1 - e^{-t/33.33}$$

$$F_{i,s} = F_{o,s}$$

$$18 = 3h_s$$

$$h_s = 6$$

$$h(t) - 6 = 1 - e^{-2/33.3}$$

$.0666$

$$\frac{A}{s} + \frac{B}{1+33.33s}$$

$A = 0.666$

~~$B = 53.33$~~

$33.33A + B = 0$

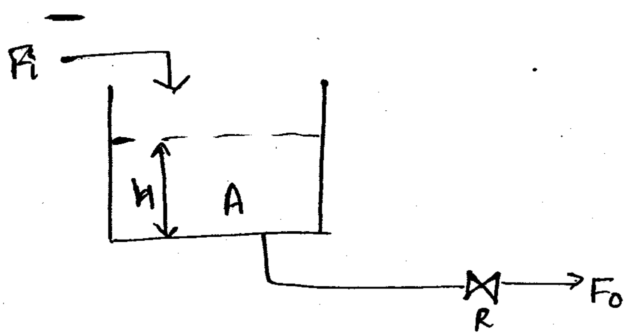
$B = -22.2$

$$\frac{0.66}{s} + \frac{-22.2}{(1+33.33s)}$$

6.5 cm.

Ans

Linearization of Non-Linear System



Fo varry non-linearly wid h

Assumptions

ρ remains contant

Flow rate ~~re~~ varry linearly with

$F = C h^{1/n}$

Apply Mass balance,

$$F_i - F_o = A\frac{dh}{dt}$$

$$F_i - C h^{1/n} = A\frac{dh}{dt}$$

$\boxed{n=2}$

Taylor series.

$F(x) = f(x_0) + (x-x_0) f'(x_0) + \frac{(x-x_0)^2}{\lfloor 2} f''(x_0)$

$$F_o(h) = F_o(h_s) + (h-h_s) f'(h_s) + \frac{(h-h_s)^2}{\lfloor 2} f''(h_s) \quad ; h = h_s$$

$$F_o(h) = F_o(h_s) + (h-h_s) f'(h_s)$$

$$F_o = C h^{1/2}$$

$$F_o' = C \tfrac{1}{2} h^{-1/2}$$

$$\boxed{F_o'(h_s) = \frac{C}{2\sqrt{h_s}}}$$

$$F_o = F_{o,s} + (h-h_s)\underbrace{\left(\frac{C}{2\sqrt{h_s}}\right)}_{1/R_1}$$

$$F_o = F_{o,s} + \frac{h-h_s}{R_1}$$

$$F_o - F_{o,s} = h - h_s / R_1$$

$$\bar{F_o} = \bar{h}/R_1$$

$$F_i - F_o = A\frac{dh}{dt}$$

$$\bar{F_i} - \bar{F_o} = A\frac{d\bar{h}}{dt}$$; in terms of deviation variable

$$\boxed{\bar{F_i} - \bar{h}/R_1 = A\,d\bar{h}/dt}$$

; $R_1 \neq$ resistance
; R_1 depends on the h_s.

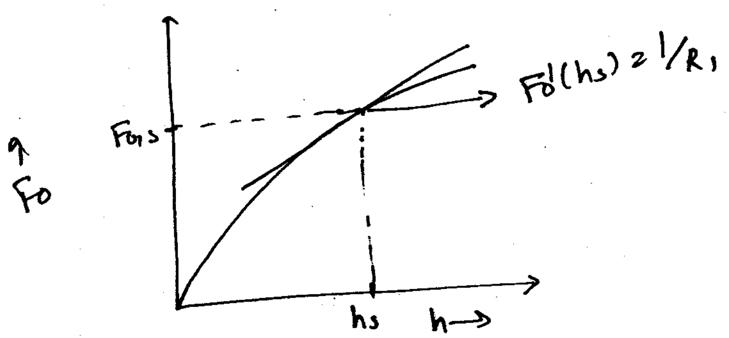
taking laplace

$$R_1\bar{F_i}(s) - \bar{h}(s) = AR[s\bar{h}(s) - \underbrace{\bar{h}'(0)}_{0}]$$

$$R_1\bar{F_1}(s) - \bar{h}(s) = ARs\bar{h}(s)$$

~~F_i/h~~

$$\boxed{\frac{\bar{h}(s)}{\bar{F_i}(s)} = \frac{R_1}{1+\tau s}}$$; $\boxed{\tau = AR_1}$



; eqn of tangent by taylor ser

$$\underbrace{(F_o - F_{o,s})}_{y} = \underbrace{\frac{1}{R_1}}_{m}\underbrace{(h-h_s)}_{x}$$

Q:) For a tank of cross-sectional area 50 cm² & inlet flowrate Fi be 9cm³/sec. The outlet flow rate Fo depends non-lineary on h $F_o = 3\sqrt{h}$, Find the transfer function $\frac{h(s)}{\bar{F_i}(s)}$ around the steady state pts. The steady state liquid level is given as 27 cm.

$A = 50\,cm^2$

$F_i = 9\,cm^3/sec$

$F_o = 3\sqrt{h}$

$A = 50 cm^2$, $F_i = 9 cm^3/sec$, $F_o = \sqrt[3]{h}$.

$\cancel{F_o = c h^{1/2}}$

$F_o = \sqrt[3]{h}$

$F_o' = \frac{1}{3} h^{-2/3}$

$\frac{1}{R_1} = F_o'(h_s) = \frac{1}{3(27)^{2/3}} = \frac{1}{27}$

$R_1 = 27$

$\tau = AR$

$= 50 \times 27$

$= 1350$ sec

$$\boxed{\frac{h(s)}{F_i(s)} = \frac{27}{1+1350 s}}$$

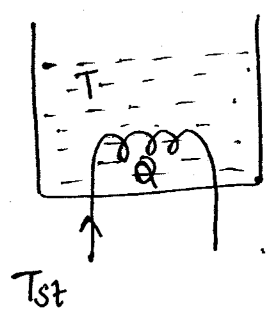
Ans.

$\frac{1}{R_1} = \frac{c}{2\sqrt{h_s}}$

$c = \frac{\sqrt[3]{27}}{\sqrt{h}}$

$\frac{1}{R_1} = \frac{3}{2\sqrt{h_s}}$

<u>First order System with a capacity for energy storage :-</u>



Apply energy balance,

Heat in = Accumulation.

Q =

$UA(T_{st} - T) = m C_p \, dT/dt$

$UA(T_{st} - T) = V \rho C_p \, dT/dt$ —— (i)

V = Vol^m of liquid in the tank.

ρ & C_p = Liquid density & heat capacity.

U = Overall heat transfer coeff b/w steam & liq.
A = total heat transfer
T_{St} = temp of the steam.

at steady state

$$UA\left[T_{st,s} - T_s\right] = V\rho C_p \, dT_s/dt \quad \text{—(ii)}$$

i − (ii)

$$UA\left[\bar{T}_{St} - \bar{T}_s\right] = V\rho C_p \, d\bar{T}/dt$$

$$\left[\bar{T}_{St} - \bar{T}\right] = \frac{V\rho C_p}{UA} \, d\bar{T}/dt$$

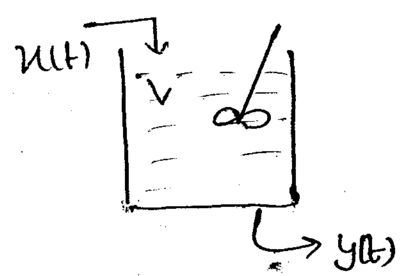
$$\bar{T}_{St}(s) - \bar{T}(s) = \tau\left[s\bar{T}(s) - \underset{0}{\bar{T}(0)}\right]$$

$$\boxed{\frac{\bar{T}(s)}{\bar{T}_{St}(s)} = \frac{1}{1+\tau s}}$$

$$\boxed{\tau = \text{storage capacitance} \times \text{Resistance offered by the system}}$$

$$\boxed{\tau = mC_p \times \frac{1}{UA}}$$

Mixing Tank System



Consider the mixing tank shown in fig., in which a stream of solution containing some dissolved salt flows at a constant volumetric flow rate of q, into a tank of constant hold up volume V.

The concentration of salt in the entering stream is $x(t)\left(\frac{\text{mass of salt}}{\text{volume}}\right)$ and depends on tym t. Assuming density & liq hold up to be constant. Find the transfer function relating the outlet concn y to the inlet concentration x.

⇒ Apply mass balance,

mass in — mass out = rate of accumulation of mass.

$$qx - qy = \frac{d(Vy)}{dt}$$

$$q(x-y) = \frac{d(Vy)}{dt} \quad \text{— (i)}$$

at steady state

$$q(x_s - y_s) = \frac{d(Vy_s)}{dt} \quad \text{— (ii)}$$

(i) − (ii) and applying deviation variable.

$$q(\overline{X_s} - \overline{Y_s}) = V\frac{d(\overline{Y})}{dt}$$

taking laplace.

$$\cancel{q}(\bar{X}(s) - \bar{y}(s)) = \frac{V}{\cancel{q}}\{s\,\bar{y}(s) - \underset{0}{\bar{y}(0)}\}$$

$$\cancel{q}\,\bar{X}(s) = \cancel{q}\,\bar{y}(s) + \frac{Vs}{q}\,\bar{y}(s)$$

$$\frac{\bar{y}(s)}{\bar{X}(s)} = \cancel{\frac{q}{q}}\left(\frac{1}{1+\tau s}\right) \qquad \tau = V/q$$

$$\boxed{\frac{\bar{y}(s)}{\bar{X}(s)} = \frac{1}{1+\tau s}}$$

Q) which processes can be modelled as first order system.

Ans> A process that posses a capacity to store mass or energy ente and thus act as a buffer b/w in flowing and out flowing stream. will be modelled as first order system.

The resistances associated wid the flow of mass or energy in reaching the required capacity also modelled as first order

for ex→ the resistance associated wid the valves in the tank system.

Second order System

Second order systems are those that can be modelled wid help of second order diff eqn

$$a_2 \frac{d^2y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y = b f(t)$$

$$\frac{a_2}{a_0} \frac{d^2y}{dt^2} + \frac{a_1}{a_0} \frac{dy}{dt} + \frac{a_0}{a_0} y = \frac{b}{a_0} f(t)$$

$a_0 \neq 0$

$$\tau^2 \frac{d^2y}{dt^2} + 2\zeta\tau \frac{dy}{dt} + y = K_p f(t) \quad \text{—(i)}$$

τ = natural period of oscillation

(zeta) ζ = damping coeff or damping factor

K_p = steady state gain

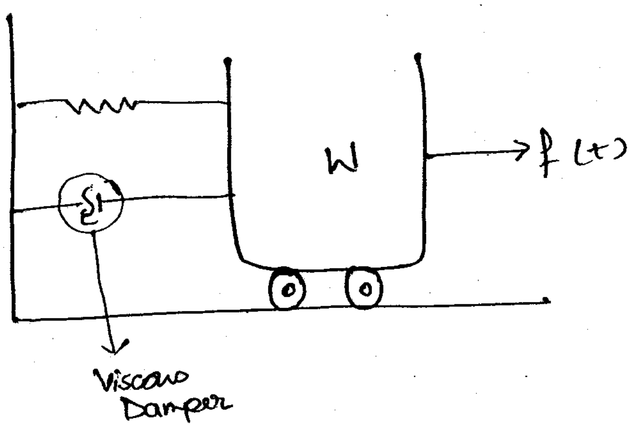
$\tau^2 \frac{d^2\bar{y}_s}{dt^2} + 2\zeta\tau \frac{d\bar{y}_s}{dt} + \bar{y}(s) = K_p \bar{F}(s)$

at steady state,

$\tau^2 s$

$$\boxed{\frac{Y(s)}{X(s)} = \frac{K_p}{\tau^2 s^2 + 2\zeta\tau s + 1}}$$

Example of second order system: —



$\frac{d^2 y}{dt^2}$

$$ma = f(t) - ky \qquad ; \text{Newton's law}$$

$\downarrow$

F_{Res}

$$F = \mu \frac{dV}{dy} A$$

$$F = \frac{\mu A}{y} V$$

$$F = CV$$

ζ = viscosity or resistance offered.

The second order system are catigorized into 3 category.

i> <u>Multi-capacity System</u>

Process when consist of 2 or more capacities in series form second order system ~~or more than 2~~

ii> <u>In-herently Second order System:-</u>

Such as fluid or mech solid components ~~of~~ of a process that posses inertia and are subjected to acceleration. Such systems are rare in chemical process industry. Some of the examples or

- ⇒ U tube manometer.
- ⇒ Externally mounted level Indicator.
- ⇒ Differential pressure transducer.
- ⇒ ~~Non~~ Pneumatic Valve.

iii> <u>A processing System wid its controller</u>

It may exhibit ~~or~~ second or higher order dynamics. in such cases the controller which has been installed on a processing unit introduces additional dynamics when coupled wid the dynamics of unit and give rise to second or higher order System.

<u>Dynamic Response of Second Order System:-</u>

$$\frac{Y(s)}{X(s)} = G(s) = \frac{K_p}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

$$Y(s) = \frac{K_p}{s\underbrace{(\tau^2 s^2 + 2\zeta\tau s + 1)}_{(s-P_1)(s-P_2)}}$$

Case - I , when $\zeta > 1$, System is over damped System

Case - II , when $\zeta = 1$, System is known as critically damped system

Case - III, when $\zeta < 1$, system is under damped System.

$$Y(s) = \frac{K_P}{s \underbrace{(\tau^2 s^2 + 2\zeta\tau s + 1)}_{(s-P_1)(s-P_2)}}$$

$$P_1 = \frac{-\zeta}{\tau} \pm \frac{\sqrt{\zeta^2 - 1}}{\tau}$$

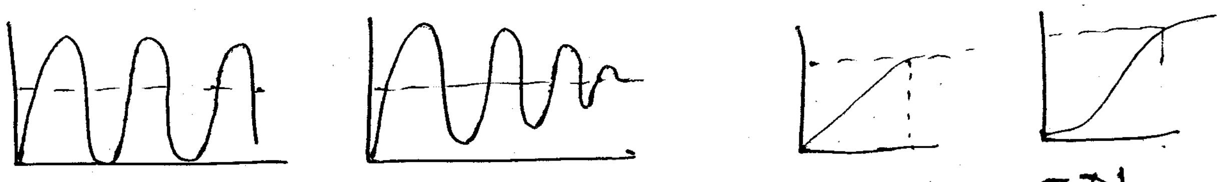
for over damped System, nature of roots are real & unequal

for critically damped System, nature of roots are real & equal.

for under damped System, nature of roots are complex

~~Two interact~~

In the higher order dynamics, system posses oscillation, for ex → the two tanks in series. Our aim is to reduce this oscillation, so that the system stays at the ultimate value. But, also system should behaves fastly. For killing oscillation. we use damping factor or the damping coeff. as ζ increases oscillation decreases and when it becomes 1. Oscillation vanish.

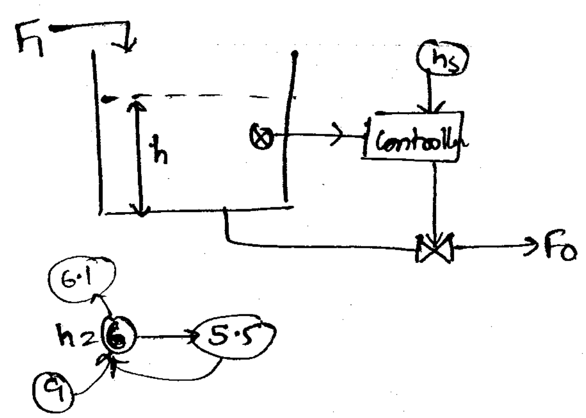


$\zeta = 0$ $\quad$ $0 < \zeta < 1$ $\quad$ $\zeta = 1$ $\quad$ $\zeta > 1$

if ζ becomes more than 1, the system becomes slow.

Tym take to reach the ultimate value increases as the resistance increase.

Carefully choose the value ζ, so that the system behaves as a critically damped system.



Oscillation minimise or remove so $\zeta = 1$.

Oct 31, 14

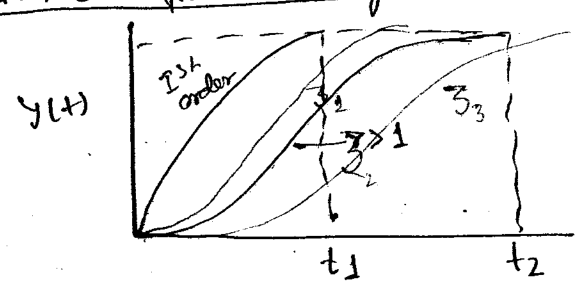
Overdamped Response (Non-Oscillatory but sluggish)

$$y(t) = K_p\left[1 - e^{-\zeta t/\tau}\left(\cosh\left(\frac{\sqrt{\zeta^2-1}}{\tau}\right)t + \frac{\zeta}{\sqrt{\zeta^2-1}}\sinh\left(\frac{\sqrt{\zeta^2-1}}{\tau}\right)t\right)\right]$$

$\zeta > 1$,

Roots are real & Unequal

Comparision b/w first order system & Second order system



$\zeta_3 > \zeta_3 > \zeta_1$

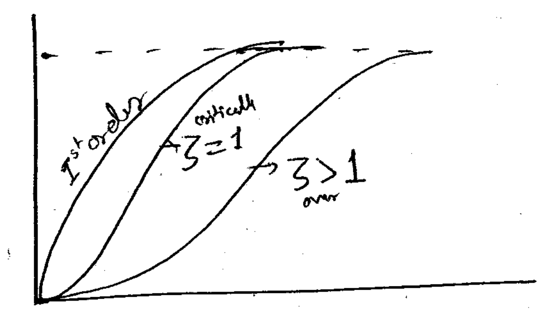
→ First order system behaves fast initially towards the same Change in the i/p and then becomes slow But in contrast Second order system behaves slowly in the initial tym and then gains speed.

→ As $\zeta \uparrow$, the process becomes more & more sluggish.

Critically damped System :-

Critically damped System are more oscillatory & faster than overdamped.

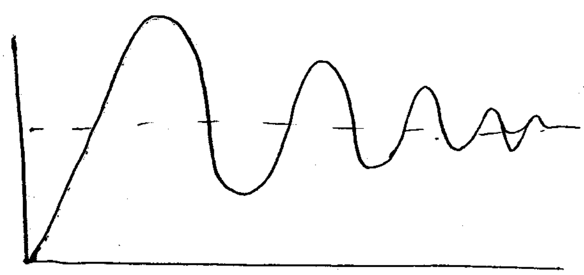
$$y(t) = K_p \left[1 - \left(1 + \frac{t}{\tau}\right) e^{-t/\tau}\right]$$



Under damped System :-

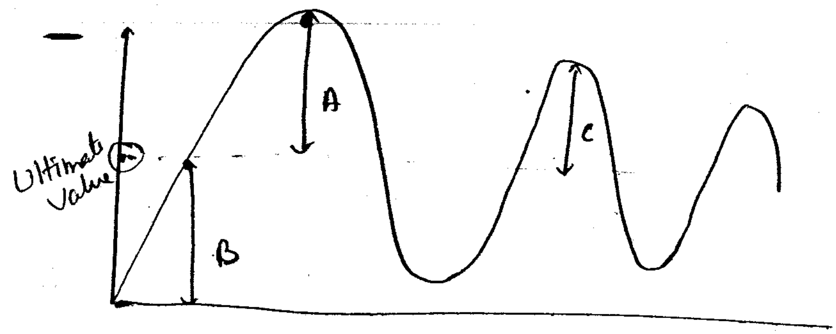
$$Y(t) = K_p \left[1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\frac{\zeta t}{\tau}} \sin\left(\frac{\sqrt{1-\zeta^2}}{\tau} t + \phi\right)\right]$$

$$\boxed{\omega = \frac{\sqrt{1-\zeta^2}}{\tau}}$$



} oscillatory.

Study of underdamped System :-



Overshoot :-

It is defined as the ratio of A/B, where B is the ultimate value of the response and A is the maxm amt by which the response ~~it~~ exceeds its ultimate value

$$\text{Overshoot} = A/B = \exp\left(\frac{-\pi \zeta}{\sqrt{1-\zeta^2}}\right)$$

it shud be low as we need less amt of oscillation

By increasing the value of ζ, overshoot decreases
at $\zeta = 1$, the overshoot approches 0.

Decay Ratio

It is defined as the ratio of any two successive peaks above the ultimate value.

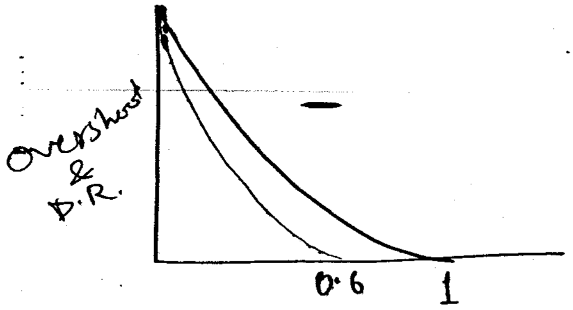
$$\text{Decay ratio} = C/A$$

$$= \left(\exp\left(\frac{-2\pi \zeta}{\sqrt{1-\zeta^2}}\right)\right)^2$$

Decay ratio remains same, if ζ is constant.
if $\zeta \uparrow$, decay ratio decreases.

and decay ratio $= (\text{overshoot})^2$

decay ratio approches zero almost at $\zeta = 0.6$



Natural period of Oscillation

$$\boxed{\omega_n = 1/\tau}$$

ω_n = natural frequency when damping is zero

at $\zeta = 0$, the system becomes fully oscillatory or we can say that it is free of any kind of damping and frequncy of

~~bod~~

$$\omega_n = 1/\tau$$

$$\omega_n = 2\pi f_n = 2\pi \, 1/T_n$$

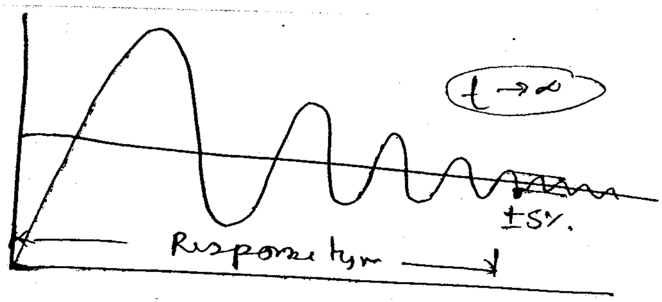
$$\boxed{T_n = 2\pi\tau}$$ Tym period of oscillation.

Resonant or damped period of Oscillation :-

$$\boxed{\omega_d = \frac{\sqrt{1-\zeta^2}}{\tau}}$$

$$\boxed{T_d = \frac{2\pi\tau}{\sqrt{1-\zeta^2}}}$$ Tym period of oscillation

Response tym :-



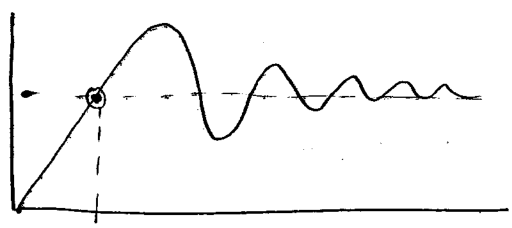
The response of an underdamp system, will reach its ultimate value in an oscillatory manner in tym $t \to \infty$

For practical purposes, it has been agreed to consider that the response its final value when it came wid in ±5% of the final steady state value and stays there.

" The tym needed for the response to reach this situation is known as the Response tym "

it shud be as low as possible
as ζ↑, response tym decreases

Rise tym :-



It is defined as the tym required for the response to reach its final value for the first tym.

If we rise the value ζ , response tym ↓

Q) A step change of magnitude 4 is introduced in a system having transfer function $\frac{10}{S^2+1.6S+4}$. ~~find~~ Calculate the value of overshoot & decay ration.

