Reciprocating Compressor – useful equations

$$Compressor\ displacement\ volume\ or\ swept\ volume\ or\ stroke\ volume = \frac{\pi}{4}D^2L, where\ D = stroke\ length\ of\ piston\ and\ D = bore\ of\ the\ cylinder$$

$$Volume\ flow\ rate\ of\ air = \frac{\pi}{4}D^2L * \frac{N}{60}, \qquad FOR\ SINGLE\ ACTING\ COMPRESSOR$$

$$Volume\ flow\ rate\ of\ air = \frac{\pi}{4}D^2L * \frac{N}{60} * 2, \qquad FOR\ DOUBLE\ ACTING\ COMPRESSOR$$

$$Piston\ Speed = 2LN\ m/minute$$

Single stage Compressor, **Equation for work input/cycle without clearance volume**

$W = \left(\frac{n}{n-1}\right) P_1V_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$, n = polytropic index

$W = \left(\frac{n}{n-1}\right) mRT_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$

$W = \left(\frac{n}{n-1}\right) (P_2V_2 - P_1V_1)$

This is also known as **indicated work input per cycle**.

For polytropic process replace 'n' by 'γ'

Indicated Power = $\left(\frac{n}{n-1}\right) P_1V_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$*N/60 , V_1 volume in m^3/cycle

For double acting air compressors multiply the work input by 2

For **isothermal process,** W = $P_1V_1 log_e \left(\frac{P_2}{P_1}\right)$

Least work input is for isothermal process

Mean Effective Pressure (P_m)

$$P_m = \frac{work\ input\ required\ per\ cycle\ (indicated\ work\ done\ per\ cycle)}{swept\ volume\ of\ the\ cylinder}$$

Indicated power (IP) = $indicated\ work\ done\ per\ cycle * number\ of\ cycles\ per\ unit\ time$

$$= P_m * L * A * \frac{N}{60} * n$$

'n=1 for single acting compressor, 'n=2 for double acting compressor

Or

IP = $Indicated\ work * \frac{N}{60} * n, n = 1\ for\ single\ acting\ and\ n = 2\ for\ double\ acting$

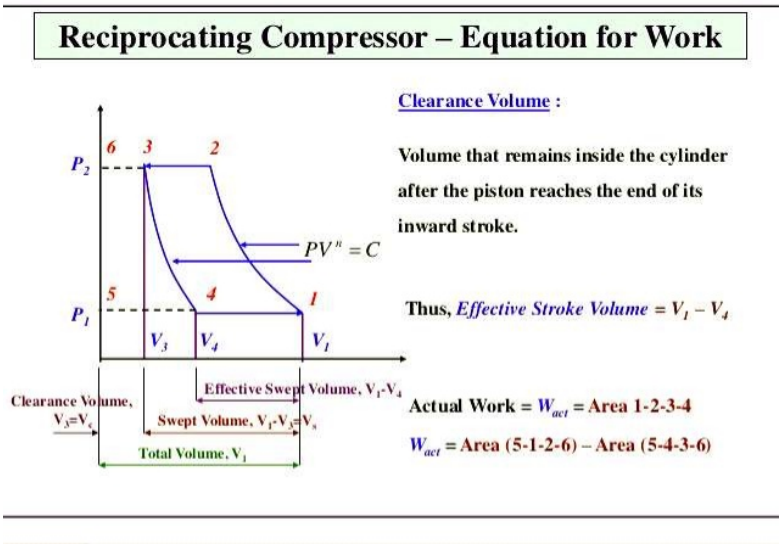
Mechanical Efficiency

$$= \frac{Indicated\ power}{brake\ Power}$$

Isothermal efficiency

$$= \frac{isothermal\ work\ input}{actual\ indicated\ work\ input}$$

Single stage Compressor, Equation for work input with clearance volume



$W = \left(\frac{n}{n-1}\right) P_1(V_1 - V_4)\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$, n = polytropic index

Clearance Ratio, $C = \frac{V_C}{V_S}$

Volumetric efficiency of reciprocating air compressor

Use only this equation for finding out the dimensions of the cylinder. Do not use volumetric efficiency corresponding to FAD for that.

$$= 1 + C - C\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}$$

Or

$$= \frac{effective\ swept\ volume}{swept\ volume} = \frac{V_1 - V_4}{V_1 - V_3}$$

Free Air Delivery (FAD)

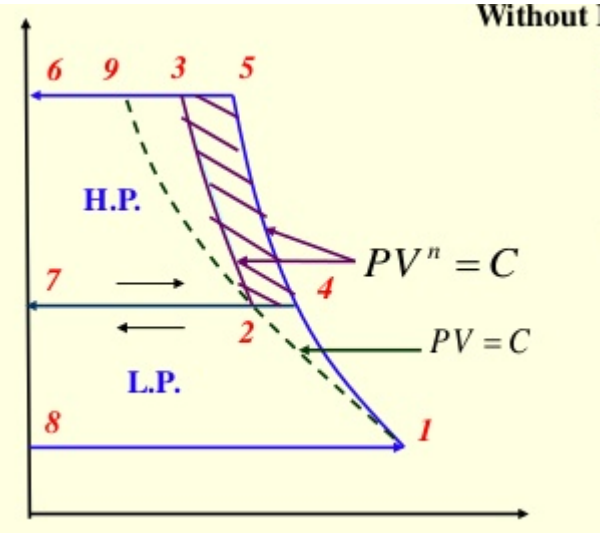
$$\left[\frac{P_{amb}(V_1 - V_4)_{amb}}{T_{amb}}\right]_{atmospheric} = \left[\frac{P_1(V_1 - V_4)}{T_1}\right]_{actual\ suction}$$

$$(V_1 - V_4)_{amb} = \frac{P_1 T_{amb}(V_1 - V_4)}{P_{amb}\ T_1}$$

$$volumetric\ efficiency\ (FAD) = \frac{P_1 T_{amb}}{P_{amb}\ T_1}\left[1 + C - C\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}\right]$$

Multistage Compression

For single acting, 2 stage compressor, Work done/cycle = W_{LP} +W_{HP}



$$Without\ Intercooling, W = W_{LP} + W_{HP}$$

$$W = \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] + \frac{n}{n-1} P_2 V_4 \left[\left(\frac{P_3}{P_2} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$\boldsymbol{With\ Perfect\ Intercooling, W = W_{LP} + W_{HP}, \quad T_2 = T_1}$$

$$W = \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] + \frac{n}{n-1} P_2 V_2 \left[\left(\frac{P_3}{P_2} \right)^{\frac{n-1}{n}} - 1 \right], P_1 V_1 = P_2 V_2 = mRT_1$$

For minimum work input to the compressor of a two stage air compressor, **Pressure ratio in each stages are same,**

$$\frac{P_2}{P_1} = \frac{P_3}{P_2} \ or \ P_2 = \sqrt{P_1 P_3}$$

Minimum Work input to a multistage compressor

For 2 stage,

$$W = \frac{2n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

Or

$$W = \frac{2n}{n-1} P_1 V_1 \left[\left(\frac{P_3}{P_1} \right)^{\frac{n-1}{2n}} - 1 \right]$$

General Equation for x stages

$$W = \frac{xn}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

Or

$$W = \frac{xn}{n-1} P_1 V_1 \left[\left(\frac{P_{x+1}}{P_1} \right)^{\frac{n-1}{xn}} - 1 \right]$$