Testing of Significance for Single Proportion :-

Let X be the number of successes in n independent trials with constant probability P of success for each trial.

$$E(X) = nP; \quad V(X) = nPQ; \quad Q = 1-P = \text{Prob. of failure.}$$

Let $p = X/n$ called the observed proportion of success.

$$E(p) = E(X/n) = \frac{1}{n}E(X) = \frac{nP}{n} = P$$

$$V(p) = V(X/n) = \frac{1}{n^2}V(X) = \frac{nPQ}{n^2} = \frac{PQ}{n}$$

Standard Error (S.E.) (p) $= \sqrt{\frac{PQ}{n}}$; $Z = \frac{p - E(p)}{S.E.(p)}$

$$= \frac{p-P}{\sqrt{PQ/n}} \sim N(0,1)$$

This Z is called test statistic which is used to test the significant difference of sample and population proportion.

Note :- 1 :→ The probable limits for the observed proportion of successes are $P \pm 3\sqrt{\frac{PQ}{n}}$.

2. If P is not known, the probable limits for the proportion in the population are $p \pm z_\alpha \sqrt{\frac{pq}{n}}$, $q = 1-p$, where sample proportion, p is taken as an estimate of P and z_α is the significant value of Z at level of significance α.

Question :→ A coin was tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbiased.

Solution :→ Null hypothesis :

H_0 : The coin is unbiased i.e., $P = 0.5$

Alternative hypothesis ;

H_1 : The coin is biased i.e. $P \neq 0.5$

Hence we use ~~two~~ two tailed test.

Here, $n = 400$, X = no. of success $= 216$

$\therefore p$ = proportion of success in the sample $= \frac{x}{n} = \frac{216}{400} = 0.54$

P = Population Proportion $= 0.5$, $Q = 1 - P = 0.5$

Test statistic;

Under Ho, test statistic $z = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$

$$|z| = \left| \frac{0.54 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{400}}} \right| = 1.6$$

Conclusion : Since $|z| = 1.6 < 1.96$ i.e. $|z| < z_\alpha$ where z_α is the significant value of z at 5% level of significance.

Here we accept Ho and conclude that the coin is unbiased.

Question :→ A machine produced 16 defective articles in a batch of 500. After over hauling it produced 3 defectives in a batch of 100. Has the machine improved?

Solution :→ $P_1 = \frac{16}{500}$, $n_1 = 500$, $P_2 = \frac{3}{100}$, $n_2 = 100$

Null hypothesis : Ho : The machine has not improved due to overhauling i.e. $P_1 = P_2$

Alternative hypothesis : $H_1 : P_1 > P_2$

Hence we use right tailed test. $\therefore P = \frac{P_1 n_1 + P_2 n_2}{n_1 + n_2} = \frac{19}{600} = 0.032$

Test statistic : Under Ho, the test statistic

$$z = \frac{P_1 - P_2}{\sqrt{PQ\left[\frac{1}{n_1} + \frac{1}{n_2}\right]}} = 0.104$$

Conclusion :→ The calculated value of $|z| < 1.645$ which is the significant value of z at 5% level of significance. ~~i.e.~~ Ho is accepted, i.e. the machine has not improved due to overhauling.