$K_p = 10 \qquad K_p = \frac{10}{4} =$

Sol $\quad \tau =$

$$2\zeta\tau = \frac{1.6}{4}$$

$$\tau^2 = 1/4 \qquad \tau = 0.5$$

$$2\zeta\tau = \frac{1.6}{4}$$

$$\zeta = \frac{1.6}{4\times0.5\times2}$$

$$= 0.4$$

$$\zeta = \cancel{\frac{0.8\ 1.6}{4\times2}} =$$

$$\exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right)$$

Overshoot $= -\cancel{4.188}, -\cancel{1.371} = 0.253$

decay ratio. $= \cancel{17.54}\ \cancel{1.879} = 0.0644$

Q) Find the maxm value of respons $(A+B)$?

$$\boxed{\cancel{\frac{A}{B} = -1.371 \qquad A = -1.371B}}$$

step change given in qustn

$$Y(s) = \frac{K_p}{\tau^2 s^2 + 2\zeta\tau s + 1} \cdot \frac{4}{s}$$

Ultimate value $\quad \lim_{t\to\infty} Y(s) = \lim_{s\to 0} sY(s)$

$$= 4K_p$$

$$= 10$$

$$\boxed{\text{Ultimate value} = \frac{A+B}{B} \times B}$$

$$= \cancel{\left(1 + B/A\right)} A \quad \boxed{\text{Ultimate value} = \left(1 + A/B\right) B}$$

$$= \quad 12.53$$

$$\boxed{\text{Ultimate Value} = AK_p(1 + \text{Overshoot})}$$

A = step change

K_p = steady state gain

Behaviour of Second Order system over impulse change :—

Impulse

$$Y(s) = \frac{K_p}{\tau^2 s^2 + 2\zeta\tau s + 1} \cdot 1$$

Step

$$Y(s) = \frac{K_p}{\tau^2 s^2 + 2\zeta\tau s + 1} \cdot \frac{1}{s}$$

$X(s) = 1$

$$Y(s)\Big|_{\text{Impulse}} = sY(s)\Big|_{\text{Step}}$$

taking laplace

$$\boxed{Y(t)\Big|_{\text{Impulse}} = \frac{dY(t)}{dt}\Big|_{\text{Step}}}$$

Behaviour of System towards Sinusoidal change :—

$$X(s) = \frac{\omega}{s^2 + \omega^2}$$

$$Y(s) = \frac{K_p}{(s+i\omega)(s-i\omega)(s-P_1)(s-P_2)}$$

here there are four factors

$$Y(t) = C_1 \cos \omega t + C_2 \sin \omega t + e^{-\zeta t/\tau}\left\{C_3 \cos\left(\frac{\sqrt{1-\zeta^2}}{\tau}\right)t + C_4 \sin\left(\frac{\sqrt{1-\zeta^2}}{\tau}\right)t\right\}$$

$$\boxed{Y(t)\Big|_{\text{Ultimate}} = \gamma \sin(\omega t + \phi)}$$

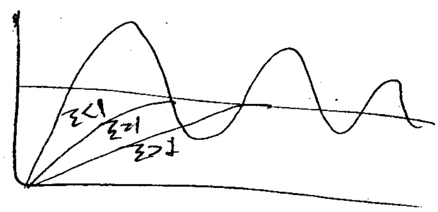
$$\Rightarrow \quad \gamma = \frac{A}{\sqrt{(1-\tau^2\omega^2)^2 + (2\zeta\tau\omega)^2}}$$

$$\Rightarrow \quad \phi = \tan^{-1}\left(\frac{-2\zeta\tau\omega}{1-\omega^2\tau^2}\right)$$

between the range of $[0, \pi]$

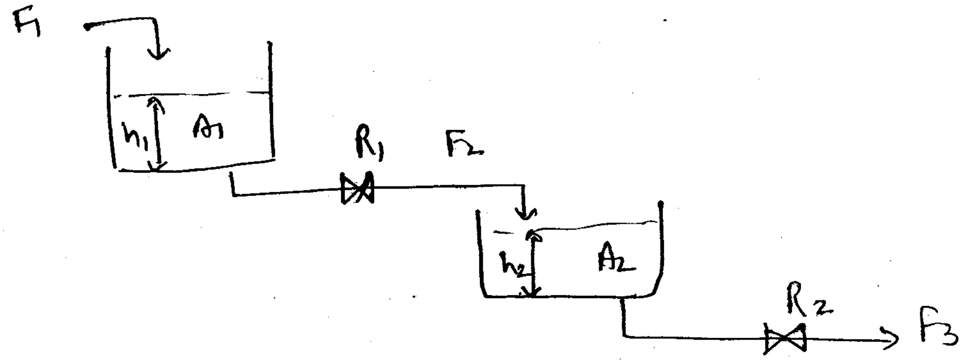
$$AR = \frac{1}{\sqrt{(1-\tau^2\omega^2)^2 + (2\zeta\tau\omega)^2}}$$

$\left.\begin{array}{l} AR < 1 \\ AR > 1 \end{array}\right\}$ depend on the values of ζ, τ, ω



Response of first order System in Series :-

Non-Interacting Systems / Capacitors.



Mass balance.

$$F_1 - F_2 = A_1\, dh_1/dt$$

$$F_2 - F_3 = A_2\, dh_2/dt$$

$$F_2 = h_1/R_1 \quad , \quad F_3 = h_2/R_2$$

$$\frac{\overline{h_1(s)}}{\overline{F_1(s)}} = \frac{R_1}{1+\tau_1 s}$$

$$\frac{\overline{F_2(s)}}{\overline{F_1(s)}} = \frac{1}{1+\tau_1 s}$$

$$\frac{\overline{h_2(s)}}{\overline{F_2(s)}} = \frac{R_2}{1+\tau_2 s}$$

$$\frac{\overline{F_3(s)}}{\overline{F_2(s)}} = \frac{1}{1+\tau_2 s}$$

$$\frac{\overline{h_2(s)}}{\overline{F_2(s)}} \times \frac{\overline{F_2(s)}}{\overline{F_1(s)}}$$

$$\frac{\overline{h_2(s)}}{\overline{F_1(s)}} = \frac{R_2}{1+\tau_2 s} \times \frac{1}{1+\tau_1 s}$$

$$\frac{\overline{h_2(s)}}{\overline{F_1(s)}} = \frac{R_2}{(1+\tau_1 s)(1+\tau_2 s)}$$

$$= \frac{R_2}{\tau_1\tau_2 s^2 + (\tau_1+\tau_2)s + 1}$$

Comparing wid general transfer function.

$$\boxed{K_p = R_2}$$

$$\tau^2 = \tau_1 \tau_2$$

$$\boxed{\tau_{eff} = \sqrt{\tau_1 \tau_2}}$$

$$2\zeta\tau = \tau_1 + \tau_2$$

$$\boxed{\zeta = \frac{2(AM)}{GM}}$$

$$\zeta = \frac{\tau_1+\tau_2}{2\sqrt{\tau_1+\tau_2}}$$

if $\tau_1 = \tau_2 = \tau$,

$$\boxed{\tau_{eff} = \tau}$$

$$\frac{R_2}{\tau_1\tau_2 s + (\tau_1+\tau_2)s + 1}$$

find roots.

$$= \frac{-(\tau_1+\tau_2) \pm \sqrt{(\tau_1+\tau_2)^2 - 4.\tau_1\tau_2}}{2\tau_1\tau_2}$$

$$= \frac{-(\tau_1+\tau_2) \pm \sqrt{\tau_1^2+\tau_2^2+2\tau_1\tau_2-4\tau_1\tau_2}}{2\tau_1\tau_2}$$

$$= \frac{-(\tau_1+\tau_2) \pm (\tau_1-\tau_2)}{2\tau_1\tau_2}$$

-ve

$$\frac{-\tau_1-\cancel{\tau_2}-\tau_1+\cancel{\tau_2}}{2\tau_1\tau_2} = \frac{-\cancel{2\tau_1}}{\cancel{2\tau_1}\tau_2} = -\frac{1}{\tau_2}$$

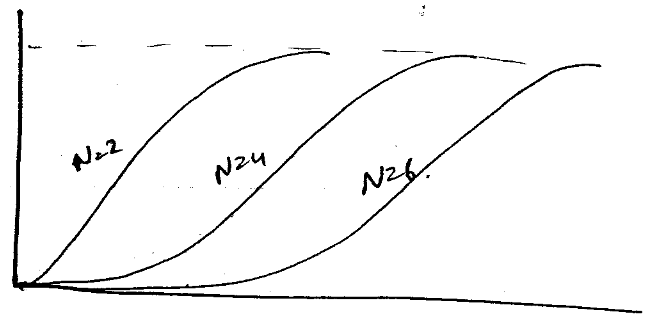
+ve

$$\frac{-\cancel{\tau_1}-\tau_2+\cancel{\tau_1}-\tau_2}{2\tau_1\tau_2} = -\frac{1}{\tau_1}$$

roots $\Rightarrow$ $-\frac{1}{\tau_1}$ ~~or~~ $\frac{-1}{\tau_2}$

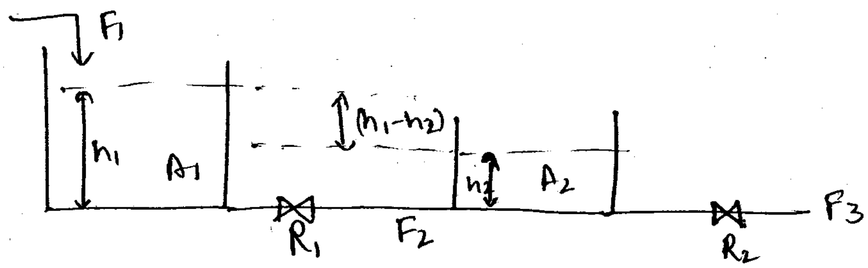
If $\overset{(N)}{n}$ capacities are in series.

$$G_M(s) = G_1(s)\, G_2(s) \, \ldots \ldots \, G_N(s)$$



as the no. of capacities increases, the response becomes more sluggish

Interacting System / capacities :—



If second is not smaller than first one, then back flow takes place.

$$F_2 = (h_1 - h_2)/R_1$$

$$F_1 - F_2 = A_1\, dh_1/dt$$

$$F_2 - F_3 = A_2\, dh_2/dt$$

$$F_3 = h_2/R_2$$

$$F_1(s) - F_2(s) = A_1\, s\bar{h}_1(s)$$

$$F_2(s) - F_3(s) = A_2\, s\bar{h}_2(s)$$

$$F_2(s) = \frac{\bar{h}_1(s) - \bar{h}_2(s)}{R_1}$$

$$F_3(s) = \frac{\bar{h}_2(s)}{R_2}$$

$$F_3(s) = \frac{\bar{h}_2(s)}{R_2}$$

$$\frac{\bar{h}_2(s)}{F_1(s)}$$

$$F_2(s) - A_2 s \bar{h}_2(s) = \frac{\bar{h}_2(s)}{R_2}$$

$$F_2(s) = \bar{h}_2(s)\left(\frac{1}{R_2} + A_2 s\right)$$

$$\frac{R_2}{\tau_1\tau_2 s^2 + (\tau_1+\tau_2+AR_2)s + 1}$$

The difference $A_1 R_2 \rightarrow$ Interaction factor.

Nov 03,14

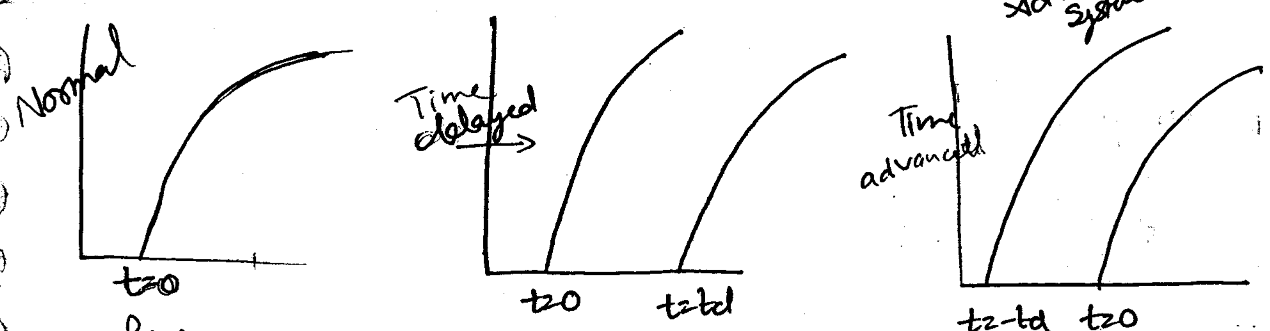
Control System:–

In the previous part our aim is not to control the process So that the system would response in a specific manner or in other words, we are not interested in controlling the behaviour of the process.

But now our main concern is to control the process, so that it exhibit a desired response.

Dead Tym :–

Practically not possible



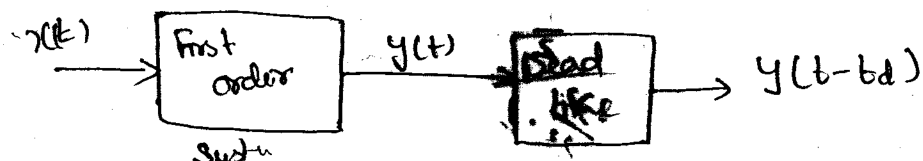
$f(t)$ $t=0,\ f(0)$

Dead time System: $f(t-t_d)$, $t=t_d$, $f(0)$

$f(t+t_d)$, $t=-t_d$, $f(0)$

$$L.T \text{ of } f(t-t_d) = e^{-t_d s}\bar{f}(s)$$

Second shifting prop: $f(t-a) = e^{-as}\bar{f}(s)$



When i/p is given till some time o/p is not observed is known dead tym.

For first order

$$L\left\{\frac{y(t)}{x(t)}\right\} = \frac{K_p}{1+\tau_p s}$$

$$L\left\{\frac{y(t-t_d)}{y(t)}\right\} = e^{-t_d s}$$

$$\therefore L\left\{\frac{y(t-t_d)}{x(t)}\right\} = \frac{K_p e^{-t_d s}}{1+\tau_p s}$$

$$\boxed{L\left\{\frac{y(t-t_d)}{x(t)}\right\} = \frac{K_p e^{-t_s}}{1+\tau_p s}}$$

as order ↑, dead tym ↑, dead tym unstablize the syst

for Second order System :—

$$\boxed{G(s) = \frac{K_p e^{-t_d s}}{\tau^2 s^2 + 2\zeta\tau s + 1}}$$

$$e^{-t_d s} = \frac{e^{-t_d s/2}}{e^{t_d s/2}} = \frac{1 - \frac{t_d s}{2} + \frac{(t_d s)^2}{2^2} \cdot \frac{1}{\underline{2}}}{1 + \frac{t_d s}{2} + \frac{(t_d s)^2}{2^2} \cdot \frac{1}{2} + \dots}$$

approximation

$$e^x = 1 + x + \frac{x^2}{\underline{2}} + \frac{x^3}{\underline{3}}$$

$$e^{-t_d s} = \frac{1 - t_d s/2}{1 + t_d s/2}$$

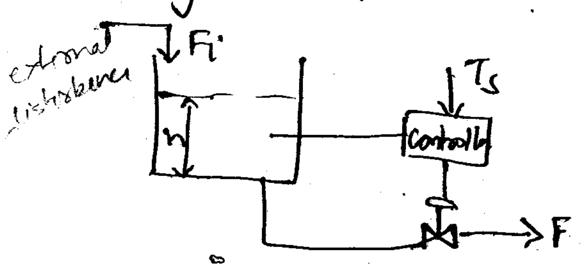
Padé's approximation for first order.

$$e^{-t_d s} = \frac{1 - t_d s/2 + \frac{(t_d s)^2}{2^2} \cdot \frac{1}{2}}{1 + t_d s/2 + \frac{(t_d s)^2}{2^2} \cdot \frac{1}{2}}$$

Second order system take terms upto square.

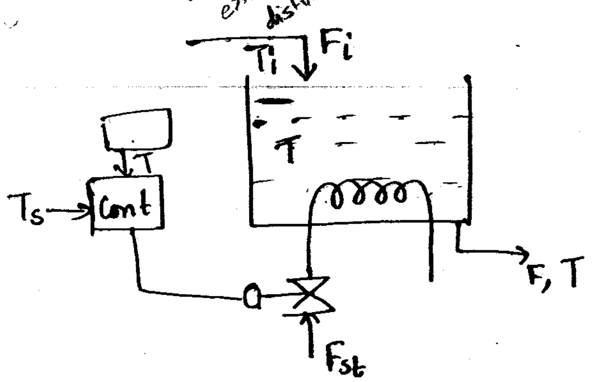
Need of the control system :—

⇒ Supressing the influence of external disturbence



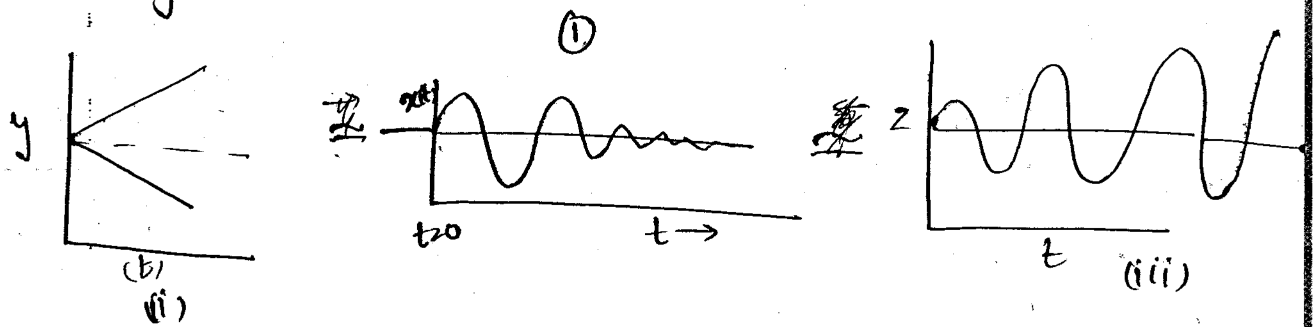
here f_i is external disturbance so the value of f_i changes, h changes so we have to set the value of controller.

or Mass
Energy Capacity System



$T = T_s$

⇒ Ensuring the stability of the process



Consider the behaviour of process at tym t=0, the constant value of x is disturbed, by some external factors. bt as the tym progresses the value of x returns to the initial value and stays there. These process are stable processes and needs no external intervention to control them.

Bt in (ii) &(iii) graph, doesn't returns to its initial value after it is disturbed by external influence. These processes are unstable processes and needs control action to stabalize them.

⇒ Optimising the performance

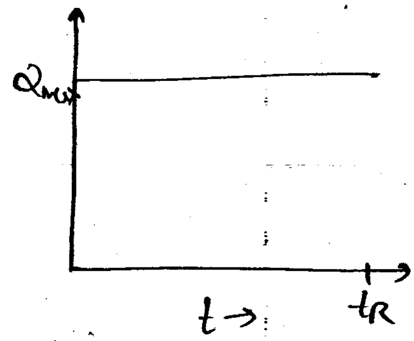
$A \to B \to C$ → undesired.
↓
Desired

both
Endothermic

total = Prod. Cost + Steam cost ; Profit = Revenue − (Total cost)

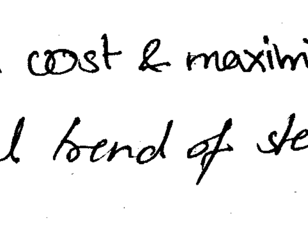
To maximise the profit the variable that we can change is the steam flow rate.

» if the steam flow rate is highes i.e Q_{max} throughout. The entires period of rxn t_R. The temp of the reacting mixture will be high.



initially when the concentration of A is large, we will have high yield of B, bt will also pay more for the steam. Also as tym goes on, C will be produced in good amt.

» if the steam flow rate is kept is at its lowest i.e $q=0$. We will have no cost of steam bt also no production B.



To optimise the cost & maximize the profit
the general trend of steam flow rate

∴ a controll system is need to compute the best steam flow rate for every tym during the rxn period.

Such problems are known as optimal control problems.

Types of Variables

I/p Variable

They denote the effect of surroundings on chemical process.

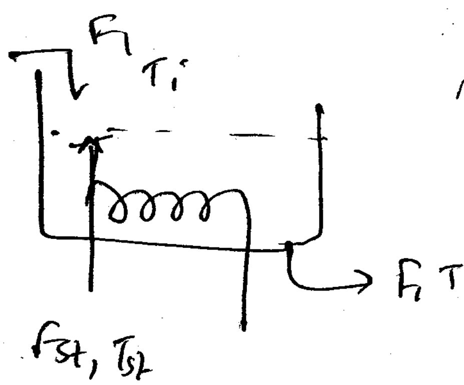
F_i, T_i, F_{st}, T_{st}

O/p Variable

They. Denote the effect of process on the surrounding.

F, T

Mass - Energy - Capacity



$, F_{st}, T_{st}, F_i, T_i$

manipulated { Take any which u can control the system and rest are disturbance.

Categories of Input Variable

- Manipulated
- Disturbances.

Manipulated

They are those variables whose value can be adjusted freely by the human operator or by a control mechanism. Therefore they are also known as ADJUSTABLE VARIABLES

Disturbances

They are those variables whose value can n't be adjusted freely by the human operator or by a control mech. ∴ They are also known as NON-ADJUSTABLE VARIABLES

Category of O/P
↳ Measured
↳ Unmeasured

Measured :-
If there value are known by directly measuring them.
for ex → Temp.

Unmeasured :-
If there value ~~are kn~~ are not measured directly
for ex → Concentration of distillate in the distillation column.

Design Elements of a Control System :—

Step - I

What are the operational objectives that a control system is called upon to achieve.

$T = T_s$; $h = h_s$; $C_A = C_{As}$

$T = T_s$
$h = h_s$
$C_A = C_{As}$

objectives which are to be controlled is known as controlled variable.

Step - II

What variable should we measure, in order to monitor the operating performance of a plant.

$T = T_s$ | to measure or maintain the temp const
$h = h_s$ |
$C_A = C_{As}$ | measure any third quan...

To keep the temp & level at desired value we will measure the temp and level of system by the help of thermocouple & level analyser. But sometimes the o/p are unmeasured. & Concentraty

In such cases, we must measure other variables that can be measured easily and reliably. Such supporting measurements are known as secondary measurement. Then we develop some mathematical relationship b/w the unmeasured o/p and secondary measurement.

$$\text{Unmeasured o/p} = f(\text{Secondary Measurement})$$

for ex → In case of distillation, the concn of distillate is nt measured wid reliablity. and therefore we measure the temp. of some selected trays in rectifying section and wid the help of them, calculate the value of distillate concentration.

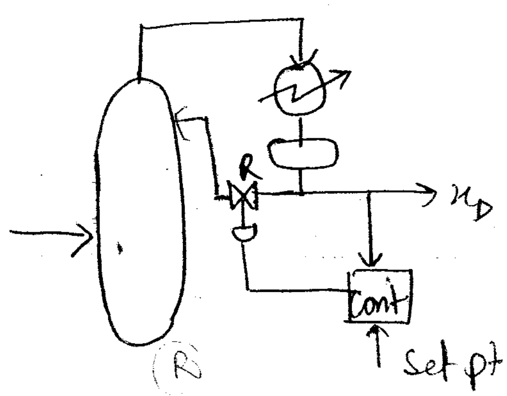
Measured Variable

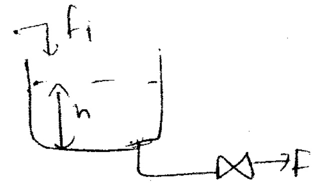
Variable whic we measure.

T, h, C_A

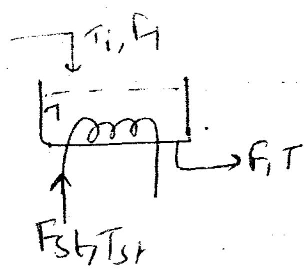
Step-III

What are the manipulated variables to be used to control the Chemical process.





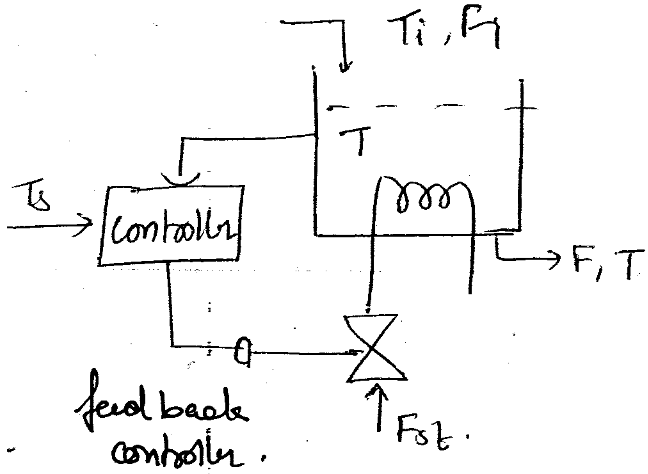
to maintain the o/p. what should be maintained



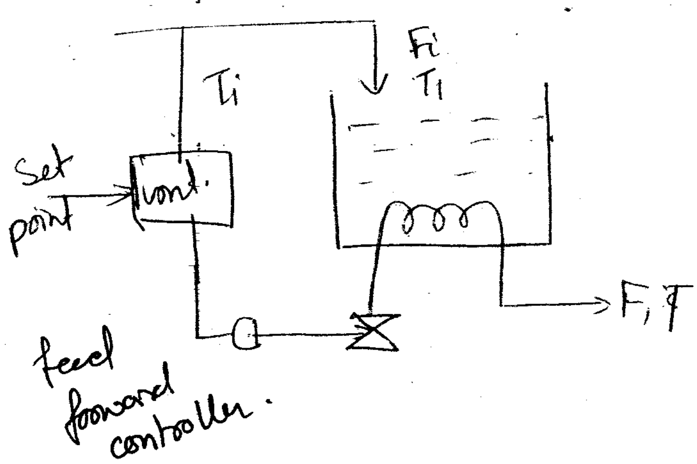
Step-IV

Choosing the best controlled configuration for a given chemical process controlled situation. —

Pehle system,



Disturbence has its effect on the system, after that the controller takes the control action. O/p signal is feed back to the controller and ∴ these controlled configurations is knows as feed back controlled configuration.

