Subj: Signal & System
Class: B.Tech (ECE) 2nd yr
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Topic: Laplace Transform

Laplace Transform: In order to Transform a given function of time f(t) into its corresponding Laplace transform firstly multiply by f(t) by e^{-st}, s being a complex number ($s = \sigma + j\omega$). Integrate this product w.r.t. time with limits as zero and infinity. This Integration results in Laplace transform of f(t), which is denoted by F(s) or $\mathcal{L} f(t)$.
The mathematically expression for Laplace Transform is:

$$\mathcal{L} f(t) = F(s) \qquad t \geq 0$$

$$F(s) = \int_{0}^{\infty} f(t) \cdot e^{-st} \, dt$$

"The term laplace transform of "f(t)" is used for the letter $\mathcal{L} f(t)$.
The time function f(t) is obtained back from the Laplace transform by a process called' Inverse Laplace transformation and denoted by $\mathcal{L}^{-1}$ thus.

$$\mathcal{L}^{-1}[\mathcal{L} f(t)] = \mathcal{L}^{-1}[F(s)] = f(t)$$

The time function f(t) and its Laplace transform F(s) are a transform pair.

example? (1) Laplace transform of e^{at}.

$$\mathcal{L} e^{at} = \int_0^\infty e^{at} \cdot e^{-st} dt.$$

$$= \int_0^\infty e^{(a-s)t} dt = \frac{1}{s-a}.$$

$$\mathcal{L} e^{at} = \frac{1}{(s-a)}$$

	$f(t)$	$F(s) = \mathcal{L}(f(t))$
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$u(t-T)$	$\frac{1}{s} e^{-sT}$
	t	$\frac{1}{s^2}$
	$\frac{t^2}{2}$	$\frac{1}{s^3}$
	t^n	$\frac{n!}{s^{n+1}}$
	e^{-at}	$\frac{1}{s+a}$

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8	e^{at}	$\frac{1}{s-a}$
9	te^{-at}	$\frac{1}{(s+a)^2}$
10	te^{at}	$\frac{1}{(s-a)^2}$
	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
2.	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
	$e^{-at} \sin \omega t$	$\frac{\omega}{(s^2+a)^2+\omega^2}$
	$\sinh at$	$\frac{a}{s^2-a^2}$