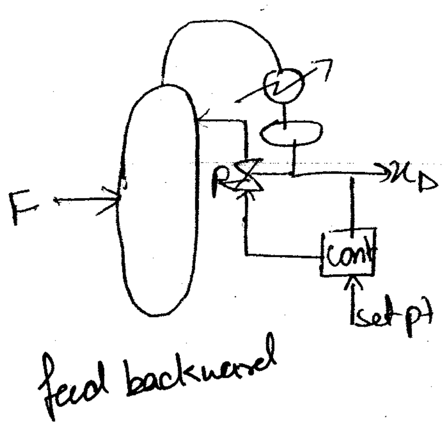


Disturbence has no effect on system, as any disturbence is introduce, error is read by controller, the temp of system decreas ~~or~~ increases as acc. to need.

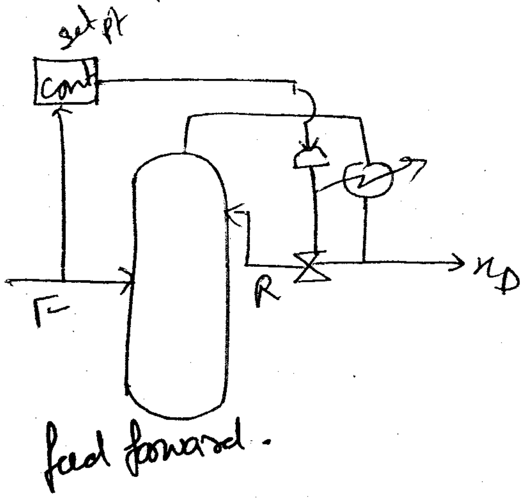
$$T_{is} - T_i = \varepsilon = 0$$

$T_{is} - T_1 = \varepsilon > 0$ increase F_{st} (open)

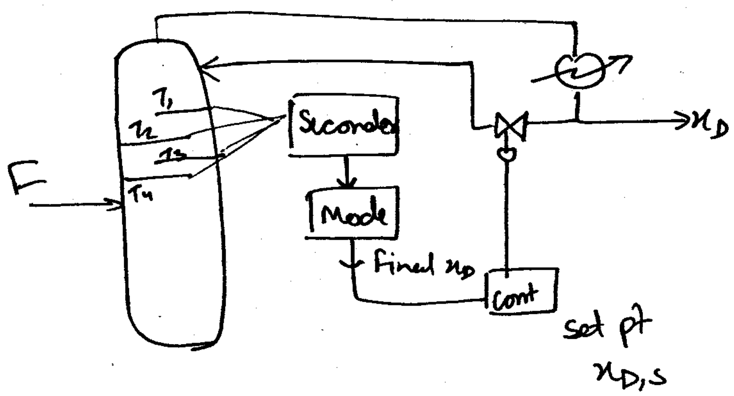
$T_{is} - T_1 = \varepsilon < 0$ decrease F_{st} (Closed, Feed forward controlled configuration has its advantage over feed backward cont. that disturbence has no effect on system but it can be employed only when the disturbences are measurable



feed backward



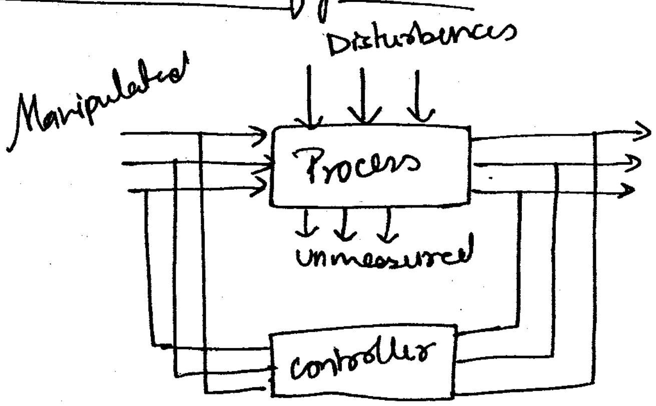
feed forward.



used in cases when the o/p or i/p is unmeasured
or
measure some second i/p or o/p & convert them to final 1s.

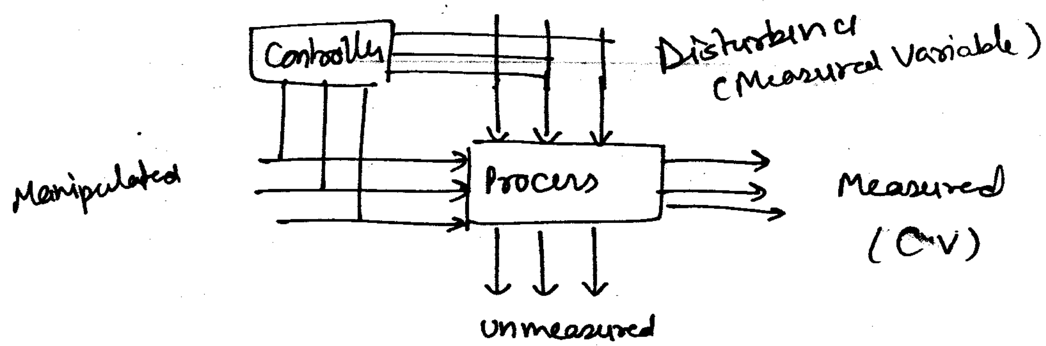
Inferential control scheme

Feed back Control Configuration

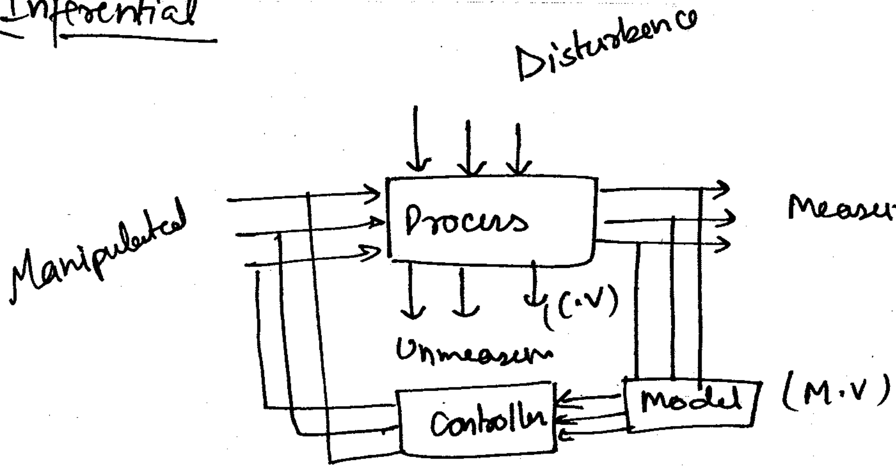


Measured
(Control variable (C.V)
(M.V)

Feed forward Controller



Inferential



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Step V

How is the info taken from the measurement used to adjust the values of manipulated variable.

It is given by the control law. It may be proportional, integral or the derivative controller or may be the combination of two or more.

Hardware Elements of a Control System

1) Chemical Process :-

2) Measuring Instruments or Sensors

Temperature

→ Thermocouple low tem.

→ Resistance thermometer

→ Bimetallic "

→ Radiation Pyrometer 525°C high temp

Pressure

→ Mano meter

→ Bourdon-tube elements

→ Bellows

→ Diaphragm elements

→ pizoelectric elements

Flow

→ orifice plates

→ Venturi flow nozzle

→ Turbine flow meter

→ Hot wire Anemometry.

Liquid level

→ Displacer devices

→ Float-Actuated devices

→ Conductivity measurements

→ Sonic Resonance

→ Dielectric Measurements.

Composition

→ Chromatographic Analyser

→ Infrared Analyser

→ ultraviolet Analyser

→ Potentiometry Analyser

→ PH meter Analyser

→ Spectrometers.

→ Coulometers Analyser.

⇒ Transducer :-

Many measurements cann't be used for control untill they are converted to physical quantities. such as electrical voltage or current known as electrical signal or compressed air or liquid, known as the pneumatic signal, which can be transmitted easily and transducer is used for this purpose.

⇒ Transmission lines :-

These are used to carry the measurement signal to the controller and from the controller to the final control element. They can be pneumatic or electrical.

⇒ Controller

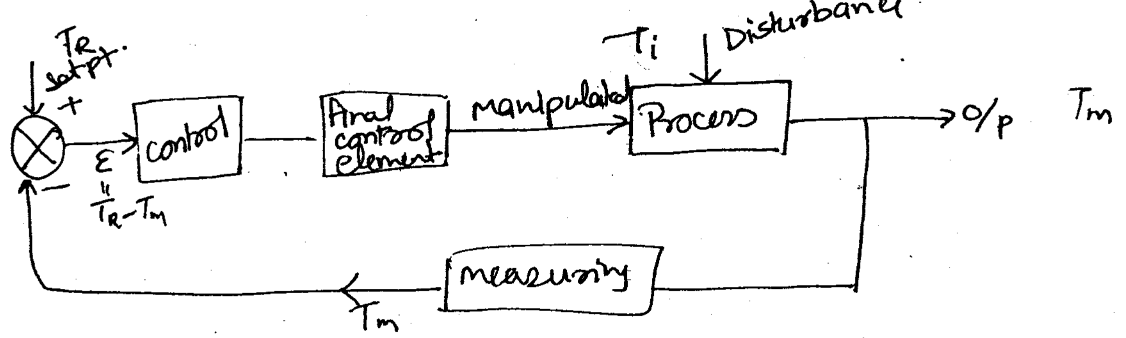
This the hardware element that has the intelligence. It recieve the information from measuring device and decides what action should be taken. (CPU)

⇒ Final Control Element :-

This the hardware element that impliment in real lyf the desision taken by the controller. for ex → valve.

⇒ Recording Elements :-

These are used to provide a visual demonstration of how a chemical process behaves.



T_R = Set point (desired value of a controlled variable)

T_i = ~~load Variable~~

load refers to a change in any variable that may cause the controlled variable of the process to change.

-ve feedback & +ve feed back :-

When we are using the diff. of the set value and the measured value, then it is known as -ve feed back control, which is desired in the chemical process industry bcz study of deviation is imp.tous

On the other hand, if i use the set pt + Measured value ($T_R + T_m$) Then it is known as +ve feed back control, which makes the process ~~uncont~~ unstable and is not desired in the chemical process industry.

Servo & Regulatory

Servo Problem:-

There is no change in the load bt the set pt, will change

not used generally as we do need not to change set pt

for ex → optimal control problem.

Regulatory Problem:-

The set pt T_r, remains the same and the load T_i changes.

Generally in chem. process industry, we want our process to remain at the steady state (set pt remains the same) and want to minimise the affect of disturbence (load changes)

Measurement Element lag :-

Measuring element may exhibit some lag

for ex → In mercury thermometer we observe the lag to be first order

$$\boxed{\frac{Y(s)}{X(s)} = \frac{1}{1+\tau_m s}}$$

Controllers :-

Various types of continuous feed back controller differs in the way, they relate C(t) wid E(t)

$$\varepsilon(t) \longrightarrow \boxed{\text{Cont}} \longrightarrow C(t)$$

Proportional Controller:-

It is the simplest type of controller, it produces an o/p signal proportional to the error signal.

$$\boxed{P = \underbrace{K_c}_{\text{controller gain}} \varepsilon + P_s} \quad \text{—(i)}$$

$$P_s = K_c \varepsilon_s + P_s \quad \text{—(ii)}$$

i−(ii)

$$P - P_s = K_c(\varepsilon - \varepsilon_s) + (P_s - P_s)$$

$$\bar{P} = K_c \bar{\varepsilon}$$

$$\boxed{\frac{\bar{P}(s)}{\bar{\varepsilon}(s)} = K_c}$$

⇒ if K_c is large, then small error signal will cause a large change in o/p & vice versa.

for the proportional controller to do the control action K_c shud be large.

The proportional controller doesn't have any kinetics, so if we add this P action in series wid the first order system, then kinetics of the whole system remains same.

On-off controller

It is a sp. case of proportional controller, which is generally employed in home heating system or domestic water heaters.

In domestic system we want the process to behave either at the maxm or at the minimum. So for that purpose we keep the K_c of controller very-very high (∞) or the system is very sensitive. When we switch on the process, the +ve deviation enters the system and bcz K_c ~~too~~ is infinite, the process is goes at its maxm & vice versa.

Proportional band :

It characterizes the range over which error must change in order to drive the actuating signal of the controller over its full range.

if $K_c \rightarrow \infty$

$P_B \rightarrow 0$

$$\boxed{K_c \propto 1/P_B}$$

$$\boxed{P_B = 100/K_c \text{ in } \%}$$

Proportional Integral Controller:-

$$\boxed{P = K_c\varepsilon + \frac{K_c}{\tau_I}\int_0^t \varepsilon dt + P_s}$$

bias value it may be zero or anything

τ_I = Integral tym or Reset tym

$$\bar{P} = K_c\bar{\varepsilon} + \frac{K_c}{\tau_I}\int_0^t \bar{\varepsilon}\, dt$$; in terms of deviation variable

taking laplace

$$\boxed{\frac{\bar{P}(s)}{\bar{\varepsilon}(s)} = K_c\left(1 + \frac{1}{\tau_I s}\right)}$$

Division result integration by s

$$L\left\{\int_0^t f(t)\,dt\right\} = \frac{\bar{f}(s)}{s}$$

It has kinetics of first order, when joined in series wid first order system the overall kinetics increases by 1, which makes the process sluggish.

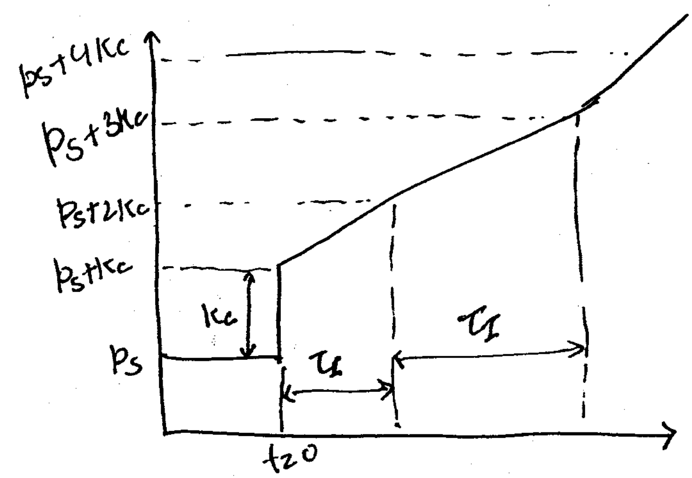
for small deviation we can't put kc large

if the error value is small

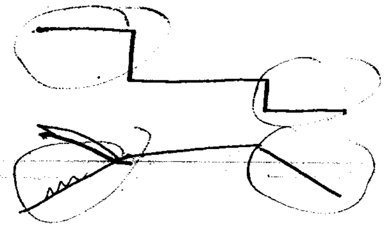
- Keep the kc large and use only P action. Bt large value of kc will make the system sensitive and after some pt. of tym if the deviation increases then the process may become unstable.
- Instead of keep your kc high, use I action in series wid P action. For the small error deviation, I action increases the proportional band which when multiplied by wid the optimal value of kc gives the optimal o/p or desired o/p. And also there is no risk ~~for~~ of unstable process as K_p is less.

By using the I action in series wid P action, the response becomes slow as the kinetics increases and therefore I action is the slowest action.

when error deviation is step i/p



we need our first P action takes place the I as P is fast & I is slow. we need our τ_I to be small



integral
Change step → RAMP

τ_I = Reset tym is the tym needed by the controller to repeat the initial proportional action change in its o/p.

Reset Rate :-

Inverse of reset tym or frequency.

Proportional Derivative Controller:-

$$\boxed{P = K_c \varepsilon + K_c \tau_d \frac{d\varepsilon}{dt} + P_s}$$

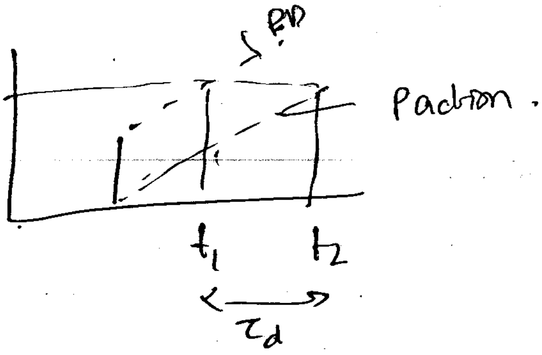
∗ ⇒ S in denominator
⇒ S in numerator increase the kinetic

$$\boxed{\frac{P(s)}{\varepsilon(s)} = K_c(1+\tau_d s)}$$

D action doesn't repeat the kinetics lyk the P action. D action work on the rate of change of error

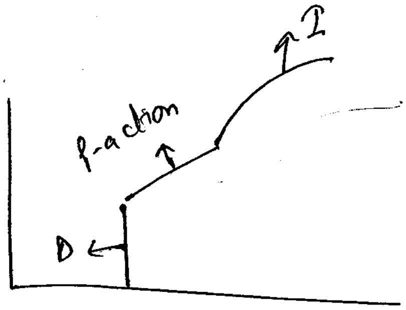
D action gives the rate of change of error and it tells us what should be the next i/p to the controller ~~for~~ Or in other words it anticipates the error and therefore this action is known as anticipatory action.

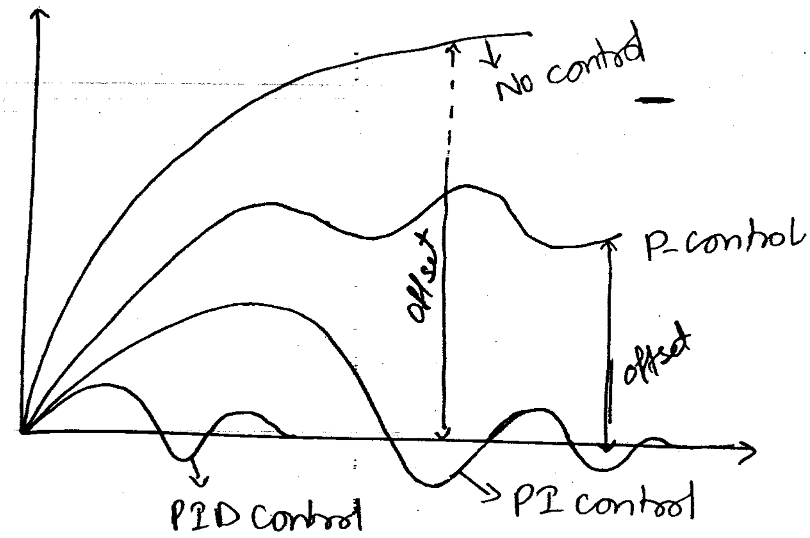
disadvant D action converts the RAMP i/p to the step i/p which in turn increases the response of the system. But for the step i/p the derivative is zero & bcz of PD action work only as the P action. D controller can't be used alone



D → fastest
I → slowest

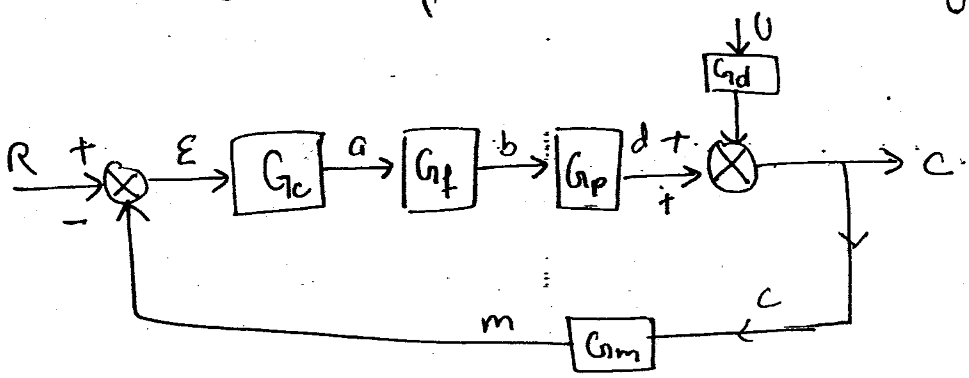
for Ramp I/p





PD controller is not used as for step change differ is zero

Block diagram of generalised closed loop System:-



for Servo ⇒ $U = 0$, C/R

for Regulatory ⇒ $R = 0$, C/U

$$a = \varepsilon G_c$$
$$b = a G_f$$
$$C = b G_p$$
$$m = C G_m$$
$$\varepsilon = R - m$$

$$C = b G_p$$
$$C = a G_f G_p$$
$$C = \varepsilon G_c G_f G_p$$

$$R = \varepsilon - m$$

$$\frac{C}{R} = \frac{b G_p}{\varepsilon - m}$$
$$= \frac{b G_p}{\frac{a}{G_c} - C G_m}$$
$$= \frac{b G_p}{\frac{a}{G_c} - b G_p G_m}$$
$$= \frac{a G_f}{\frac{a}{G_c} - a G_f G_p G_m}$$
$$= \frac{G_f G_c}{1 - G_f G_p G_m G_c}$$

$$C = \varepsilon G_c G_f G_p$$

$$C = (R - m) G_c G_f G_p$$

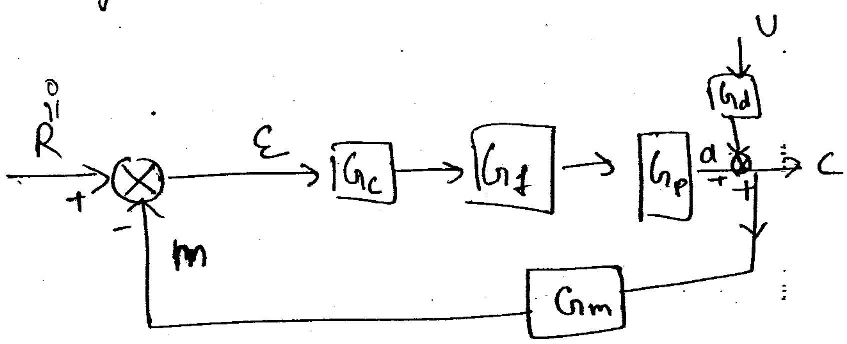
$$C = (R - C G_m) G_c G_f G_p$$

$$C[1 + G_m G_c G_f G_p] = R\, G_c G_f G_p$$

$$\boxed{\frac{C}{R} = \frac{G_c G_f G_p}{1 + G_c G_f G_p G_m}}$$

$C = (R - \underset{C G_m}{\underset{\downarrow}{m}})\, G_c G_f G_p$

for Regulatory Mode



$R = 0, \quad C/U$

$$C = d + U G_d$$

$$d = b G_p$$

$$b = a G_f$$

$$a = \varepsilon G_c$$

$$\varepsilon = (0 - m)$$

$$m = C G_m$$

$$C = b G_p + U G_d$$

$$= a G_f G_p + U G_d$$

$$C = -C G_m G_c G_f G_p + U G_d$$

$$C[1 + G_m G_c G_f G_p] = U G_d$$

$$\boxed{\frac{C}{U} = \frac{G_d}{1+G_cG_fG_pG_m}}$$

Observations

⇒ The denominator of closed loop transfer function is 1 + (multiplication of all the transfer funct^n in the loop)

$$1 + G_{OLTF}$$; OLTF = overall Transf. funct^n

$$1 + \Pi_{Loop}$$; π = multiplication

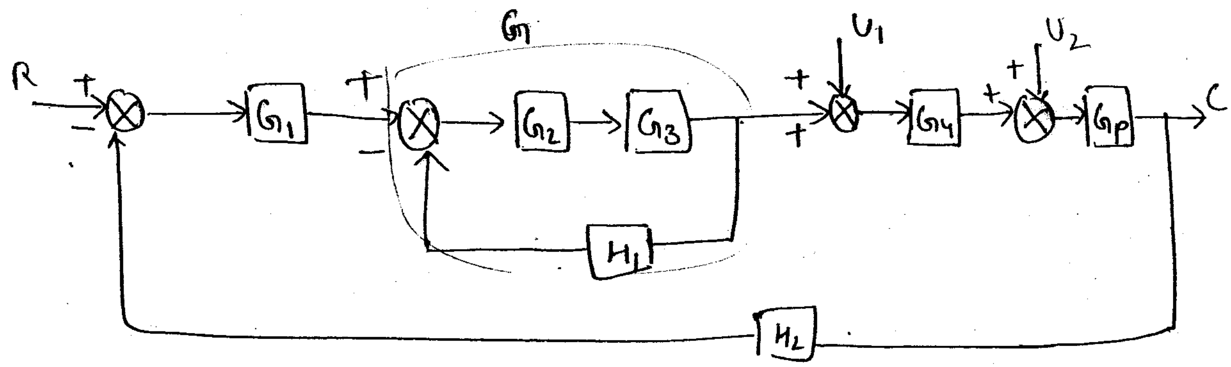
This is Characteristic eq^n the roots of which gives info. abt stability.

⇒ The numerator of both the transfer function is also under the same concept. i.e (transfer function between the o/p & i/p)

$\Pi_{Forward}$ = multiplication of all the transfer funct^n in forward path btw o/p & i/p

$$\boxed{G(s) = \frac{\Pi_{Forward}}{1+\Pi_{Loop}}}$$

Q.)



$$\frac{C}{R}, \frac{C}{U_1}, \frac{C}{U_2}$$

$$G = \frac{G_2 G_3}{1 + G_2 G_3 H_1}$$

i)

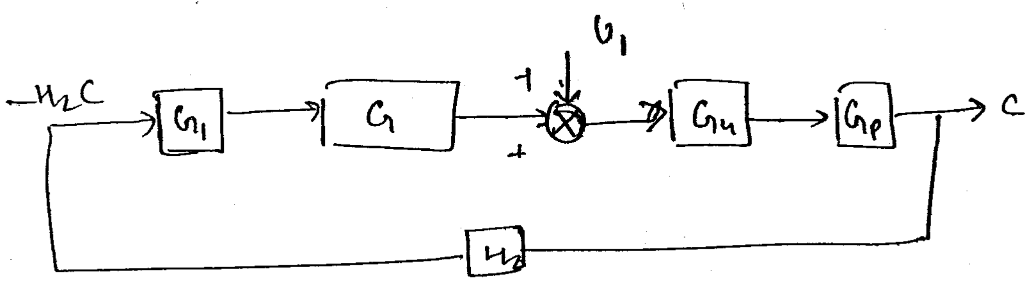
$$C = G_p G_4 G G_1 (R - H_2 C)$$

$$= \frac{C}{R} = \frac{G_1 G G_4 G_p}{1 + G_1 G G_4 G_p H_2}$$

Ans.

ii)

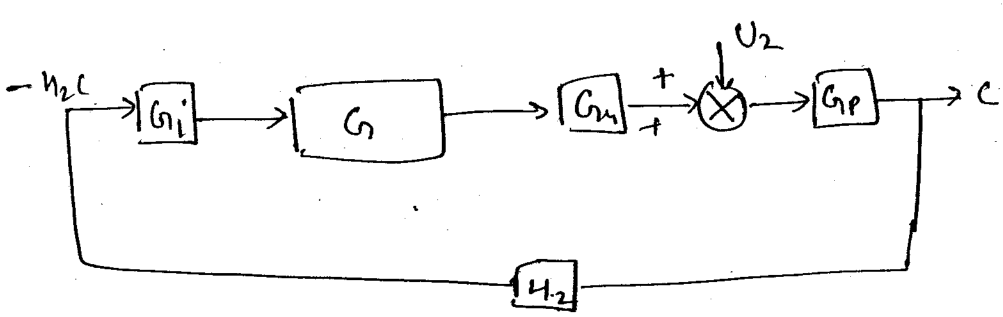
$$\frac{C}{U_1} =$$



$$C = (-CH_2 G_1 G + U_1) G_4 G_p$$

$$\frac{C}{U_1} = \frac{G_4 G_p}{1 + G_1 G G_4 G_p H_2}$$

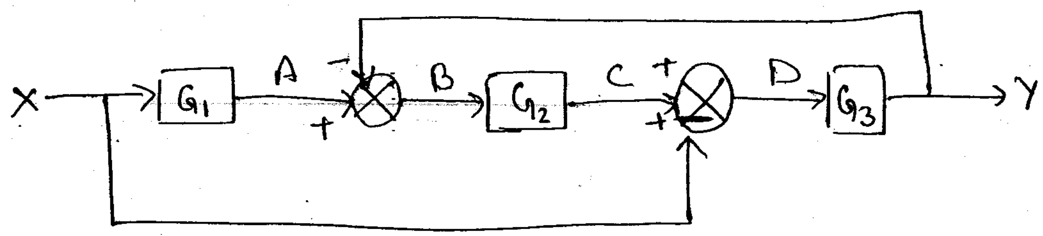
iii)



$$C = (-CH_2 G_1 G G_4 + U_2) G_p$$

$$\frac{C}{U_2} = \frac{G_p}{1 + H_2 G_1 G G_4 G_p}$$

Q.)



Calculate Y/X

$\frac{G_1 G_2}{1 - G_1 G_2}$

$A = XG_1$ $\quad$ $D = C + X$

$B = A - Y$ $\quad$ $Y = DG_3$

$C = BG_2$

$$Y = DG_3 = (C + X)G_3$$

$$Y = (BG_2 + X)G_3$$

$$Y = ((A - Y)G_2 + X)G_3$$

$$Y = ((XG_1 - Y)G_2 + X)G_3$$

$$Y = XG_1G_2G_3 - YG_2G_3 + XG_3$$

$$Y(1 + G_2G_3) = X(G_1G_2G_3 + G_3)$$

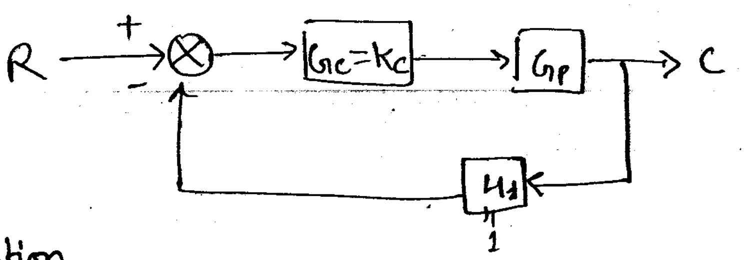
$$\frac{Y}{X} = \frac{G_1G_2G_3 + G_3}{1 + G_2G_3}$$

Offset

Offset is defined as the diff b/w the ultimate value of set pt to the ultimate value of the o/p (Response). In diag, we represent R as set pt

$$\text{Offset} = \underset{\text{set pt}}{R(\infty)} - \underset{\text{Response}}{C(\infty)}$$

$$= R(\infty) - Y(\infty)$$



Assumption

- Kinetic of measuring element is unity
- " , final control " " "
- Controller is proportional controller
- we r working for the servocase.

$$\frac{C}{R} = \frac{K_c G_p}{1 + K_c G_p}$$

$$G_p = \frac{K_p}{1 + \tau_p s}$$

$$\frac{C}{R} = \frac{\frac{K_c K_p}{1 + \tau_p s}}{1 + \frac{K_c K_p}{1 + \tau_p s}}$$

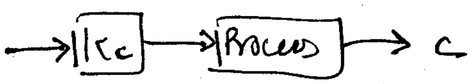
$$\frac{C}{R} = \frac{K_c K_p}{1 + K_c K_p + \tau_p s}$$

$$\frac{C}{R} = \frac{K_c K_p}{(1 + K_c K_p)\left(1 + \frac{\tau_p s}{1 + K_c K_p}\right)}$$

closed loop steady state gain

closed loop time constant

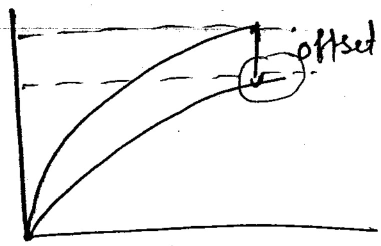
$$\boxed{\frac{C}{R} = \frac{K_p'}{1 + \tau_p' s}}$$



$$\frac{C}{R} = \frac{K_p K_c}{1+\tau_p s}$$

$$K_p' = \frac{K_p K_c}{1+K_p K_c}$$

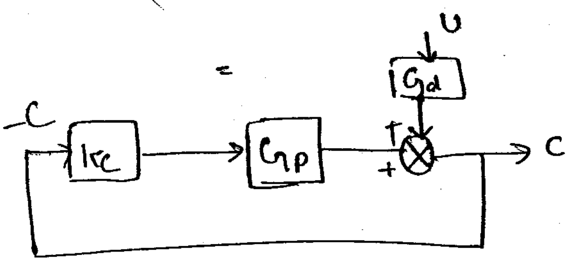
$$\tau_p' = \frac{\tau_p}{1+K_p K_c}$$



offset $= \frac{1}{1+K_c K_p}$

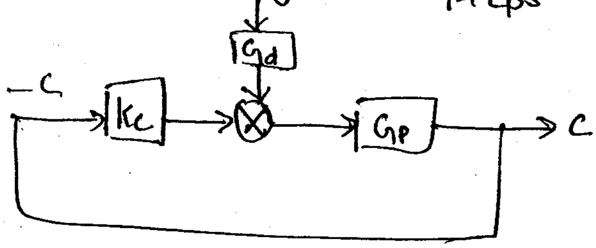
$$\frac{C}{U} =$$

$$C = (-C H_1 G_c G_p + U)$$



$$\frac{C}{U} = \frac{G_d}{K_c G_p + 1}$$

$$\frac{C}{U} = \frac{K_d}{1+\frac{K_c K_p}{1+\tau_p s}}$$



$G_d = K_d$

$$\frac{C}{U} = \frac{K_p K_d}{1+K_c K_p + \tau_p s}$$

$$\frac{C}{U} = \frac{K_p'}{1+\tau_p' s}$$

In general case,

We conclude that the ~~first~~ closed loop response of a first order System has following characteristic for P-controller.

→ It remains first order w.r.t servo or Regulatory mode.

→ The tym constant has been reduced or $\tau_p' < \tau_p$ which means that closed loop response has become faster than the open loop response to changes ~~the~~ in the set pt & the load.

→ The stead state gains has been decreased by a factor of $(1+K_pK_c)$

$$\frac{C}{R} = \frac{K'_p}{1+\tau_p' s}$$

$$R(t) = 1$$

$$R(s) = 1/s \quad , R(\infty) = 1$$

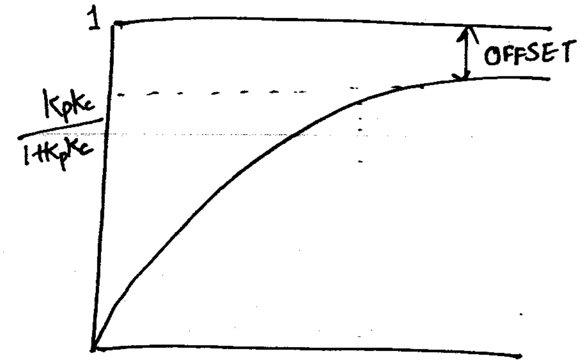
$$C = \frac{K'_p}{1+\tau_p' s} \cdot \frac{1}{s}$$

$$C(\infty) = \lim_{s \to 0} s\,C(s)$$

$$C(\infty) = K_p' = \frac{K_cK_p}{1+K_pK_c}$$

$$\text{OFFSET} = R(\infty) - C(\infty)$$

$$= 1 - \frac{K_pK_c}{1+K_cK_p} = \frac{1+K_pK_c - K_pK_c}{1+K_pK_c} = \frac{1}{1+K_pK_c}$$



Regulatory Mode

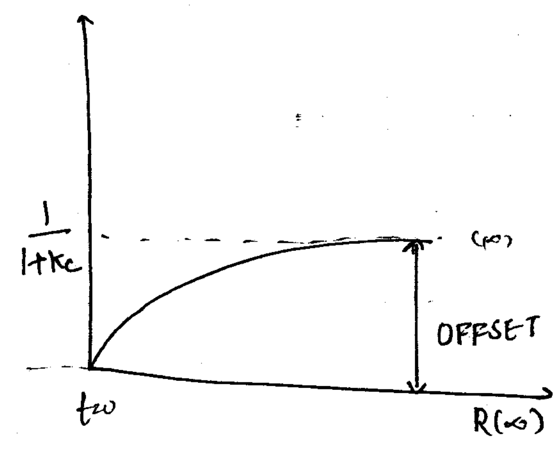
take all gain = 1, except K_c.

$$OFFSET = R(\infty) - C(\infty)$$

$$= 0 - \frac{1}{1+K_c}$$

$$= -\frac{1}{1+K_c}$$

in Regulatory case the set pt is const so $R(\infty) = 0$, therefore offset $= -C(\infty)$



for P.I

$= K_c\left(1+\frac{1}{\tau_I s}\right)$

$\frac{C}{R} = \frac{K_cK_c(1+\frac{1}{\tau_I s})}{1+K_cK_c(1+\frac{1}{\tau_I s})}$

$\frac{K_c^2(1+\frac{1}{\tau_I s})}{1+K_c^2(1+\frac{1}{\tau_I s})}$

for P.I controller.

$$\frac{C}{R} = \frac{K_c\left(1+\frac{1}{\tau_I s}\right)\frac{1}{1+\tau_p s}}{1+K_c\left(1+\frac{1}{\tau_I s}\right)*\left(\frac{1}{1+\tau_p s}\right)}$$

$$C = \frac{k_c\left(1+\frac{1}{\tau_I s}\right)\left(\frac{1}{1+\tau_p s}\right)}{1+K_c\left(1+\frac{1}{\tau_I s}\right)\left(\frac{1}{1+\tau_p s}\right)} \times \frac{1}{s}$$

$$C(\infty) = 1$$

$$\text{Offset} = R(\infty) - C(\infty)$$
$$= 1 - 1 = 0$$

$\text{OFFSET} = \frac{1}{1+K_c}$

$K_c \to \infty$

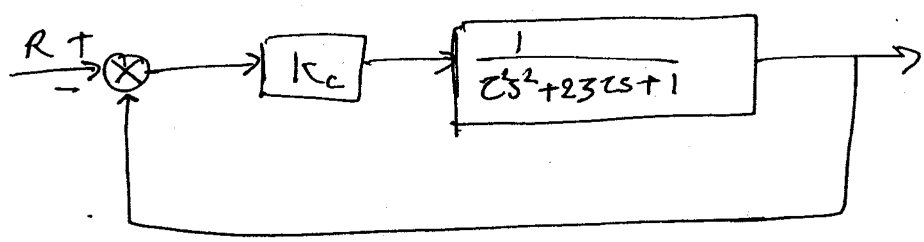
$\text{OFFSET} = 0$

Not preferred

$$\boxed{\begin{array}{ll} 1 - \dfrac{K_c K_f K_p}{1+K_c K_f K_p K_m} & \text{for servocase} \\ 0 - \dfrac{K_d}{1+K_c K_f K_p K_m} & \text{for Regulator} \end{array}}$$

$R(s) = \frac{1}{s}$

Nov 06, 14



$$G(s) = \frac{\frac{K_c}{\tau^2 s^2 + 2\zeta\tau s + 1}}{1 + \frac{K_c}{\tau^2 s^2 + 2\zeta\tau s + 1}}$$

$$= \frac{K_c}{\tau^2 s^2 + 2\zeta\tau s + (1+K_c)}$$

$$= \frac{K_c'}{(\tau')^2 s^2 + 2\zeta'\tau' s + 1}$$

$$\boxed{K_c' = \frac{K_c K_p}{1+K_c K_p}}$$

$$\boxed{\tau' = \frac{\tau}{\sqrt{1+K_c K_p}}}$$

$$\boxed{\zeta' = \frac{\zeta}{\sqrt{1+K_c K_p}}}$$

In case of IInd order also process behaves faster than the open loop.

ζ ~~value~~ only depend on Kc

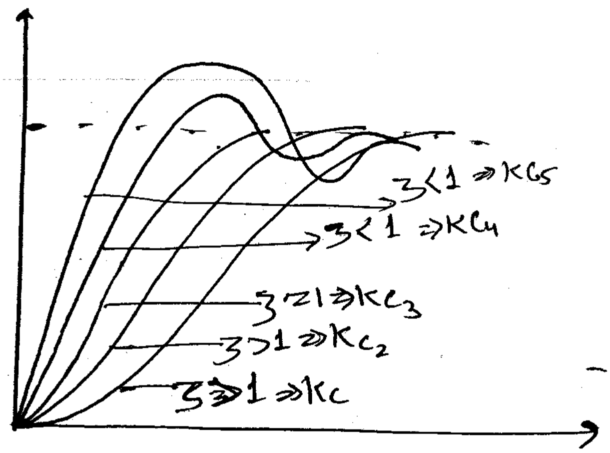
When second order system is series wid P controller

→ Kinetics remains the same → Second order remains second order.

→ Tym constant of the overall process decreases which means it will take less tym to response in comparision to open loop System.

→ Damping factor for the overall system decreases which makes the system faster. Or in other words, a overdamp system in the open loop may behaves as a critically damped or under damped in the closed loop system.

→ The steady state gain of the overall process decreases which also makes the response faster



$$K_{c_5} > K_{c_4} > K_{c_3} > K_{c_2} > K_{c_1}$$

Offset

for servo

$$\frac{C}{R} = \frac{\dfrac{K_c}{\tau^2 s^2 + 2\zeta\tau s + 1}}{1 + \dfrac{K_c}{\tau^2 s^2 + 2\zeta\tau s + 1}}$$

Offset

for servo

$$C(\infty) = K_c/1+K_c$$

$$R(\infty) = 1$$

$$\text{OFFSET} = 1/1+K_c$$

for Regulatory

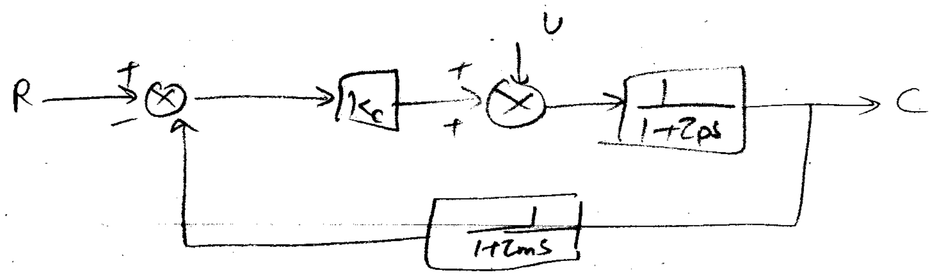
$$= -1/1+K_c$$

In the case of second order system the offset is same as that of the first order bt they are sluggish in comparision to the first order. To make the response fast we ↑ the value of K_c, which decreases the offset, tym constant, steady state gain & also the damping factor. Bt this action in turn increases the oscillation. To remove the oscillations we use the D action.

P	PI	PD
The kinetics remains the same	Kinetics increases by 1	Kinetics remains the same
It is fast & has fast response as $\tau' < \tau$	It makes the system sluggish by increasing the order.	Makes the system fast.
As the controller is fast no need to increase the Kc value. Oscillations are there bt are less. $\xi' < \xi$	The response is slow to make it fast Kc have to be increased, which in turn increases the oscillation.	The D-action have -ve impact on the oscillation. Or in other word they eliminate the oscilli

Gives offset $\pm\frac{1}{1+K_c}$ | Offset is zero | Offset is same as that of the P controller.

Q7



for servo case, If $\tau_p = 8$, $\tau_m = 1$, find the value of K_c to provide critically damped system.

$K_c = 1.53$

$$\tau^2 = \frac{8}{1+K_c}$$

$$2\xi\tau = 9/1+K_c$$

for critical $\xi = 1$

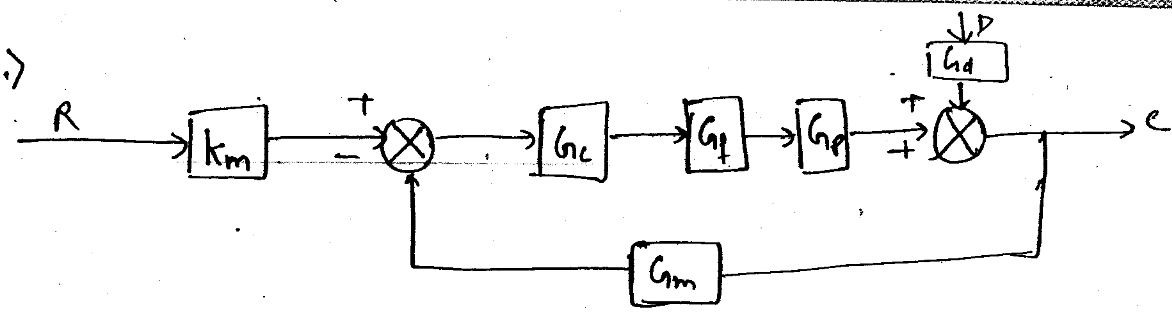
$$\frac{C}{R} = \frac{\frac{K_c}{1+\tau_p s}}{1+\frac{K_c}{(1+\tau_p s)(1+\tau_m s)}} = \frac{K_c(1+\tau_m s)}{(1+\tau_p s)(1+\tau_m s)+K_c}$$

$$= \frac{K_c(1+s)}{(1+s)(1+8s)+K_c}$$

$$1-\frac{1}{1+K_c} =$$

$$=$$

Q.)



i.) Find C/R & C/D

ii.) If $k_m = 1.5$

$G_c = k_c = 4$

$G_f = 0.5$, $G_m = 2$

$G_p = G_d = -2/s$

Then calculate OFFSET for step change of magnitude 2 in disturbance.

Ans = 0.5

$D(t) = 2$

$D(s) = 2/s$

i.)

Sol^n

$$C = (R k_m - C G_m) G_c G_f G_p$$

$$\frac{C}{R} = \frac{R k_m G_c G_f G_p}{1 + G_m G_c G_f G_p}$$

$$= \frac{1.5 \times 4 \times 0.5 \times -2/s}{1 + 2 \times 4 \times 0.5 \times -2/s} \quad \frac{-4/s}{1 + -8/s} = \frac{\frac{-4}{s}}{\frac{s-8}{s}}$$

$$= \frac{+4}{8-s} =$$

$$C = ((0 - C G_m) G_c G_f G_p + D G_D)$$

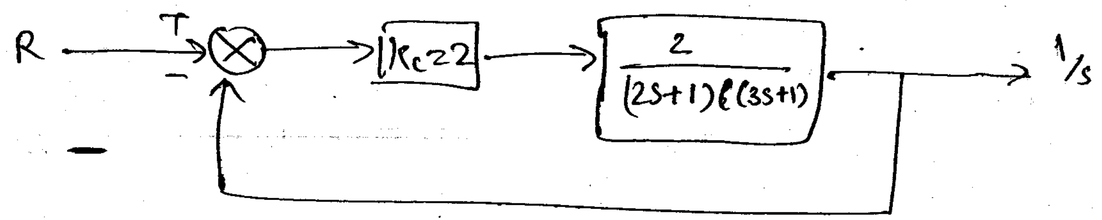
$$\frac{C}{D} = \frac{G_D}{1 + G_m G_c G_f G_p} = \frac{-2/s}{1 + 2 \times 4 \times 0.5 \times -2/s} = \frac{-2/s}{\frac{s-8}{s}}$$

$$= \frac{2}{8-s}$$

$$\frac{2}{8-s} \cdot \frac{2}{s}$$

$$\frac{4}{8} = 0.5$$

Q.)

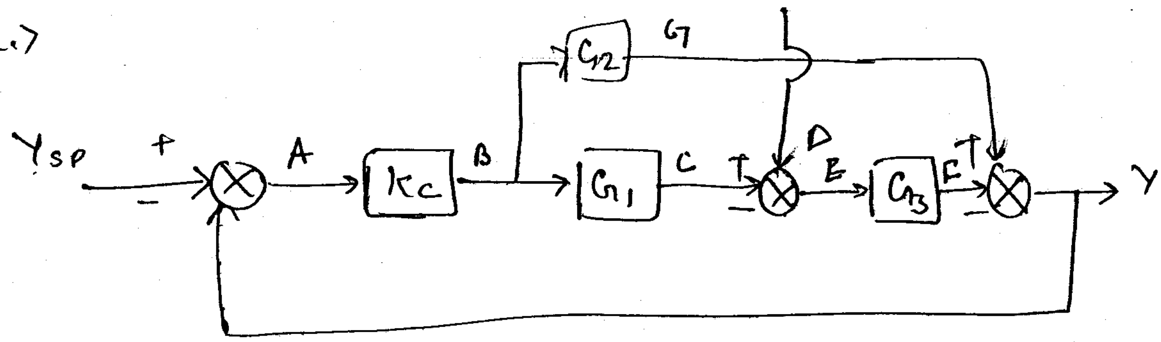


for unit step change in R. find off set

$1 - \frac{4}{1+4}$

$= 0.2$

Q.)



Find Y/Y_{sp} & Y/D

$$\frac{Y}{Y_{sp}} = \frac{K_c G_1 G_3}{1 + K_c G_1 G_3}$$

$$\frac{Y}{D} = \frac{G_3}{1 + K_c G_1 G_3}$$

$A = Y_{sp} - Y$ $\quad$ $F = E G_3$

$B = A K_c$ $\quad$ $Y = F + G$

$C = B G_1$ $\quad$ $G = B G_2$

$E = C + D$

for D=0

$$Y = \left((Y_{sp} - Y) K_c G_1 G_3 + (Y_{sp} - Y) K_c G_2 \right)$$

$$\cancel{Y_{sp} K_c G_1 G_3} \; (Y_{sp} - Y) K_c \left(G_1 G_3 + G_2 \right)$$

$$\frac{Y}{Y_{sp}} = \frac{K_c (G_1 G_3 + G_2)}{\ldots}$$

for $Y_{sp} = 0$

$$Y = ((-YK_cG_1 + D)G_3 + (-YK_c)G_2)$$

ii) If $G_1 = 1$, G_2 & $G_3 = \frac{1}{(S+1)}$. A step change of magnitude M is made in the set point. Find the value of offset.

$$\frac{Y}{Y_{sp}} = \frac{K_c(G_1G_3 + G_2)}{1 + K_c(G_1G_3 + G_2)}$$

$$= \frac{K_c\left(\frac{1}{(S+1)} + \frac{1}{(S+1)}\right)}{1 + K_c\left(\frac{1}{(S+1)} + \frac{1}{(S+1)}\right)}$$

$$= \frac{2K_c}{(S+1)} \Big/ 1 + \frac{2K_c}{(S+1)}$$

$$\frac{C}{R} = \frac{2K_c}{S+1+2K_c} = \frac{2K_c}{2K_c + (S+1)}$$

$$C = \frac{2K_c}{2K_c + (S+1)} \cdot \frac{M}{S}$$

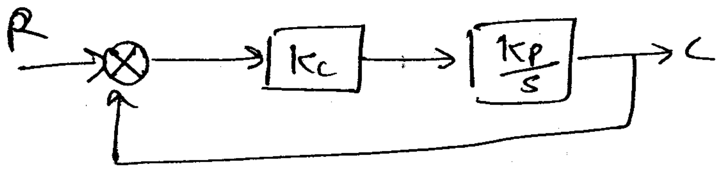
$$C(\infty) = \frac{2K_cM}{2K_c + 1}$$

$$R(\infty) - C(\infty) = M - \frac{2K_cM}{2K_c+1} = \frac{\cancel{2K_cM} + M - \cancel{2K_cM}}{2K_c + 1}$$

$$\text{offset} = \frac{M}{2K_c + 1}$$

(Non-Self Regulatory)

i) Purely capacitive system in series wid P controller. find offset and other transfer function have kinetics of 1.



Sol^n for servo mode

$$\frac{C}{R} = \frac{K_c \frac{K_p}{s}}{1 + \frac{K_c K_p}{s}} = \frac{K_c K_p}{s + K_c K_p}$$

$C(\infty) = k\,1$

$R(\infty) = 1$

$= 1 - 1 = 0$

for Regulator m

$$\frac{C}{U} = \frac{1\ \frac{K_p}{s}}{1 + \frac{K_c K_p}{s}} = \frac{K_p}{s + K_c K_p}$$

$C(\infty) = 1/K.$

→ P controller not every time the offset value if it added in series wid the purely capacitive system, then bcz of the kinetic of system, the offset is zero. for the step change in set pt.

Nov 07, 14

Stabilities

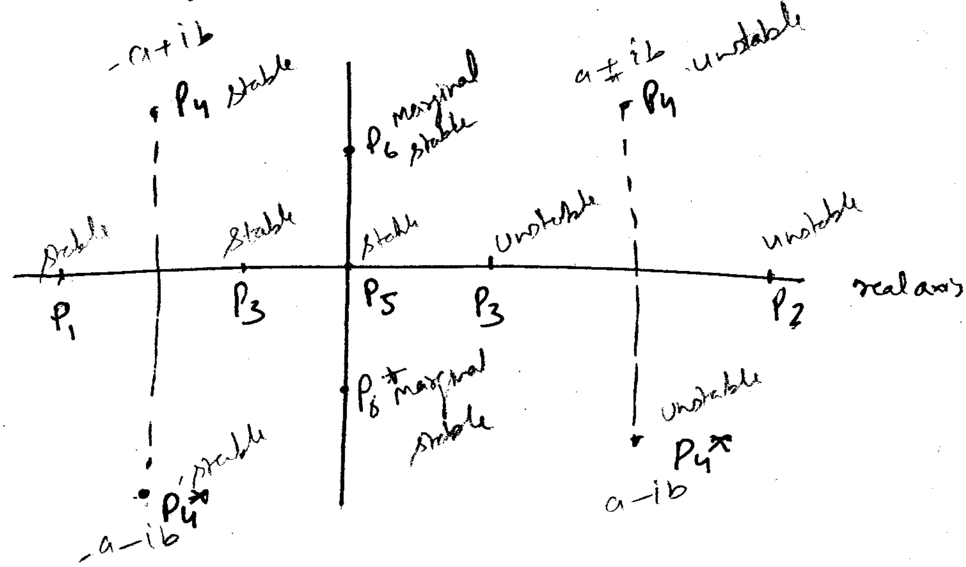
Poles & zeros of a Transfer function :-

$$G(s) = \frac{P(s)}{Q(s)}$$

where $P(s)$ & $Q(s)$ are polynomial

The values of s at which the t.f becomes zero is known as zeros of transfer function i.e roots of $P(s)$

The values of s at which the t.f becomes infinite is known as Poles of transfer function i.e roots of $Q(s)$



Let denominator of trans. funct^n can be written as

$$Q(s) = (s-P_1)(s-P_2)(s-P_3)(s-P_4)(s-P_4^*)(s-P_5)$$

$\rightsquigarrow$ for the P_1 & P_2,

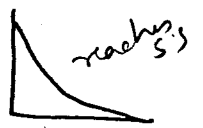
$\Rightarrow$ $P_1 = e^{-P_1 t}$

$\Rightarrow$ $P_2 = e^{P_2 t}$

P_1 & P_2 are the real nos. Such that P_1 ~~&~~ is $-$ve & P_2 is +ve
The laplace of P_1 & P_2 will give terms $\underline{e^{-P_1 t} \; \& \; e^{P_2 t}}$.

Since $P_1 < 0$

$t \to \infty \Rightarrow e^{P_1 t} \to 0$



; ultimate value

but $P_2 > 0$

$t \to \infty \quad e^{P_2 t} \to \infty$

Hence, distinct poles on the $-$ve ~~ax~~ real axis produce terms that decays to zero wid tym. {*Stable systems*} while real +ve poles makes the response of the system grow towards ∞ wid tym {*unstable systems*}

$\Rightarrow \quad (C_1 t + C_2)\, e^{P_3 t}$ real & equal.

unstable $\quad P_3 > 0, \quad e^{P_3 t} \to \infty$

Stable $\quad P_3 < 0, \quad e^{P_3 t} \to 0$; exponential rate is fast then linear rate

Unstable $\quad P_3 = 0, \quad e^{P_3 t} \to 1$

- In first case, System is unstable.
- " third " , " " , bcz $t \to \infty$, Response $\to \infty$
- But for the second case, the exponetial term at the infinite tym decays to zero, before ~~t~~ when $t \to \infty$, as the expontenial rate is more than the algebraic rate. The system show <u>stable</u> response.

For the case of real & equal poles, the poles must be −ve for stable system.

⇒ P_4 & P_4^* ⟶ Complex Conjugates

$$e^{P_4 t}\underbrace{(C_1\cos\omega t + C_2 \sin\omega t)}_{\sin(\beta \pm \phi)}$$

$P_4 = \underset{real}{a} + ib$

$e^{at}(C_1\cos bt + C_2 \sin bt)$

$$P_4 = a + ib$$
$$P_4^* = a - ib$$

This term $(C_1 \cos\omega t + C_2 \sin\omega t)$ introduced into the sys

$$\Downarrow$$

$$e^{at}\sin(\beta \pm \phi)$$

when real part is +ve

$$a>0, \quad e^{at} \to \infty$$

System becomes unstable

when real part is −ve

$$a<0, \quad e^{at} \to 0$$

System decays to zero (stable)

when real part is $= 0$

$$a=0, \quad e^{at} \to 1$$

System is unstable. Oscillatory due to presence of sin terms

⇒ P_5, roots lie on the origin :—

for the factor P_5, we get a constant term.

Constant remains constant for $t \to \infty$! stable

Principle of Superposition :—

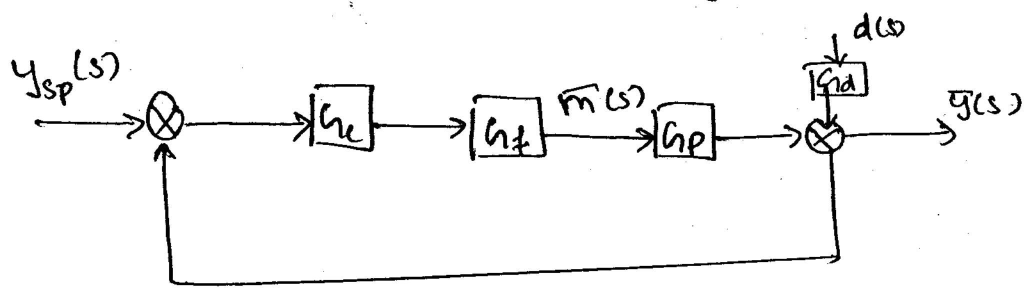
for u=0

$$\frac{C'}{R} = \frac{G_c G_f G_p}{1 + G_c G_f G_p G_m}$$

$$\frac{C}{U} = \frac{G_d}{1 + G_c G_f G_p G_m}$$

$$C = \frac{G_c G_p G_f}{1 + G_c G_p G_f G_m} R + \frac{G_d}{1 + G_c G_p G_f G_m} U$$

Q) $\bar{y}(s) = \frac{10}{s-1} \bar{m}(s) + \frac{5}{(s-1)} \bar{U}(s)$



$G_f = 1$

$G_c = K_c$

$$\frac{\bar{y}_s}{y_{sp}} = \frac{G_c G_f G_p}{1 + G_c G_f G_p}$$

Acc. :

when $\bar{U}(s) = 0$

$$\bar{y}(s) = \frac{10}{s-1} \bar{m}(s)$$

from comp.

$$G_p = \frac{10}{s-1}$$

when $\bar{m}(s) = 0$

$$\bar{y}(s) = \frac{5}{(s-1)} \bar{U}(s)$$

$$G_d = \frac{5}{s-1}$$

$$= \frac{K_c \cdot 1 \cdot \frac{10}{s-1}}{1 + K_c \cdot 1 \cdot \frac{10}{s-1}} = \frac{\frac{10K_c}{s-1}}{\frac{s-1+10K_c}{s-1}} = \frac{10K_c}{10K_c + (s-1)}$$

$$\frac{\bar{Y}(s)}{d(s)} = \frac{G_d}{1 + G_c G_f G_p}$$

$$= \frac{5/(s-1)}{1 + k_c \cdot \frac{10}{s-1}} = \frac{5}{10k_c + (s-1)}$$

$$Y(s) = \frac{10k_c}{10k_c + (s-1)} y_{sp}(s) + \frac{5}{10k_c + (s-1)} d(s)$$

$$Y(s) = \frac{10k_c}{s - (1-10k_c)} y_{sp}(s) + \underbrace{\frac{5}{s - (1-10k_c)}}_{G_d} d(s)$$

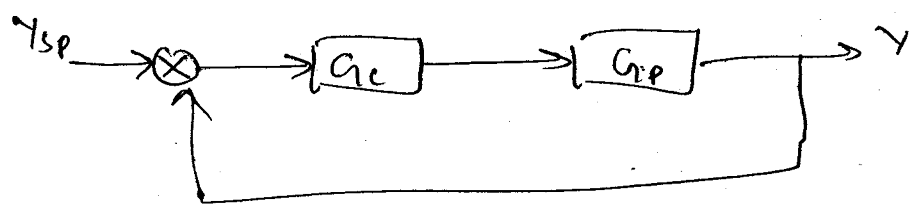
as we compare above $P = (1-10k_c)$ the $(1-10k_c)$ should be $-ve$

$$s - (1-10k_c)$$

$$1 - 10k_c < 0$$

$$\frac{1}{10} < k_c$$

Q.) $\bar{y}(s) = \frac{1}{s^2+2s+2}$, if we add PI controller in series with it in servo mode. Find transfer function.



$$\frac{Y}{Y_{sp}} = \frac{G_c G_p}{1 + G_c G_p}$$

$s^3 + 2s^2 + (2+k_c)s + \frac{k_c}{\tau_I}$

$s^3 + 2s^2 + (2+k_c)s + k_c/\tau_I$

i) $k_c = 100$, $\tau_I = 0.1$

roots -7.185, $2.54 \pm i\,11.5$

2) $k_c = 10$, $\tau_I = 0.5$

$-$ve real parts of all 3 roots

thus system remains stable.

As we keep $k_c \uparrow$, the system becomes unstable.

Routh Criterion for Stability :—

1) Characteristic Equation.

$$\boxed{1 + \Pi_{Loop} = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_n = 0}$$

write in form of polynomial.

2) In charc. eqn, if any of the coeff. $a_0, a_1, a_2 \dots a_n$ is $-$ve the system becomes unstable.

Routh Array

arrange coeff. in below form.

$$\begin{array}{lll} a_0 & a_2 & a_4 \\ a_1 & a_3 & a_5 \\ b_1 & b_2 & \\ \overset{c}{c_1} & \overset{d}{c_2} & \\ d_1 = 0 & & \end{array}$$

$(n-1)^{th}$ — c_1

$(n)^{th}$ — d_1

$$b_1 = \frac{a_1 a_2 - a_3 a_0}{a_1}$$

$$b_2 = \frac{a_1 a_4 - a_5 a_0}{a_1}$$

$$c_1 = \frac{b_1 a_3 - b_2 a_1}{b_1}$$

$$c_2 = \frac{b_1 a_5}{b_1}$$

$$d_1 = \frac{c_1 b_2 - c_2 b_1}{c_1}$$

» In first column if any element is -ve, the system is unstable.

» If the coeff. of the last row is zero, (means $d_1 = 0$) then system possess 2 roots which lies on imaginary axis. And the location of those roots can be find out by using an eqn

$$cs^2 + d = 0$$

where c & d are the coeff. of $(n-1)^{th}$ row

Q.) $0.5s^3 + 3s^2 + 5.5s + (3k_c + 3) = 0$

$$\begin{array}{ccc} 0.5 & 5.5 & 3 \\ 3 & 3k_c+3 & \end{array}$$

16.

$$b_1 = \frac{(3 \times 5.5) - 0.5\,(3k_c + 3)}{3}$$

$$= \frac{16.5 - 1.5k_c \mp 1.5}{3}$$

$$= \frac{15 - 1.5k_c}{3}$$

$$= 5 - 0.5k_c$$

$\Rightarrow \quad b_1 > 0$

$5 - 0.5k_c > 0$

$5 < 0.5k_c$

$10 < k_c$

if $b_1 = 0$, find location of roots.

$cs^2 + d = 0 \qquad , b_1 = 0 , \; k_c = 10$

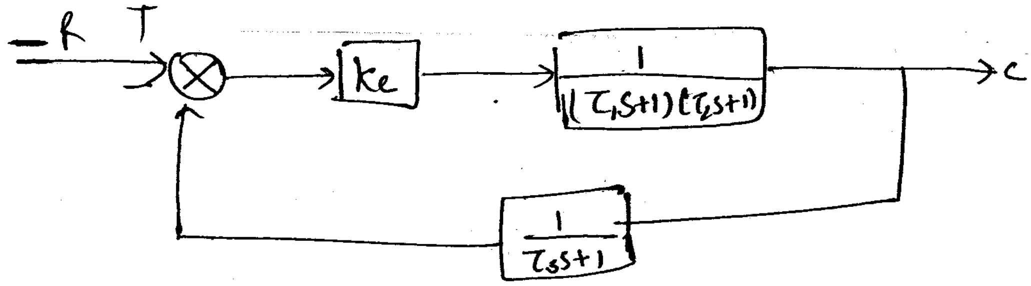
$3s^2 + 33 = 0$

$s^2 = -11$

if the roots are on the imaginary axis, the system is on the verge of instablity which means any deviation can make the system unstable.

Q.)

$\tau_1 = 1, \ \tau_2 = 2, \ \tau_3 = 3$



$$1 + \frac{K_c}{(\tau_1 s+1)(\tau_2 s+1)(\tau_3 s+1)} = 0$$

$$\frac{(s+1)(2s+1)(3s+1) + K_c}{(s+1)(2s+1)(3s+1)} = 0$$

$2s^2+s+2s+1$

$(2s^2+3s+1)$
$\cdot(3s+1) \Rightarrow$
$6s^3+9s^2+3s$
$2s^2+3s+1$
$6s^3+11s^2+6s+1$

$$\frac{6s^3+11s^2+6s+1+K_c}{\cancel{6s}} = 0$$

$6 \quad\quad 6$

$11 \quad\quad (K_c+1)$

$$b_1 = \frac{66 - 6(K_c+1)}{11} > 0$$

$$= \frac{66 - 6K_c - 6}{11} = \frac{60 - 6K_c}{11} > 0$$

$60 - 6K_c > 0$

$60 > 6K_c$

$\cancel{K_c < 60}$

$\therefore K_c < 10$

$Cs^2 + d = 0$

$11s^2 + 11 = 0$

$11s^2 = -11$

$s^2 = -1$

$s = \pm\sqrt{-1} = \pm i$

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Q.> $G_c = K_c$, $G_p = \frac{1}{(S+1)(0.5S+1)}$, $G_m = \frac{3}{S+3}$, $G_f = 1$.

Find the value of K_c for stability & also location of imaginary roots for the case of verge of instability.

$U=0$, servo mode.

$$1 + \frac{K_c}{(S+1)(0.5S+1)} \cdot \frac{3}{S+3} \cdot 1 = 0$$

$$(S+1)(0.5S+1)(S+3) + 3K_c = 0$$

$(S+1)(S+3)$
$S^2+3S+S+3$
(S^2+4S+3)
$(0.5S+1)$
$0.5S^3 + S^2$
$+2S^2+4S$
$+1.5S+3$

$$0.5S^3 + 3S^2 + 5.5S + 3 + 3K_c = 0$$

$$0.5S^3 + 3S^2 + 5.5S + 3(K_c+1) = 0$$

0.5	5.5
3	$3(K_c+1)$

$$b_1 = \frac{16.5 - 1.5(K_c+1)}{3}$$

$$16.5 - 1.5(K_c+1) > 0$$

$$\frac{16.5}{1.5} = K_c + 1$$

$$K_c = 10$$

$$CS^2 + d = 0$$

$$3S^2 + 3(K_c+1) = 0$$

$$3S^2 + 33 = 0 \Rightarrow 3S^2 = -33$$

$$S^2 = -11$$

Check

Q.) $G_c = K_c$, $G_p = \frac{20}{s-2}$, $G_m = 1$, $G_f = 1$

$$1 + \frac{20K_c}{(s-2)} = 0$$

$$(s-2) + 20K_c = 0$$

$$\boxed{1.s + \underline{2(10K_c - 1)} = 0}$$

Acc. to Routh Array Routh stability test each element shud be +ve
S. waalo to +ve
So $2(10K_c - 1) > 0$

$$2(10K_c - 1) > 0$$

$$10K_c > 1$$

$$K_c > \frac{1}{10}$$

Ans
greatest than 1/10

42
~~504~~
~~12~~

$\frac{-204 - 60}{5}$

12/5

Q.) $s^4 + 5s^3 - s^2 - 17s + 12 = 0$

See the no. of sign changes in characteristic eqn & there is no need to find the Routh Array. no

Columns → Row

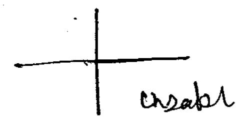
1	-1	12
5	-17	0
$\frac{12}{5}$	$b_2 = 12$	
42	0	

$$b_1 = \frac{-5 + 17}{5} = \frac{12}{5}$$

$$b_2 = \frac{60 - 0}{5} = 12$$

$$\frac{\frac{12}{5} \times -17 - 12 \times 5}{12/5}$$

Select the no. of roots which will be located to the right of imaginary axis.



* The no. of roots lies on the R.H.S of imaginary axis is equals to the no. of sign changes in the first column of Routh Array.

Sol^n no. of roots = 2.

Q.) Routh Array, find nature of root.

2	2
4	4
0	

$4s^2+4=0$

$s^2=-1$

$s=\pm i$

$2s^3+4s^2+2s+4=0$

$2s^2(s^2+2)+2(s+2)=0$

$(2s^2+2)(s+2)=0$ $\quad 2s^2=-2$

$s=-2, \pm i$

verge of stability.

if any roots present on imaginary axis ⇒ verge of instability

" " " " " R.H.S " ⇒ unstable.

Q:) $12s^3 + 14s^2 + 8s + (1+K_c) = 0$, $K_c = ?$. location of roots for verge of insatability case.

$$\begin{matrix} 12 & 8 \\ 14 & (1+K_c) \end{matrix}$$

$\pm \frac{\sqrt{2}}{3} i$

$$b_1 = \frac{112 - 12(1+K_c)}{14}$$

$b_1 > 0$

$112 - 12(1+K_c) > 0$

$112 > -12(1+K_c)$

$9.333 > -1 \mp K_c$

$-9.33 < 1+K_c$

$K_c = 10.333$

14

Q.) $G_c = K_c(1 + 2/s)$ Servo mode.

$G_p = \frac{1}{2s+1}$, $G_m = \frac{1}{s+1}$, $G_f = 1$

find the case of stability.

$$1 + \frac{K_c(1+2/s)}{(2s+1)(s+1)} = 0$$

$$s(2s+1)(s+1) + K_c(s+2) = 0$$

$$2s^3 + 3s^2 + s + K_c(s+2) = 0$$

$$2s^3 + 3s^2 + (K_c+1)s + 2K_c = 0$$

$(2s+1)(s+1)$
$2s^2 + s +$
$2s + 1$
$2s^2 + 3s + 1$

2	(K_c+1)
3	$2K_c$

$$b_1 = \frac{3(K_c+1) - 4K_c}{3} = \frac{-K_c+3}{3}$$

$b_1 > 0$

$-K_c + 3 > 0$

$K_c < 3$

$K_c - 3 < 0$

$K_c + 3 > 0$

$K_c < 3$

$j \pm 2i$

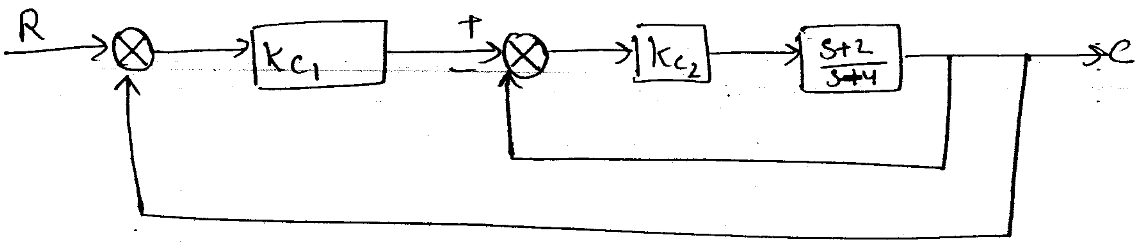
$3s^2 + 6 = 0$

$3s^2 = -6$

$s^2 = -2$

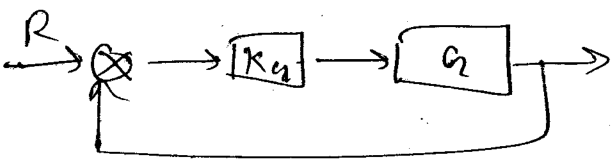
$s = \pm\sqrt{2}\,i$

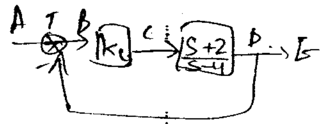
Q7



Find the value of K_{c_2}, for which the system is stable.

$K_{c_1} = 0.5$





$E = (A-D)\, K_{c_2} \frac{(S+2)}{(S-4)}$

$= (R-D)\, K_{c_1} K_{c_2} \frac{(S+2)}{(S-4)}$

$0.5(R-D) + (A-D) K_{c_2} \frac{(S+2)}{S-4}$

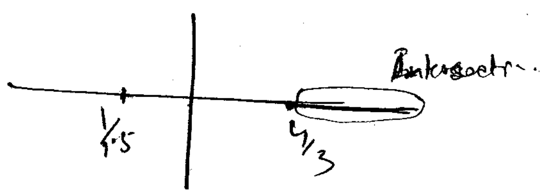
$0.5R - 0.5D + (A-D) K_{c_2} (S+2)$
$(S-4)$

$0.5R$

$$(1 + 1.5K_{c_2})\, s + (3K_{c_2} - 4) = 0$$

$\Downarrow$

$K_{c_2} > -\frac{1}{1.5}$, $K_{c_2} > 4/3$



$\therefore\ K_c > 4/3$

$$\frac{\frac{K_c(S+2)}{(S-4)}}{1 + \frac{K_c(S+2)}{S+4}}$$

$$\frac{K_c(S+2)}{(S-4) + K_c(S+2)}$$

$$\frac{K_c(S+2)}{(K_c+1)S + 2K_c - 4}$$

$C =$

$$\frac{K_{c_1} K_{c_2} (S+2)}{1 + \frac{K_{c_1} K_{c_2}(S+2)}{(K_c+1)S + 2K_c - 4}}$$

$K_{c_2} S + S + 2K_{c_2} - 4$

$1.5 K_{c_2} S -$

$(K_c+1)S + 2K_c - 4 + K_{c_1} K_{c_2} (S+2)$

$(K_{c_2}+1)S + 2K_{c_2} - 4 + 1.5K_{c_2}(S+2) = 0$

Frequency Response :—

— $A \sin \omega t \rightsquigarrow$ Process $\rightsquigarrow A_0 \sin(\omega t + \phi)$

The response of the process towards any sinusoidal change can be determined by graphical representation.

These graphs are known as Bode Plots / Bode diagrams

Bode diagram consist of 2 graphs.

→ $\ln AR$ vs $\ln \omega$ or AR vs ω on ln-ln Paper

→ MR vs ω

MR ↓ Magnitude Ratio

⇓

$\frac{AR}{K_c}, \frac{AR}{K_p}$

→ ϕ vs $\ln \omega$ or ϕ vs ω on semi-log paper.

transfer function

Amplitude Ratio

$$G(i\omega) = a + ib = re^{i\phi} = AR\, e^{i\phi}$$

$$r = \sqrt{a^2+b^2}$$

$$\phi = \tan^{-1}(b/a)$$

for ex

for first order system

$$G(s) = \frac{1}{1+\tau s}$$

$$AR = \frac{1}{\sqrt{1+\tau^2\omega^2}}$$

$$\phi = \tan^{-1}(-\omega\tau)$$

$$G(s) = \frac{1}{1+\tau s}$$

$$G(i\omega) = \frac{1}{1+\tau(i\omega)} \times \frac{1-\tau(i\omega)}{1-\tau(i\omega)}$$

; Since imaginary no. can't be in deno.

$$= \frac{1}{1+\tau^2\omega^2} - \frac{i(\omega\tau)}{(1+\tau^2\omega^2)}$$

$$AR = \sqrt{\left(\frac{1}{1+\tau^2\omega^2}\right)^2 + \frac{i(\omega\tau)^2}{(1+\tau^2\omega^2)^2}}$$

$$= \sqrt{\frac{1+\omega^2\tau^2}{(1+\tau^2\omega^2)^2}}$$

$$\frac{\cancel{1+\omega^2\tau^2}}{\cancel{1+\tau}}$$

$$= \frac{\sqrt{\cancel{1+\omega^2\tau^2}}}{1+\tau^2\omega^2}$$

$$= \sqrt{\frac{1}{1+\tau^2\omega^2}} = \boxed{\frac{1}{\sqrt{1+\tau^2\omega^2}} = AR}$$

$$\theta = \tan^{-1}\left(\frac{\frac{-\omega\tau}{1+\tau^2\omega^2}}{1+\omega^2\tau^2}\right)$$

$$\boxed{\theta = \tan^{-1}(-\omega\tau)}$$

$$G(s) = G(i\omega)^{ARe^{i\theta}}$$

$$\frac{P(s)}{Q(s)} = \frac{P(i\omega)}{Q(i\omega)} = \frac{\overset{\text{magnitude}}{|P(i\omega)|}\, e^{i \arg P(i\omega)}}{|Q(i\omega)|\, e^{i \arg Q(i\omega)}}$$

$$\boxed{AR = \frac{|P(i\omega)|}{|Q(i\omega)|}}$$

$$\boxed{\phi = \arg P(i\omega) - \arg Q(i\omega)}$$

forex first order system

$$G(s) = \frac{1}{1+\tau(s)}$$

$$G(i\omega) = \frac{1}{1+\tau(i\omega)} = \frac{1 \cdot e^{i0}}{\sqrt{1+\tau^2\omega^2}\ e^{i\tan^{-1}(\tau\omega)}}$$

$$AR = \frac{1}{\sqrt{1+\tau^2\omega^2}}$$

$$\phi = 0 - \tan^{-1}(\tau\omega)$$

$$= \tan^{-1}(-\omega\tau)$$

$$\frac{1 \cdot e^{i0}}{\sqrt{a^2+b^2}}\ e^{i\tan^{-1}(b/a)}$$

$AR\,e^{i\phi}$

$AR = \infty$, $\phi = \ominus$

Overall amplitude Ratio & phase Angle :—

Non-interacting

$$G(s) = G_1(s)\,G_2(s) \dots G_N(s)$$

$$AR\,e^{i\phi} = (AR_1 e^{i\phi_1})(AR_2 e^{i\phi_2}) \dots (AR_N)\,e^{i\phi_N}$$

$$AR\,e^{i\phi} = \underbrace{(AR_1\,AR_2\,AR_3 \dots AR_N)}_{\text{multiplication}}\ e^{\overbrace{i(\phi_1+\phi_2+\phi_3 \dots +\phi_N)}^{\text{summation}}}$$

$$\therefore \boxed{AR = AR_1\,AR_2 \dots AR_N}$$

$$\boxed{\ln AR_{overall} = \ln AR_1 + \ln AR_2 + \dots + \ln AR_N}$$

$$\boxed{\phi_{overall} = \phi_1 + \phi_2 + \phi_3 + \dots + \phi_N}$$

Bode plot for the first order System:-

$$AR = \frac{1}{\sqrt{1+\tau^2\omega^2}} \qquad 10k$$

If

$$AR = \frac{K_c}{\sqrt{1+\tau^2\omega^2}}$$

$$MR = \frac{AR}{K_c} = \frac{1}{\sqrt{1+\tau^2\omega^2}}$$

Corner frequency:-

It is defined as the pt. where the two asymptotes meets.

$$\omega = \frac{1}{\tau}$$

$$\boxed{\tau\omega_c = 1}$$

LFA

$\omega \to 0$ $\qquad AR \Rightarrow 1$

$\ln AR \to \ln 1$

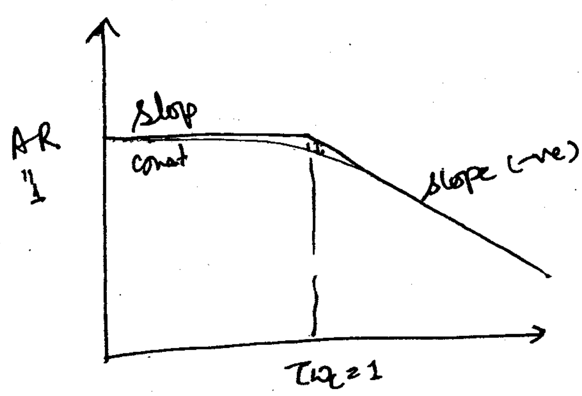
HFA

$\omega \to \infty$ (very high not infinity.) $\qquad AR \simeq \frac{1}{\tau\omega}$

$$\ln AR \Rightarrow \ln\left(\frac{1}{\tau\omega}\right)$$

$$\Rightarrow \ln(\tau\omega)^{-1}$$

$$\to -\ln(\tau\omega)$$



$1 - 0.707$

at corner freque:

$$AR = \frac{1}{\sqrt{1+\tau^2\omega^2}}$$

$\omega = \omega_c \qquad \tau\omega_c = 1$

$$AR = \frac{1}{\sqrt{1+1}}$$

$$= \frac{1}{\sqrt{2}} = \frac{1}{1.414} = 0.707$$

LFA has a constant slope (or zero slope) & HFA has -ve slope
At the corner frequency the amplitude ratio is 0.707 & deviation obtained in bode plote is $(1-0.707)$ & deviation is max^m at the corner frequency.

Phase Angle Frequency :-

$$\phi = \tan^{-1}(-\omega\tau)$$

$\frac{LFA}{\omega \to 0}$ $\quad \phi \to 0$

$\frac{HFA}{\omega \to \infty}$ $\quad \phi \to -\pi/2$

at corner frequency.

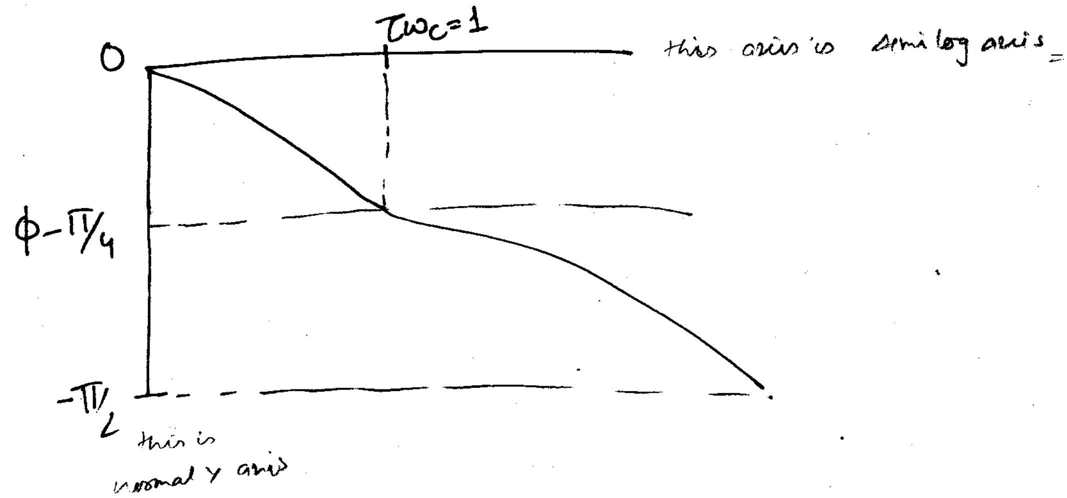
$$\omega = \omega_c$$

$$\phi = \tan^{-1}(-(\omega_c\tau))$$

$$= \tan^{-1}(-1)$$

range $[0, -\pi/2]$

$$\phi = -\pi/4$$



Purely Capacitive System :—

$$G(s) = K_p/s$$

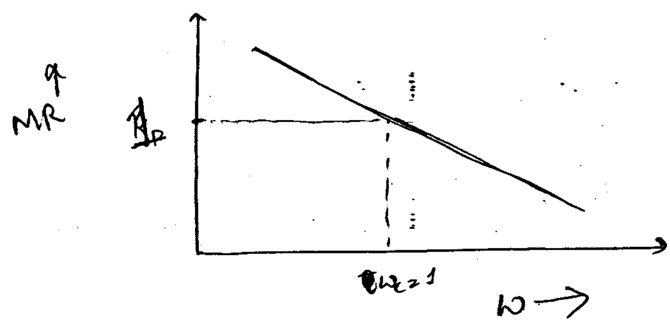
$$G(i\omega) = \frac{K_p}{i\omega} \times \frac{i}{i} = -\frac{iK_p}{\omega}$$

$$AR = \sqrt{0^2 + \left(-\frac{K_p}{\omega}\right)^2} = K_p/\omega$$

$$\phi = \tan^{-1}\left(\frac{-K_p/\omega}{0}\right)$$

$$\phi = \tan^{-1}(\infty)$$

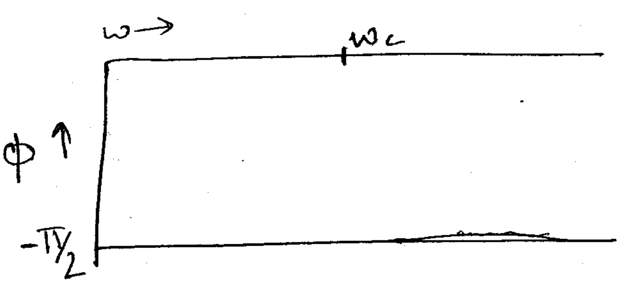
$$= -\pi/2$$



magnitude Ratio

$$MR = \frac{AR}{K_p} = \frac{1}{\omega}$$

$$\boxed{\ln MR = \ln 1/\omega = -\ln \omega}$$



⇒ AR value ↑ as ω ↓ or vice-versa

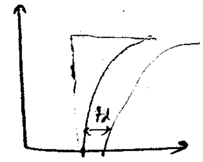
⇒ φ lag remains constant.

Dead tym System :-

$G(i\omega) = r e^{i\theta}$
$r = AR$

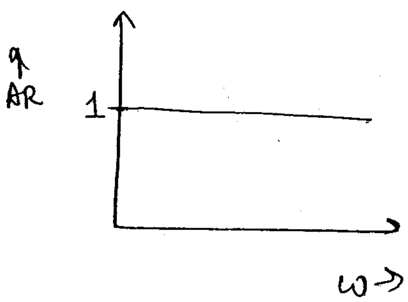
$$G(s) = e^{-t_d s}$$

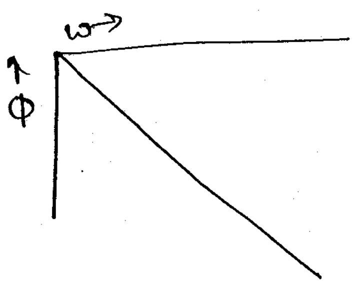
$$G(i\omega) = e^{-i t_d \omega}$$



$$AR = 1 \quad , \quad \phi = -t_d\omega$$

$\ln AR = \ln 1$





decreases infinetly

$\Rightarrow$ AR is constant for dead tym system but ϕ ~~are~~ $\uparrow$ wid $\downarrow$ ω

P-controller

$$G(s) = K_c$$

$$G(i\omega) = K_c$$

There is no need to study the bode plot as both AR, ϕ are const.

$$\Rightarrow \boxed{\begin{matrix} AR = K_c \\ \phi = 0 \end{matrix}}$$

Bode plot has no significance for P-controller

P.I controller :-

$$G(s) = K_c \left(1 + \frac{1}{\tau_I s}\right)$$

$$G(i\omega) = K_c \left(1 + \frac{1}{\tau_I i\omega}\right)$$

$$= K_c \left(\frac{\tau_I i\omega + 1}{\tau_I i\omega} \right)$$

$$= K_c \left(\frac{\tau_I i\omega + 1}{\tau_I i\omega} \times \frac{\tau_I \omega}{\tau_I i\omega} \right)$$

$$= K_c \left(\frac{-\tau_I^2 \omega^2 + i\omega}{-\tau^2 \omega^2} \right)$$

$$= K_c \left(\frac{\tau_I^2 \omega \mp i\omega}{\tau^2 \omega^2} \right)$$

$$=$$

$$AR = K_c \sqrt{1 + \frac{1}{(\tau_I \omega)^2}}$$

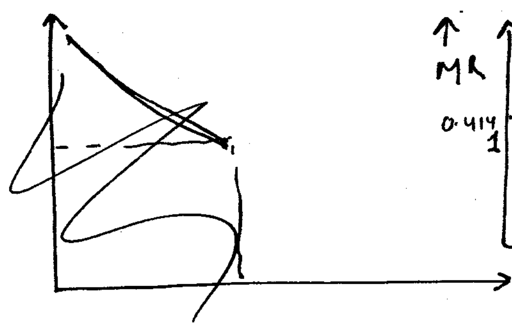
$\underline{LFA}$ $\omega \to 0$ $\quad MR = \frac{AR}{K_c} \to \frac{1}{\tau_I \omega}$ $\quad$ | $\quad$ $\underline{HFA}$ $\omega \to \infty$ $\quad MR \to 1$

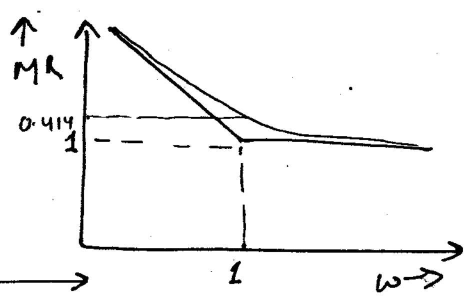
$\ln MR \to -\ln \tau_I \omega$ $\quad$ | $\quad$ $\ln MR \to \ln 1$

at corner frequency

$$\tau_I \omega_c = 1$$

$$MR = \sqrt{2} = 1.414$$





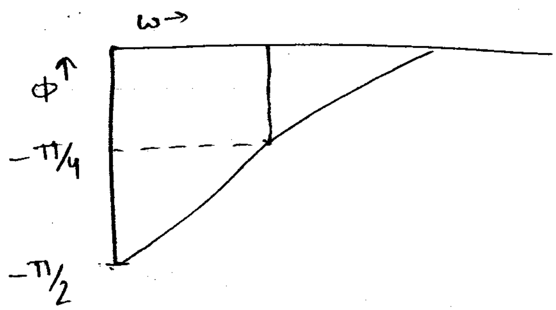
$$\phi = \tan^{-1}\left(\frac{-1}{\tau_I \omega}\right)$$

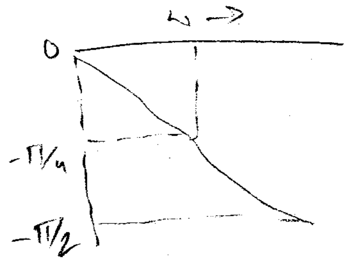
ϕ angle lags, so o/p becomes slow

LFA $\omega \to 0$ $\phi \to -\pi/2$ — **HFA** $\phi \to 0$

at corner frequency

$\phi = -\pi/4$





PD controller

In PD controller there is no phase lag there is phase lead.

$$G(s) = K_c(1+\tau_D s)$$
$$G(i\omega) = K_c(1+\tau_D i\omega)$$

$$\boxed{\tau_D \omega_c = 1}$$

$$AR = K_c\sqrt{1+\tau_D^2\omega^2}$$

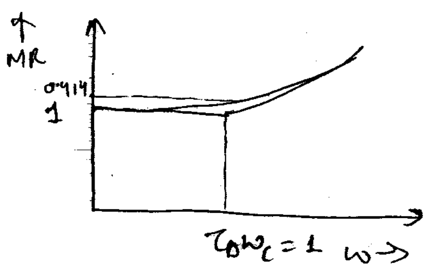
$$MR = \frac{AR}{K_c} = \sqrt{1+\tau_D^2\omega^2}$$

LFA $\omega \to 0$, $MR \to 1$

$\ln MR \to \ln 1$

HFA $\omega \to \infty$ $MR \to \tau_D \omega$

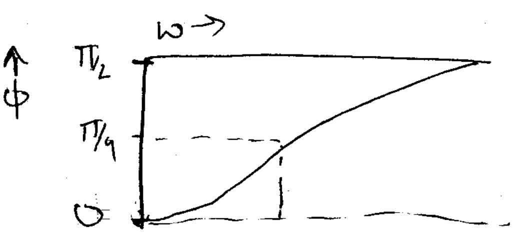
$\ln MR \to \ln \tau_D \omega$



$\phi = \tan^{-1}(\tau_D \omega)$

$\phi \to 0$

$\phi \to \pi/2$



as derivative term ↑, lead ↑.

PID - Controller

$$G(s) = K_c (1+\tau_d s)\left(1+\frac{1}{\tau_I s}\right)$$

$\boxed{\tau_I \omega_1 = 1}$

$\boxed{\tau_D \omega_2 = 1}$

$$\cancel{\frac{AR}{K_c} = \sqrt{1+\left\{\frac{1}{(\tau_I \omega)^2}+(\tau_D \omega)\right\}^2}}$$

$$\frac{AR}{K_c} = \sqrt{1+\left\{\tau_D \omega - \frac{1}{\tau_I \omega}\right\}^2}$$

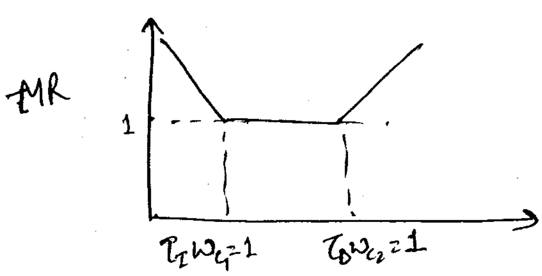
$$\phi = \tan^{-1}\left(\tau_D \omega - \frac{1}{\tau_I \omega}\right)$$

At LFA $\omega \to 0$

$$MR \to \frac{1}{\tau_I \omega} \Rightarrow \ln MR = -\ln \tau_I \omega$$

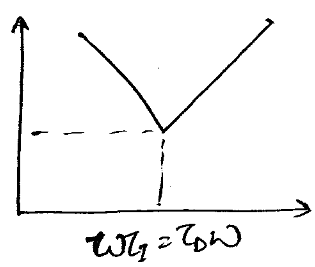
At HFA $\omega \to \infty$

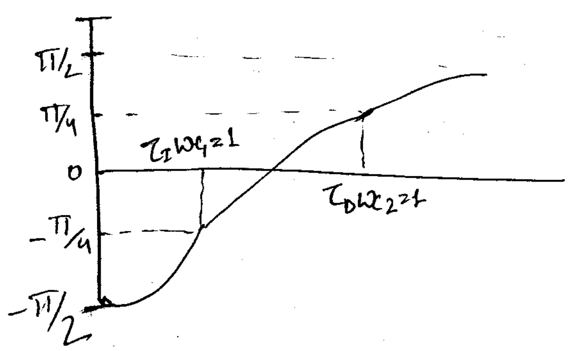
$$MR \to \tau_D \omega \Rightarrow \ln MR = \ln \tau_D \omega$$



if $\tau_I = \tau_D$

$\Rightarrow$





Nov 12, 14

Bode Stability Criterion

applied to these system (ϕ & AR ↓ with ↑ ω)

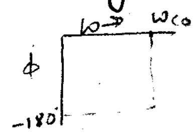
If cross over frequency doesn't exist, then the system is stable.

Cross over frequency:– value of frequency when $\phi = -180$

It is the value of the frequency for which the phase lag is 180°

$$\phi = -180^\circ$$

$$\omega = \omega_{co} \longrightarrow \text{cross over frequency.}$$



in Routh array last elm –ve: Marginal stability

first order Systems

$$G_p(s) = \frac{K_p}{1+\tau_p s}$$

$$AR = \frac{1}{\sqrt{1+(\tau_p\omega)^2}}$$

$$\phi = \tan^{-1}(-\tau_p\omega)$$

$$\phi \in [-\pi/2, 0]$$

ϕ can never be -180°, the cross over frequency doesn't exist.
hence the system is always stable.

Second Order System

$$G_p(s) = \frac{K_p}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

$$\phi = \tan^{-1}\left(\frac{-2\zeta\tau\omega}{1-\tau^2\omega^2}\right).$$

$$\phi \in [-\pi, 0] \quad ; \omega \to \infty.$$

for the finite value of ω, the phase angle remains more than -180°.
In other words, the cross over frequency doesn't exist.
Hence second order system are also stable.

Third Order System

$G_C = K_C \quad G_P = \frac{K_P}{1+\tau_P s} \quad G_F = \frac{K_F}{1+\tau_F s} \quad , G_m = \frac{K_m}{1+\tau_m s}$

$$= \frac{K_C K_P K_F K_m/(1+\tau_P s)(1+\tau_F s)}{1 + \frac{K_C K_P K_F K_m}{(1+\tau_P s)(1+\tau_F s)(1+\tau_m s)}}$$

$$= \frac{\frac{K_C K_P K_F}{(1+\tau_P s)(1+\tau_F s)}}{\frac{(1+\tau_P s)(1+\tau_F s)(1+\tau_m s) + K_C K_P K_F K_m}{(1+\tau_P s)(1+\tau_F s)(1+\tau_m s)}}$$

$$= \frac{K_C K_P K_F (1+\tau_m s)}{(1+\tau_P s)(1+\tau_F s)(1+\tau_m s) + K_C K_P K_F K_m}$$

$$AR = \frac{1}{\sqrt{1+(\tau_P \omega)^2}} \cdot \frac{1}{\sqrt{1+(\tau_m \omega)^2}} \cdot \frac{1}{\sqrt{1+(\tau_F \omega)^2}}$$

$$\phi = \tan^{-1}(-\omega\tau_P) + \tan^{-1}(-\omega\tau_F) + \tan^{-1}(-\omega\tau_m)$$

$\omega \to \infty , \quad \phi = -270^\circ$
$\phi = 0^\circ$

for finite value of ω, ϕ can be -180° or may be more than that,
$\therefore$ third or higher order kinetics are generally unstable

for n system of first order

$$(AR)_{ovrl} = AR_1 \times AR_2 \times \cdots AR_n$$

$$(\phi)_{ovrl} = \phi_1 + \phi_2 + \phi_3 \cdots + \phi_n$$

First Order in Series wid dead tym System

$e^{i\phi}$

$$G(s) = \frac{K_p e^{-\tau_d s}}{1+\tau^2\omega^2}$$

$$\underbrace{\phi}_{\text{degree}} = \underbrace{\tan^{-1}(-\tau\omega)}_{\text{degree}} + \overbrace{\underbrace{(-\tau_d\omega)}_{\text{radian}}}^{\text{degree}} \times \underbrace{\frac{180^\circ}{\pi}}_{57.295}$$

$\omega \to \infty$, $\phi = \infty$

for the infinite value of ω, $\phi \to \infty$ which means when ω is finite the ϕ can be more than -180° which means the system (first order system wid dead tym) is unstable.

Second Order System in Series wid dead tym System

$$G(s) = \frac{K_p e^{-\tau_d s}}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

$$\cancel{\phi =} \tan^{-1}\left(\frac{-2\zeta\tau\omega}{1-\tau^2\omega^2}\right) + (-\tau_d\omega)\times\frac{180}{\pi}$$

$$\phi = 2\tan^{-1}(-\tau\omega) + (-\tau_d\omega)\times\frac{180^\circ}{\pi}$$

$$\phi = \tan^{-1}(-\tau_p\omega) + \tan^{-1}(-\tau_f\omega) + (-\tau_d\omega)\times\frac{180}{\pi}$$

→ If cross over frequency exist, then there are chances of unstability

→ If ϕ becomes more than 180, then the system surely be unstable

→ If at cross over frequency, the amplitude ratio of the system is more than 1, then system ~~be~~ surely be unstable. If it is 1, then the system marginal stable. But if it ~~stat~~ less than 1 then the system will be stable.

Q:> . $G_p = \frac{2}{(S-1)}$, It is to be controlled wid help of PI controller. If the transfer functⁿ of all other elements are unity. Then what is the value of K_c for which system produce a stable response.

$\tau_I = 2$

Solⁿ

$$\phi = \tan^{-1}(-\cancel{\tau}\omega) + \tan^{-1}\left(\frac{-1}{\tau_I\omega}\right)$$

$$\frac{2}{(i\omega-1)}$$

$$\frac{2}{(i\omega-1)} \times \frac{(i\omega+1)}{(i\omega+1)}$$

$$\frac{2+2i\omega}{-\omega^2-1}$$

$$\frac{2}{-\omega^2-1} + i\frac{2\omega}{-\omega^2-1}$$

$$= \frac{2}{-(1+\omega^2)} + \frac{i\,2\omega}{-(1+\omega^2)}$$

as $\phi = 180°$

$$-180° = \tan^{-1}(\omega) + \tan^{-1}\left(\frac{-1}{2\omega}\right)$$

$$\tan^{-1}\left(\omega + \frac{1}{2\omega}\right)$$

$$\omega_{co} = 0.707$$

$$AR = \sqrt{}$$

$$AR = \frac{2}{\sqrt{1+\omega^2}} \cdot K_c\sqrt{\left(1+\left(\frac{1}{2\omega^2}\right)^2\right)}$$

$=$

$$1 \cancel{\times} \quad \frac{\cancel{2}}{} \; 1.6330 \cdot K_c \; 1.2248$$

$$K_c \cancel{\times}\; 0.499$$

Steps for checking the Stability of System (Analytical Procedure)

→ Write the expression of ϕ angle

$$\phi = \phi_1 + \phi_2 + \phi_3 + \dots + \phi_n$$

→ Put $\phi = -180°$ & calculate the value of frequency, this frequency is cross over frequency.

→ Write the expression for overall amplitude ratio.

$$AR = AR_1 \cdot AR_2 \cdot AR_3 \cdots AR_n$$

→ Put the value of cross over frequency in the expression of AR, & Calculate the value of AR. $\boxed{\omega = \omega_{co}}$

if
AR > 1 , unstable
AR = 1 , Marginal Stable
AR < 1 , Stable.

Q.) $G(s) = \dfrac{5}{(2s+1)^4}$. Check whether system is stable or not.

4 first order system are in series

$$\frac{5}{(2i\omega+1)^4}$$

$$\phi = 4\tan^{-1}(-2\omega)$$

$$-180° = 4\tan^{-1}(-2\omega)$$

$$\omega = 0.5$$

$$\omega_{co} = \omega = 0.5$$

$$AR = \frac{5}{(1+4\omega^2)^2} = 1.25$$

if P controller is added, find value of K_c for stable syst.

$$\frac{5k}{4} \qquad 1.25K_c < 1$$

$$K_c < 0.8$$

Q) $G(s) = \frac{K_c e^{-1.23s}}{0.1s+1}$. Find the value of K_c for the marginal stable system.

$$-180 = \tan^{-1}(-0.1\omega) + (-1.23\,\omega) \times \frac{180}{\pi}$$

$$= \qquad - 70.50955\omega$$

$$\omega c_0 = 17 \text{ rad/min.}$$

$$AR = \frac{K_c}{\sqrt{1+(0.1\omega)^2}}$$

$$K_c = 1.972$$

$$K_c \simeq 2$$

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Safety Parameters

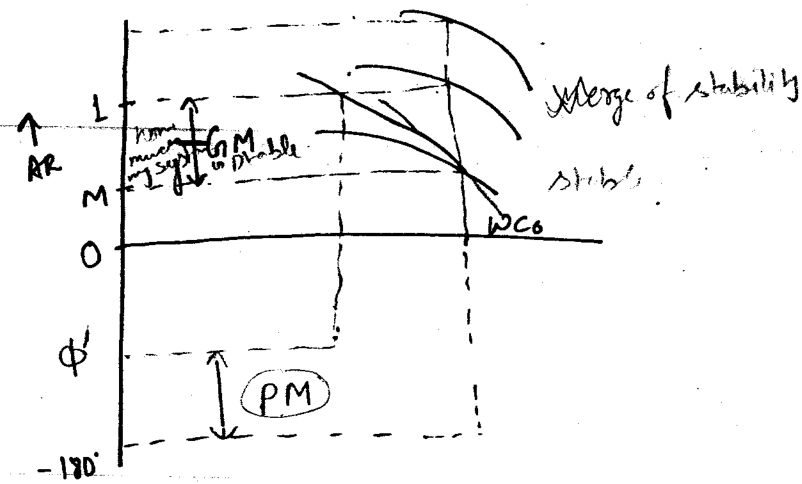
Gain Margin :— It is denoted as a safety factor, we know that we can calculate the cross over frequency for a phase angle of $-180°$ for checking the stability of the system. We calculate the value of amplitude ratio and if AR is 1, then the system is Marginal stable or on the verge of instability. but if $AR < 1$, then the system is stable. Gain Margin is the difference b/w these two conditions.

$$\ln 1 - \ln M$$

$$\ln \frac{1}{M}$$

$$\boxed{GM = \frac{1}{M}}$$

M = Value of AR at ωc_0.



Stability: $GM > 1$, $M < 1$

$$\boxed{PM = 180 - |\phi'|}$$

ϕ' = value of lag at ω_{co}

GM

$\boxed{GM \simeq 1.7}$ safer value — if not given any value of GM in question then we take $GM \simeq 1.7$

1) $\phi = \phi_1 + \phi_2 + \phi_3 + \cdots + \phi_N$

$\phi = -180°$ $\quad \omega = \omega_{co}$

2) $AR = AR_1 \, AR_2 \cdots AR_N$

$\omega = \omega_{co}$ $\quad AR = \text{Value} = M$

3) $GM = 1/M$

PM

$\boxed{PM = 30°}$ Safer value

1) $AR = AR_1 \, AR_2 \cdots AR_N$

2) $AR = 1$, $\quad \omega = \omega_{co} = ?$

3) by putting $\omega = \omega_{co}$, $\quad \phi = \phi'$

$$PM = 180 - |\phi'|$$

Q7 $G(s) = \dfrac{K_c e^{-2s}}{0.5s+1}$

$$\phi = \tan^{-1}(-0.5\omega) + (-2\omega) \times \frac{180}{\pi}$$

$-180 =$

$-65.408 = \tan^{-1}(-0.5\omega)$

$\omega =$ ~~4.872~~ 1.285 rad/min

$\omega_{co} =$ ~~4.872~~ 1.285 rad/min

$$AR = \frac{K_c}{\sqrt{1+(0.5\omega)^2}}$$

$\cancel{\dfrac{K_c}{2.4029} = AR}$ if $AR = 1$

$\cancel{K_c = 2.402}$

$$1 = AR = \frac{K_c}{\sqrt{1+(0.5\omega)^2}}$$

$K_c = 1.188$ for (Marginal stable)

if $\boxed{K_c = 2}$

$$AR = \frac{2}{\sqrt{1+(0.5\omega)^2}}$$

$M = AR = 1.6826$

$$GM = \frac{1}{1.6826} = 0.594$$

Q7 $G = \dfrac{0.8\,K_c\,e^{-1.74s}}{(5s+1)(10s+1)(15s+1)}$. If GM = 1.7, Calculate K_c?

$$-180^\circ = \tan^{-1}(-5\omega) + \tan^{-1}(-10\omega) + \tan^{-1}(-15\omega)$$
$$(-1.74\,\omega) \times \frac{180}{\pi}$$

$$\omega_{C_0} = 0.16.$$

4.12

$$AR = \frac{0.8\,K_c}{\sqrt{1+(5\omega_{c_0})^2}\,\sqrt{1+(10\omega_{c_0})^2}\,\sqrt{1+(15\omega_{c_0})^2}}$$

1.2806 1.886. 2.6

$$GM = \frac{1}{M}$$

$$1.17 = \frac{1}{\quad}$$

$$M = \frac{1}{1.17} = 0.8547$$

$$\boxed{\begin{matrix}\omega = \omega_{co} \\ AR = M\end{matrix}}$$

error
$G_c = 80\%$ ...
1.17.
find K_c is ...

$$GM = 1.17$$
$$GM' = 1.17 + 0.585$$
$$= 1.755$$

$$M = \frac{1}{GM'} = 0.569800$$

$0.569800 =$

1.) $\phi =$

put $\phi = -180°$

$\omega = 0.147$

2.) AR =

Put $\omega = 0.147$ & check AR.

GM = 1.44

AR = 0.689

% $\downarrow$ = 1.5293

Q.) PM = 30°, calculate K_c value.

$$AR = \frac{0.8 K_c e^{-1.74s}}{(1+5s)(1+10s)(1+15s)}$$

$$AR = \frac{0.8\ K_c}{\sqrt{1+(5\omega)^2} + \sqrt{1+(10\omega)^2} + \sqrt{1+(15\omega)^2}}$$

1.1525   1.5209   1.988

$PM = 180 - |\phi'|$

$30° = 180 - |\phi'|$

$\cancel{\phi'} = 180$

$$30° = 180 - \left| \tan^{-1}(5\omega) + \tan^{-1}(10\omega) + \tan^{-1}(15\omega) + 1.74\omega \times \frac{180}{\pi} \right|$$

$\omega = ?$

$\omega = \omega_{co} = 0.1146$

$\Rightarrow$ $\phi = \phi'$

$\omega = \omega_{co}$.

$1 = AR = 0.1716\ K_c$

$K_c = \cancel{5.826}$
$= 4.358$ Ans

Q) If dead tym error is 100%, then check for this K_c value the system is stable or not.

1.74 + 1.74

stable

$$\phi = \tan^{-1}(-5\omega) + \text{-----} \left(\quad \right) \frac{180}{\pi}$$

$\phi = -180$

$\omega = \omega_{co}$ $\qquad$ $\omega_{co} = 0.1374$

$AR = \frac{\quad 4.357}{}$

$AR < 1$

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Q)
$$G(s) = \frac{K_c}{S(\cancel{5S}+1)^2}$$

1) Find K_c for stable system
2) Find GM for $K_c = 3$
3) If GM = 1.7, Find K_c.

(frequency-response)

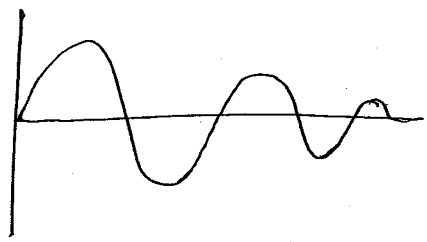
Zeigler Nichols tunning:- online tunning method

~~It comes under frequency~~

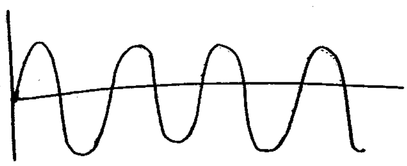
It is frequency response analysis and comes under closed loop tunning method.

Method

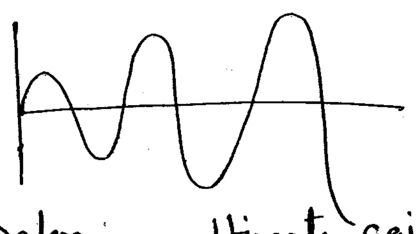
- Bring the System to the steady state | Process is in operation.
- Use only the P-controller | K_c
- Close the feed back loop.
- Introduce a set pt change.
- Varry the value of K_c, untill the system Oscillates wid constant amplitude.



Stable



Marginal stable



Unstable

- Determine ultimate gain K_u
- Determine ultimate period P_u

Determine Ultimate gain K_u:- in simple words $= \cancel{K_u} =$ reciprocal of AR

It is the process gain that provides the AR $=1$, at cross-over frequency.

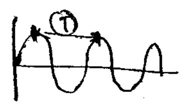
It is the the controller gain that provides the AR$=1$, at cross-over frequency. which we get sustained oscillation i.e. (marginal stability)

It is also defined as the reciprocal of AR of the System response at the cross-over frequency. | $GM = K_u$

Ultimate Period:- (P_u):-

It is defined as the tym taken between two successive peaks in the continuosly oscillating process o/p.

$T = 2\pi/\omega$, $\omega = \omega_{co}$



$$\boxed{T = \frac{2\pi}{\omega_{co}} = P_u}$$ unit = cycles/min

Controllers	K_c	τ_I	τ_D
P	$K_u/2$		
PI	$K_u/2.2$	$P_u/1.2$	
PID	$K_u/1.7$	$P_u/2$	$P_u/8$

K_c is more bcz of Δ action system remains stable.

Q.) For the transfer functn $G(s) = \dfrac{1}{(5s+1)(2s+1)(10s+1)}$

Limitations

- It is not used when the system becoming frequently unstable.
- It is essential to obtain accurate open loop transfer functh along with the accurate value of dead tym.

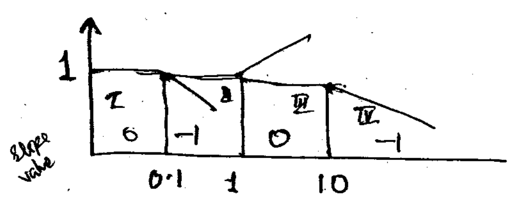
Graphical AR additional Method
OR
Rules for Obtaining Bode Plot for Multicapacity System :-

- The overall amplitude ratio for the controll system is obtained by the multiplying the individual AR and take logarithm on BS.
- The overall ϕ angle is obtained by simple addition of the individual phase angle.
- For graphical amplitude ratio addition, An individual AR, which is above 1 is taken as +ve & AR which is below 1 is taken as -ve.
- The overall AR curve can then be obtained.
- Overall ϕ, obtained by addition of different ϕ angles for diff. systems can be used to plot the second bode plot.

Q) $$G(s) = \frac{(s+1)}{(0.1s+1)(10s+1)}$$

Sol^n $\tau \rightarrow$ 10 1 0.1 $\quad \omega_c = \frac{1}{\tau}$

$\omega_{c_1} = 0.1 \quad \omega_{c_2} = 1 \quad \omega_{c_3} = 10$

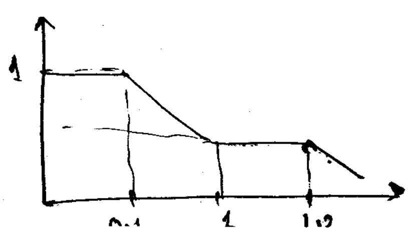


0 - 0.1 ⇒ 0

0.1 - 1 ⇒ 0 - 1 = -1

1 - 10 ⇒ -1 + 1 = 0

(10) ⇒ 0 - 1 = -1



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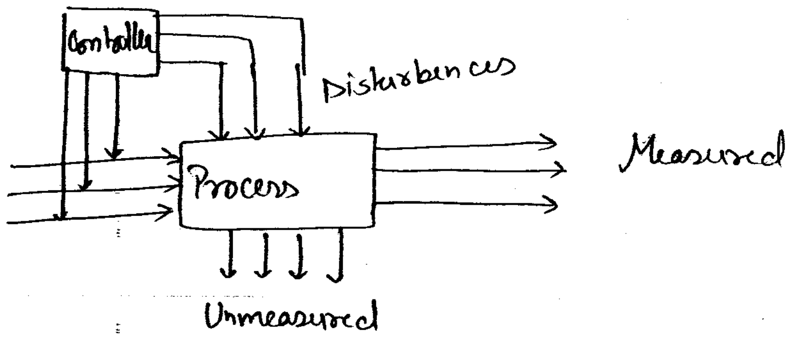
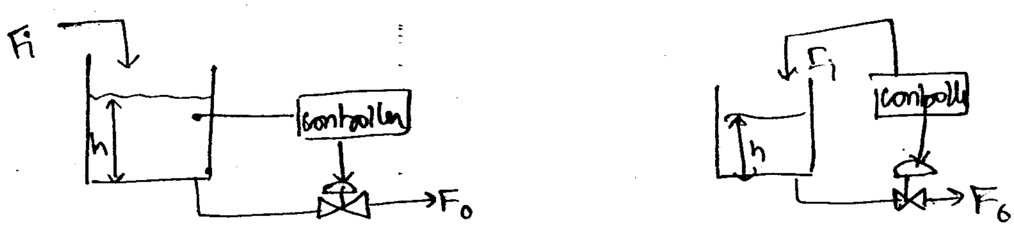
Feed forward Controller :—



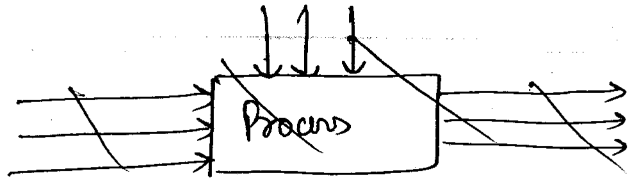


A feed back controller responds only after it detects deviation in the value of the controlled o/p from its desired set pt. &

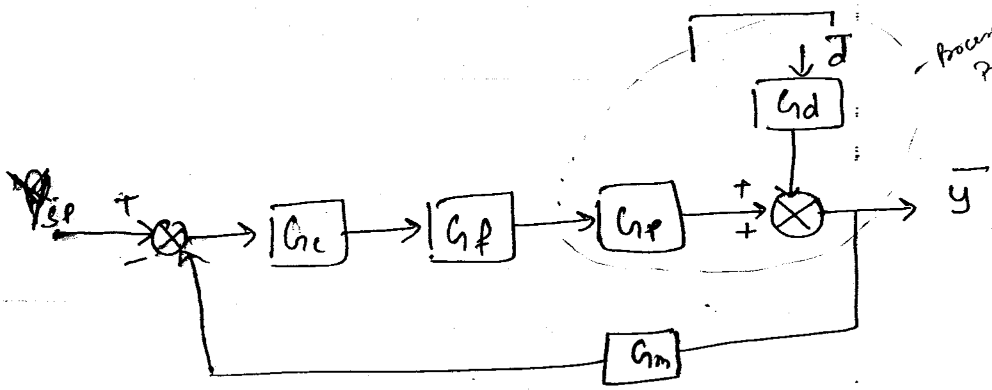
On the other hand a feed forward controller detects the disturbance directly and takes an appropriate control action in order to eliminate its controll effect on the process o/p.

In other words, feed back action starts after the disturbance is felt through the changes in the process o/p, the feed forward controll action starts immidiately after the disturbance is measured directly.

Hence, feed back controller acts in a compensatory manner, while feed forward controller acts in a anticipatory manner.

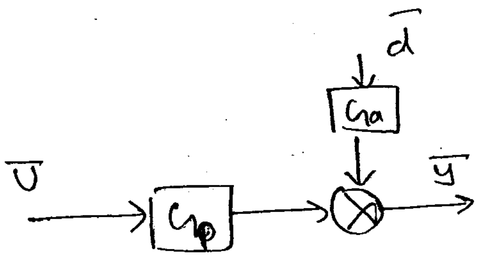


Feed backward



- Process contains Process system & disturbance system

Feedforward

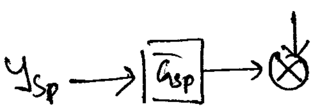


$$\bar{y} = \bar{u} G_p + \bar{d} G_d$$

$$\boxed{\bar{y}_{sp} = \bar{u} G_p + \bar{d} G_d}$$

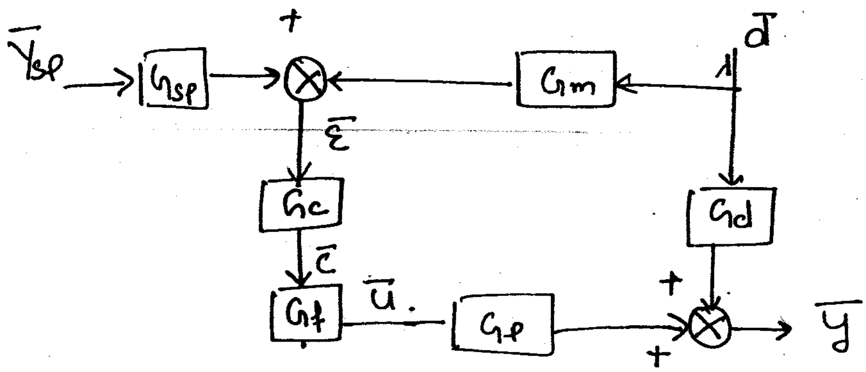
Assumption.

$y_{sp} \sim y$



In case of F.B, Value of $G_{sp} = 1$, but in case of FF G_{sp} has some value

$$\bar{u} = (\bar{y}_{sp} \bar{G}_{sp} - \bar{d} G_{d}) G_c G_f$$



— = represent deviation variable.

$$\bar{y}_{sp} = \bar{u}\, G_p + \bar{d}\, G_d$$

Comparing the eqn.

$$\bar{y}_{sp} = \left(\frac{\bar{u}}{G_c G_f} + \bar{d}\, G_m \right) \frac{1}{G_{sp}}$$

$$\cancel{\frac{\bar{u}}{G_c G_f} + \bar{d}\, G_m} = \cancel{G_{sp}\left(\bar{u} G_p + \bar{d}\, G_d\right)}$$

$$\bar{y}_{sp} = \frac{\bar{u}}{G_c G_f G_{sp}} + \frac{\bar{d}\, G_m}{G_{sp}}$$

by comparing. finding the values of G_{sp} & G_c

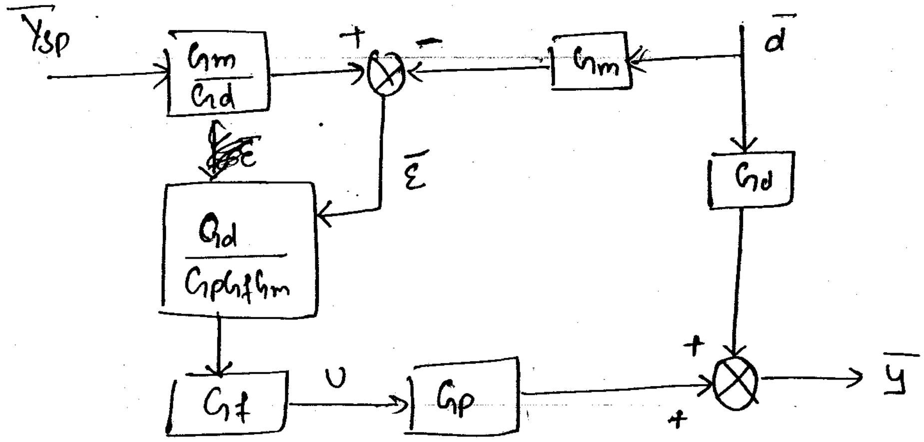
$$G_d = \frac{G_m}{G_{sp}}$$

$$\boxed{G_{sp} = \frac{G_m}{G_d}}$$

$$G_p = \frac{1}{G_c G_f G_{sp}}$$

$$G_p = \frac{G_d}{G_c G_f G_m}$$

$$\boxed{G_c = \frac{G_d}{G_p G_f G_m}}$$



for servo mode

$$\bar{y} = \left(\bar{y}_{sp}\frac{G_m}{G_d} \mp G_m\bar{d}\right)\frac{G_d}{G_pG_fG_m}G_fG_p$$

$$= \bar{y}_{sp}\frac{\cancel{G_mG_dG_fG_p}}{\cancel{G_dG_fG_mG_p}} - \bar{d}\frac{G_mG_dG_fG_p}{G_pG_fG_m}$$

for Regulatory mode

$$y = (-\bar{d}G_m)G_cG_fG_p + \bar{d}G_d$$

Overall

$$\bar{y} = \left(\bar{y}_{sp}\frac{G_m}{G_d} - \bar{d}G_m\right)\frac{G_d}{G_pG_fG_m}G_fG_p + \bar{d}G_d$$

$$\bar{y} = \bar{y}_{sp}\left(\frac{G_mG_dG_fG_p}{G_dG_pG_fG_m}\right) + \bar{d}\left(G_d - \frac{G_mG_dG_fG_p}{G_pG_fG_m}\right)$$

$$y = \overline{y}_{sp} + \overline{d}\,(G_d - G_d)$$

$$y = \overline{y}_{sp}$$

The contol objective is to maintain $\overline{Y}$ at desired set pt., $\overline{y}_{sp}$ and $\therefore$ above eq^n comes out at $\boxed{\overline{y} = \overline{y}_{sp}}$

For perfect disturbence Rejection in case of Regulatory Mode :-

$$\overline{y} = G_p \overline{u} + \overline{d}\, G_d$$

$$\overline{y} = G_p \{ (\overline{y}_{sp} G_{sp} - \overline{d}\, G_m) G_c G_f \} + \overline{d}\, G_d \}$$

for regulatory mode

$$\overline{y}_{sp} = 0$$

for perfect Rejection

$$\boxed{0 = \overline{d}\,(G_d - G_m G_c G_f G_p)}$$

for regulatory mode $\overline{d} \neq 0$

So $(G_d - G_m G_c G_f G_p)\ 0$

$$\boxed{G_c = \frac{G_d}{G_m G_f G_p}}$$

Perfect Disturbance Rejection

$$\boxed{G_c = \frac{G_d}{G_p}}$$

$(G_m = G_f = 1)$

For Servo Problem

when $d = 0$, the controller should be able to ensure that o/p tracks the set pt.

$$\boxed{\bar{y} = \bar{y}_{sp}}$$

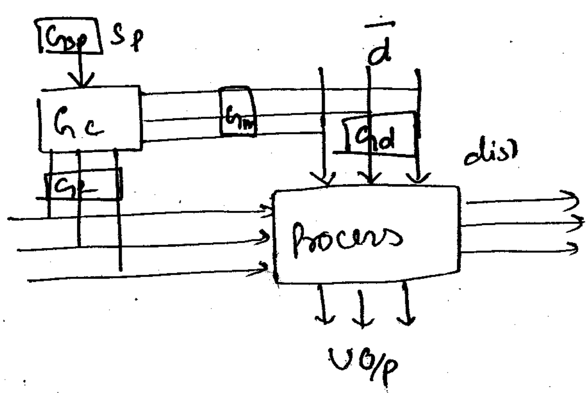
$$\bar{y} = G_p\left((\bar{y}_{sp} G_{sp} - \bar{d}\, G_m) G_c G_f \right) + \bar{d}\, G_d$$

$$y_{sp} = (G_p G_{sp} \bar{y}_{sp})\, G_c G_f$$

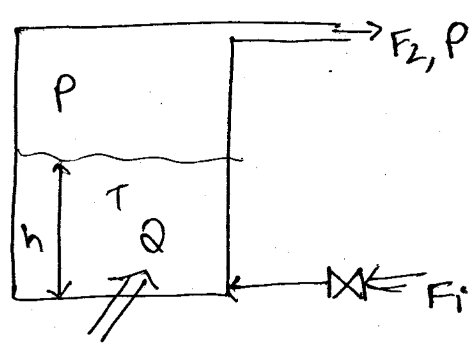
$\bar{d} = 0$
$\bar{y} = \bar{y}_{sp}$

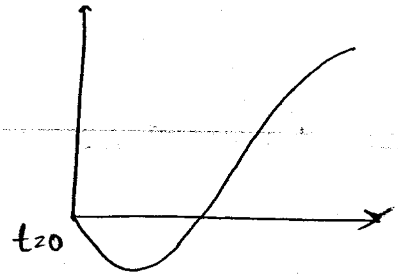
$$\boxed{G_p G_{sp} G_c G_f = 1}$$

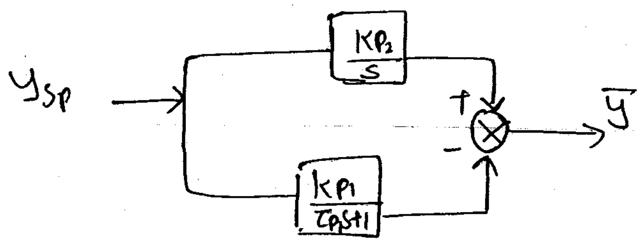
$$\boxed{G_c = \frac{1}{G_p G_{sp} G_f}}$$



Inverse Response







$$\bar{y} = \bar{y}_{sp}\left(\frac{K_{P_2}}{s} - \frac{K_{P_1}}{\tau_{P_1}s+1}\right)$$

$$\frac{\bar{y}}{\bar{y}_{sp}} = \frac{K_{P_2}}{s} - \frac{K_{P_1}}{\tau_{P_1}s+1}$$

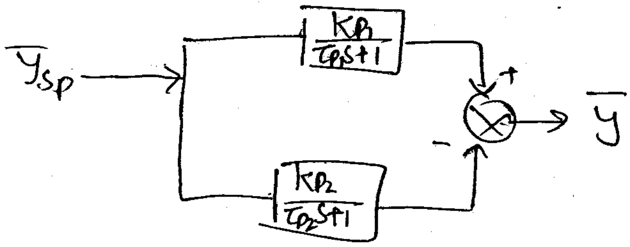
$$= \frac{K_{P_2}(\tau_{P_1}s+1) - sK_{P_1}}{s(\tau_{P_1}s+1)}$$

$$\frac{\bar{y}}{\bar{y}_{sp}} = \frac{s(K_{P_2}\tau_{P_1} - K_{P_1}) + K_{P_2}}{s(\tau_{P_1}s+1)}$$

$$\boxed{K_{P_2}\tau_{P_1} \lessgtr K_{P_1}}$$

jis kisi bhi sys^m ka gain jada hoga pehle hi steady state achieve karega jo pehle wala jada hai if not min.

if two first order system are in parallel



$$\frac{\bar{y}}{\bar{y}_{sp}} = \frac{K_{P_1}}{\tau_{P_1}s+1} - \frac{K_{P_2}}{\tau_{P_2}s+1}$$

$$= \frac{K_{p_1}(\tau_{p_2}s+1) - K_{p_2}(\tau_{p_1}s+1)}{(\tau_{p_1}s+1)(\tau_{p_2}s+1)}$$

$$= \frac{(K_{p_1}\tau_{p_2} - K_{p_2}\tau_{p_1})s + K_{p_1} - K_{p_2}}{}$$

for the inverse response, the transfer funct^n exhibit a +ve zero

$$s = \frac{-(K_{p_1} - K_{p_2})}{K_{p_1}\tau_{p_2} - K_{p_2}\tau_{p_1}} \quad ; = \text{shud be } -ve$$

$$\boxed{K_{p_2}\tau_{p_1} > K_{p_1}\tau_{p_2}}$$

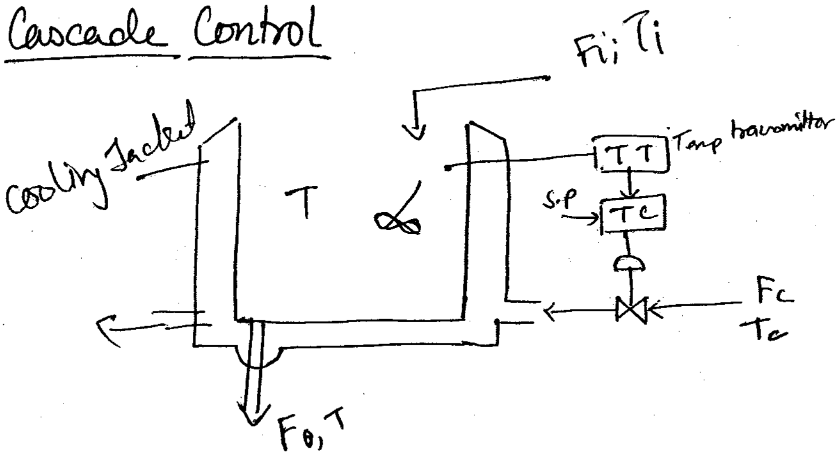
$$\boxed{\frac{\tau_{p_1}}{\tau_{p_2}} > \frac{K_{p_1}}{K_{p_2}}}$$

Initially the response is in opposite direction, bcz τ_{p_2} is less. to where its eventually ends up on +ve side.

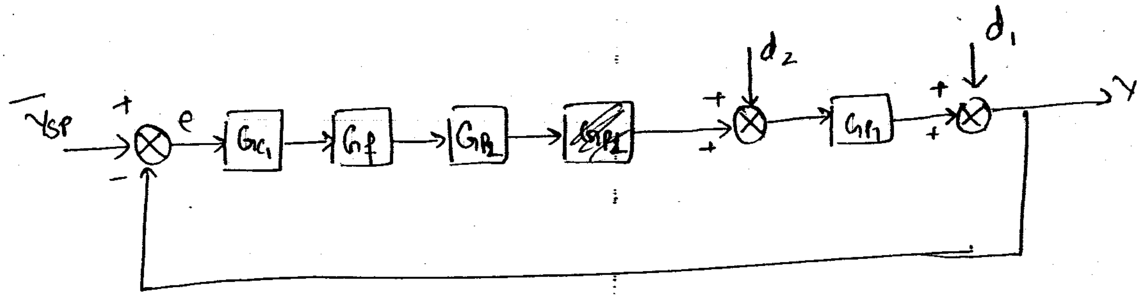
for ex⇒ liquid level in a boiler system.

Nov 19, 14

Cascade Control

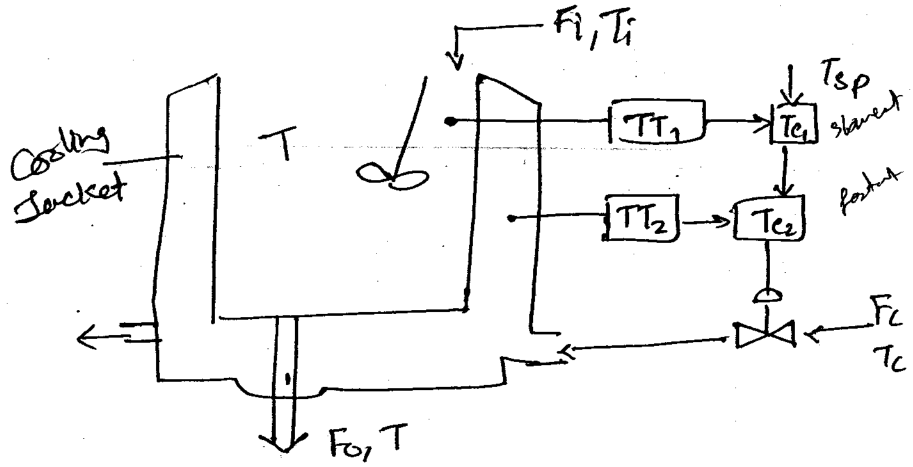


In the given fig, there is a CSTR, in which an exothermic rxn is taking place. To maintain the temp. of the reactor a cooling jacket is provided. The inlet stream have the flow rate F_i & temp T_i. The temp of the rxn at any tym is T, the coolants enters wid a flow rate F_c, & temp T_c

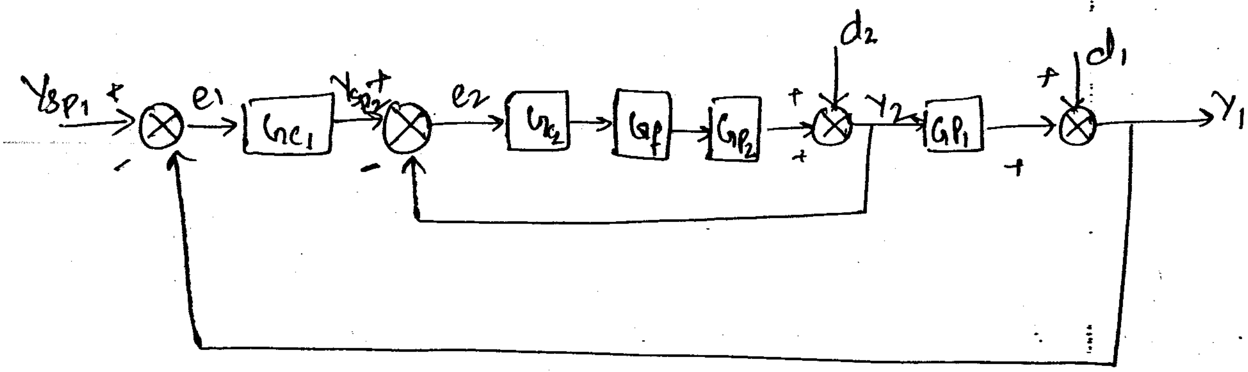


If the temp of the rxn is not at the desired set pt, then the error same ~~controller~~ generated, and controller will do the required control action wid the help of valve. The valve then changes the flow rate of coolant and then accordingly the coolant reduces the error. and temp of rxn track the desired set of temp.

If there are some changes in the inlet feed temp. then the process temp. changes at the same tym, bt if there is some changes in coolant temp. then the process temp changes slowly, which means that the process is more sensitive toward the changes in T_i & less sensitive towards the changes in T_c. In the same way any deviation caused in the rxn temp. by T_i, is controlled by the controller at the same tym and process will reach the steady state in less tym, ~~Any~~ deviation caused by T_c in the rxn will nt be corrected by the controller at the same tym or the process will take more tym to reach the steady state. Hence controller is more sensitive towards the changes in T_i bt less sensitive towards the changes in T_c.



These controller is faster which is attached to Manipulated variable or valve.



$$Y_1 = Y_2 G_{P_1} + d_1$$

$$Y_2 = [e_2 G_{c_2} G_{P_2} + d_2]$$

$$e_2 = (Y_{sp_2} - Y_2)$$

$$Y_{sp_2} = e_1 G_{c_1}$$

$$\& \; e_1 = Y_{sp_1} - Y_1$$

$$Y_1 = [e_2 G_{c_2} G_{P_2} + d_2] G_{P_1} + d_1$$

$$= \left[\left\{(Y_{sp_1} - Y_1) G_{c_1} - Y_2\right\} G_{c_2} G_{P_2} + d_2\right] G_{P_1} + d_1$$

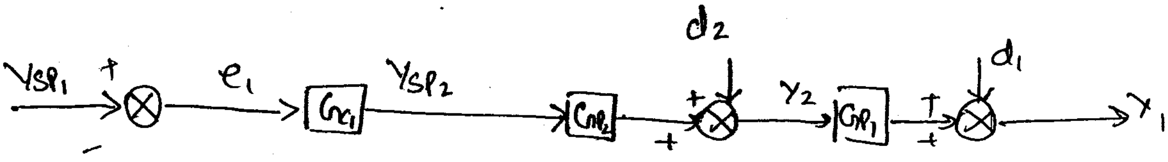
$$= \cancel{Y_{sp_2} - Y_1}$$

$$= (Y_{sp_1} G_{c_1} - Y_1 G_{c_1} - Y_2) G_{c_2} G_{P_2} G_{P_1} + d_2 G_{P_1} + d_1$$

$$Y_1 = Y_{sp_1} G_{c_1} G_{c_2} G_{p_2} G_{p_1} - Y_1 G_{c_1} G_{c_2} G_{p_2} G_{p_1} - Y_2 G_{c_2} G_{p_2} G_{p_1} + d_2 G_{p_1} + d_1$$

$$Y_1 = \underbrace{\frac{G_{p_1} G_{p_2} G_{c_1} G_{c_2}}{1 + G_{p_2} G_{c_2} + G_{p_1} G_{p_2} G_{c_1} G_{c_2}}}_{\zeta} Y_{sp_1} + \frac{G_{p_1}}{\zeta} d_1 + \frac{1 + G_{p_2} G_{c_2}}{\zeta} d_2$$

$\zeta = ?$



$$Y_1 = \left[\left((Y_{sp_1} - Y_1) G_{c_1} G_{p_2} + d_2 \right) G_{p_2} + d_1 \right]$$

$$Y_1 = \underbrace{\frac{G_{c_2} G_{p_1} G_{p_2}}{1 + G_{c_2} G_{p_1} G_{p_2}}}_{\zeta} Y_{sp_1} + \frac{G_{p_1}}{\zeta} d_1 + \frac{1}{\zeta} d_1$$

characteristic eqn

w/o cascade present.

$$1 + G_{c_2} G_{p_1} G_{p_2} = 0$$

if cascade is present

$$1 + G_{c_2} G_{p_2} + G_{c_1} G_{c_2} G_{p_1} G_{p_2} = 0$$

\# $G_{c_1} = K_{c_1}$, $G_{c_2} = K_{c_2}$, $G_{p_1} = \frac{1}{(s+1)^2}$, $G_{p_2} = \frac{1}{s+1}$. Find the stability.

if cascade is present

$$1 + \frac{K_{c_2}}{(s+1)} + \frac{K_{c_1} K_{c_2}}{(s+1)^2 (s+1)} = 0$$

$$(s+1)^3 + K_{c_2} (s+1)^2 + K_{c_1} K_{c_2} = 0$$

$$s^3 + 3s^2 + 3s + K_{c_2} (s^2 + 2s) + K_{c_2} + 1 + K_{c_1} K_2 = 0$$

$\cancel{1+KG}$

$$s^3+s^2(3+K_{c_2})+s\,(3+2K_{c_2})+1K_{c_2}+K_{c_1}K_{c_2}=0$$

1	$3+2K_{c_2}$
$3+K_{c_2}$	$1+K_{c_2}+K_{c_1}K_{c_2}$

$$b_1=(9+6K_{c_2}+3K_{c_2}+2K_{c_2}^2)$$
$$-(1+K_{c_2}+K_{c_1}K_{c_2})>0$$
$$8+8K_{c_2}+2K_{c_2}^2>K_{c_1}K_{c_2}$$

---

if secondary loop is absent

$$1+\frac{KG}{(S+1)^3}=0$$

$$S^3+1+3S^2+3S+K_{c_1}$$

1	3
3	$1+K_{c_1}$

$$b_1>0$$
$$9-1-K_{c_1}>0$$
$$8>K_{c_1}$$

$$K_{c_1}<8+\frac{8}{K_{c_2}}+2K_{c_2}$$

$$K_{c_1}<8+\left(\frac{8}{K_{c_2}}+2K_{c_2}\right)$$

Cascade controller are more stable than the single controller bcz the K_c (controller gain) range has increased.

if secondary loop is present, then find Y_2 in terms of Y_{SP_2}

$$Y_2=(Y_{SP_2}-Y_2)\,G_{c_2}G_{P_2}$$

$$Y_2=\frac{G_{c_2}G_{P_2}}{1+G_{c_2}G_{P_2}}Y_{SP_2}$$

$$\frac{Y_2}{Y_{SP_2}}=G_{P_2}=\frac{K_{P_2}}{1+\tau_{P_2}S}$$

If secondary loop is absent, then find Y_2 in term of Y_{sp_2}

$$Y_2 = G_{P_2} Y_{sp_2}$$

$$\frac{Y_2}{Y_{sp_2}} = \frac{G_{P_2} G_{c_2}}{1+G_{c_2} G_{P_2}} = \frac{K_{c_2} K_{P_2}}{1+\tau_{P_2} s + K_{c_2} K_{P_2}}$$

controller - proportional. ~~cont~~ process = first order.

$$G_{P_2} = \frac{K_{P_2}}{\tau_{P_2} s+1}$$; when secondary loop is absent.

$$G_{P_2} = \frac{K_{c_2} K_{P_2}/(\tau_{P_2} s+1)}{1+K_{c_2} K_{P_2}/(\tau_{P_2} s+1)}$$; when secondary loop is present

$$= \frac{K_{c_2} K_{P_2}}{1+K_{c_2} K_{P_2} + \tau_{P_2} s} = \frac{K_{c_2} K_{P_2}}{(1+K_{c_2} K_{P_2})(1+\tau_{P_2}' s)}$$

$$= \frac{K_2'}{1+\tau_{P_2}' s}$$

$$\tau_{P_2}' = \frac{\tau_{P_2}}{1+K_{c_2} K_{P_2}}$$

Stability ↑
Response becomes fast.

So cascade controller are better, but cost increases as two controller are used.

» Tym constant ↓, Response ↑ (fast), stability ↑.

NOV 20, 14

Valve Characteristics defined as the prop. of valve & gives relationship b/w the flow & stemp position.

1) Linear
2) Equal %age (Increasing Sensitivity)
3) Quick Opening (Decreasing Sensitivity) initially full flow

Linear:-

Sensitivity is constant.

Sensitivity

It is defined as the ratio of fractional change in flow to the fractional change in stemp position for fixed upstream & downstream pressure.

The diff rate of change of the flow wrt to stemp position

$$\boxed{S = \frac{df}{dx}}$$

Linear

$$\frac{df}{dx} = \alpha$$

$$\int df = \int \alpha \, dx$$

$$\boxed{f = \alpha x + c}$$

when $\boxed{x = 0, \ f = 0}$

when valve is fulley closed, flow is zero / no flow then valve can be shut tightly.

$$\boxed{f = \alpha x}$$

$\Rightarrow$ $\boxed{x = x_{max}}$ $\boxed{f = f_{max}}$, when the valve is fully open.

$$\boxed{f_{max} = \alpha x_{max}}$$

% change in flow $\frac{f}{f_{max}} = \frac{x}{x_{max}}$ % change in lift. fraction of full stem position

$$\boxed{F = X}$$

Equal %age Value :— Sensitivity is proportional to flow.

$$\frac{df}{dx} = \alpha f$$

$$\frac{df}{f} = \alpha dx$$

$$\int df/f = \alpha \int dx$$

$$\boxed{\ln f = \alpha x + C}$$

when $x = 0$, $f = f_0$

Valve can not be shut tight.

$$C = \ln f_0$$

$$\boxed{\ln f/f_0 = \alpha x}$$

$$\boxed{f = f_0 e^{\alpha x}}$$ Characteristic eqn

$x = x_{max}$

$f = f_{max}$

$$\boxed{f_{max} = f_0 e^{\alpha x_{max}}}$$

$x = 1$
$f = 1$

$$\boxed{F = F_0^{1-x}}$$ final eqn /characteristic eqn.

also known as logarithmic characteristic /valve. or increasing sensitivity valve.

Quick opening

(Sensitivity $\propto \frac{1}{\text{flow rate}}$)

$$\frac{df}{dx} = \frac{\alpha}{f}$$

$$\int f df = \int \alpha dx$$

$$\boxed{\frac{f^2}{c} = \alpha x + C}$$

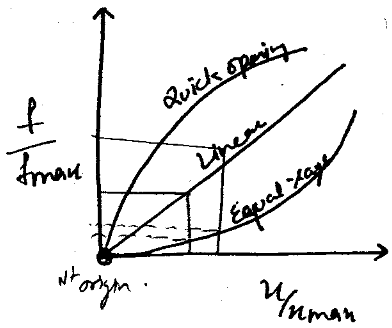
$x = 0$, f

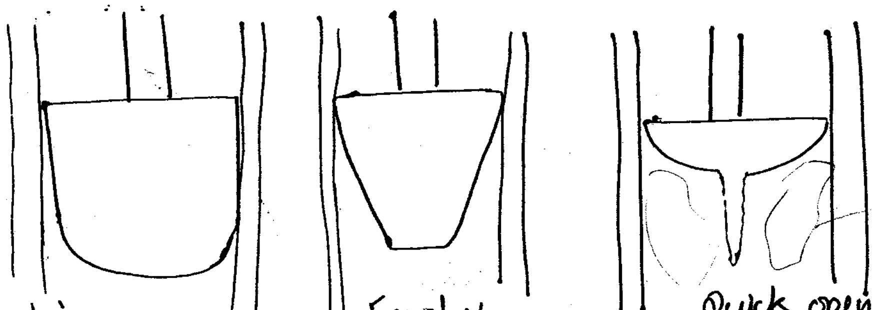
when flow is max$_m$, valve is fully open.

$$f = \sqrt{2\alpha x}$$

$$\boxed{f = \beta\sqrt{x}}$$ Characteristic eqn

also known as square root characteristic valve. or decreasing sensitivity.



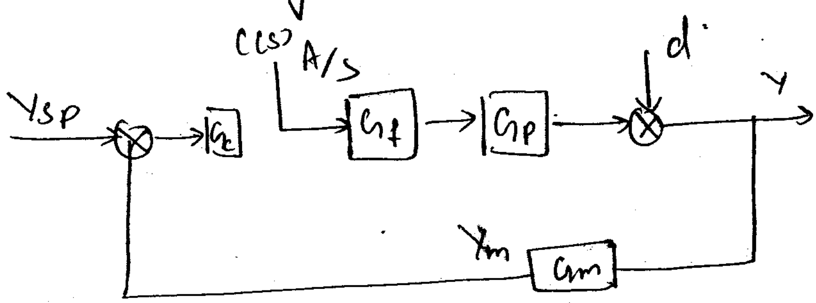


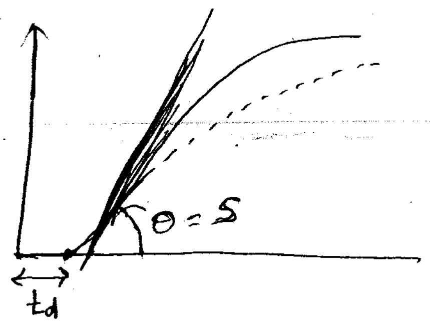
Linear   Equal %   Quick openin

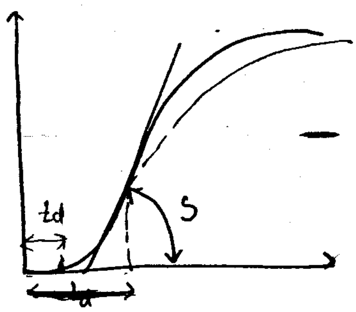
initially area ↑ and very small change later on.

Controller Tunning

Cohen-Conn Tuning :-







Sigmoidal curve

Cohen - coon approximated the rxn curve for most of the system wid dead tym process.

(Steady state gain) $K_p = \frac{B}{A}$

$\tau_p = B/S$ = slope

B = o/p value of amplitude

A = i/p value of amplitude

S = slope.

K_p & τ_p = Parameters for the first order system

Process Rxn Curve = 310
Stepnopoulos