

Test of Significance:- An important aspect of the sampling theory is to study the test of Significance which will enable us to decide, on the basis of the results of the sample, whether

- (i) the deviation between the observed sample statistic and the hypothetical Parameter value or
- (ii) the deviation between two sample statistics is significant or might be attributed due to chance or the fluctuations of the sampling.

Testing of Statistical Hypothesis:-

Step I: Null hypothesis:- For applying the tests of Significance, we first set up a hypothesis which is a definite statement about the Population Parameter called Null Hypothesis. It is denoted by  $H_0$ .

Null hypothesis is the hypothesis which is tested for possible rejection under the assumption that it is true. First, we set up  $H_0$  in clear terms.

Step 2: Alternative hypothesis: Any hypothesis which is complementary to the null hypothesis ( $H_0$ ) is called alternative hypothesis. It is denoted by  $H_1$ .

For example, if we want to test the null hypothesis that the population has a specified mean  $\mu_0$  then we have

$$H_0: \mu = \mu_0$$

then the alternative hypothesis will be

- (i)  $H_1: \mu \neq \mu_0$  (Two tailed alternative hypothesis)
- (ii)  $H_1: \mu > \mu_0$  (right tailed alternative hypothesis or single tailed)

(iii)  $H_1: \mu < \mu_0$  (left tailed alternative hypothesis (or) single tailed)

Hence alternative hypothesis helps to know whether the test is two tailed test or one tailed test. Therefore, we set up  $H_1$  for this decision.

Step III: Level of Significance: - The Probability of the value of the variate falling in the critical region is known as level of significance. A region corresponding to a statistic  $t$  in the sample space  $S$  which amounts to rejection of the null hypothesis  $H_0$  is called as **critical region** or region of rejection while which amounts to acceptance of  $H_0$  is called acceptance region. The probability  $\alpha$  that a random value of the statistic  $t$  belongs to the critical region is known as the level of significance.

Step 4: - Test statistic (or test criterion):

We compute the test statistic  $z$  under the null hypothesis. For larger samples corresponding to the statistic  $t$ , the variable  $z = \frac{t - E(t)}{S.E.(t)}$  is normally distributed with mean 0 and variance 1. The value of  $z$  given above under the null hypothesis is known as test statistics.

Step 5: Conclusion: We compare the computed value of  $z$  with the critical value  $z_\alpha$  at level of significance  $(\alpha)$ . which is given by  $P(|z| > z_\alpha) = \alpha$

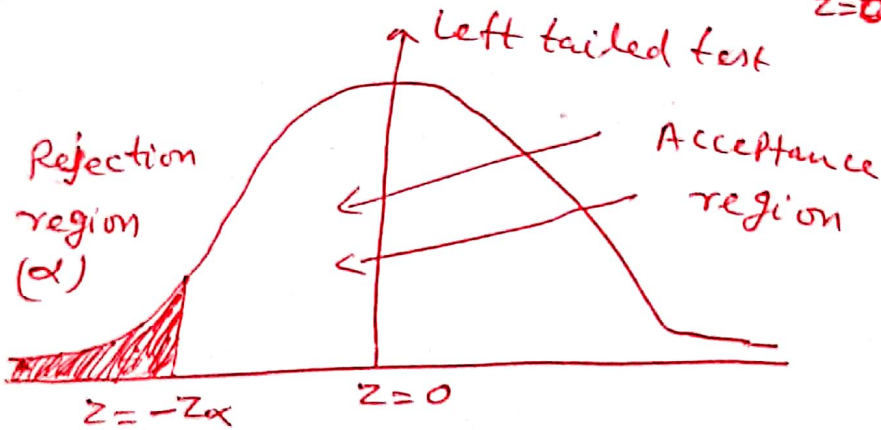
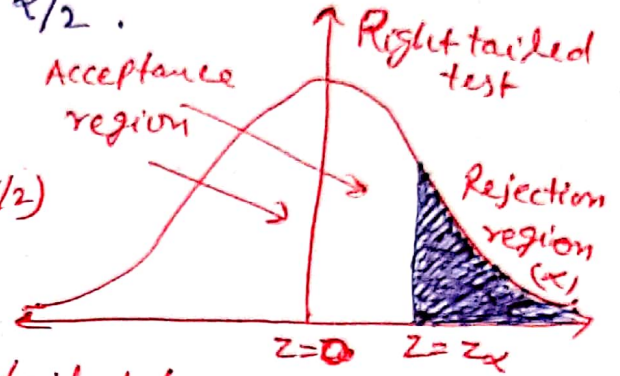
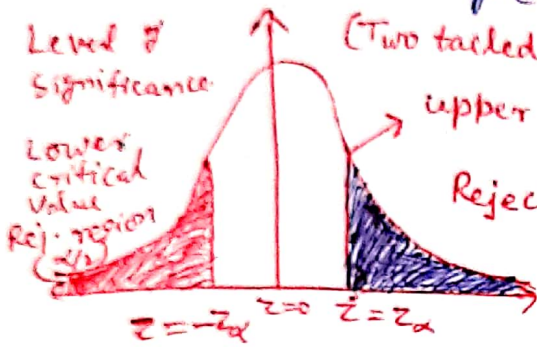
i.e.  $z_\alpha$  is the value of  $z$  so that the total area of the critical region on both tail is  $\alpha$ . ①

Since the normal curve is symmetrical, from equation (1), we get

$$P(Z > z_\alpha) + P(Z < -z_\alpha) = \alpha; \dots$$

i.e.  $2P(Z > z_\alpha) = \alpha$ ; i.e.  $P(Z > z_\alpha) = \alpha/2$

i.e., the area of each tail is  $\alpha/2$ .



The critical value  $z_\alpha$  is that value such that the area to the right of  $z_\alpha$  is  $\alpha/2$  and the area to the left of  $-z_\alpha$  is  $\alpha/2$ .

In the case of one tailed test,

$P(Z > z_\alpha) = \alpha$  if it is right tailed;  $P(Z < -z_\alpha) = \alpha$  if it is left tailed.

The critical value of  $z$  for a single tailed test (right or left) at level of significance  $\alpha$  is same as the critical value of  $z$  for two tailed test at level of significance  $2\alpha$ .

Using the equation, also using the normal tables, the critical value of  $z$  at different level of significance ( $\alpha$ ) for both single tailed test are calculated and listed below.

The equations are

$$P(|Z| > z_\alpha) = \alpha; \quad P(Z > z_\alpha) = \alpha; \quad P(Z < -z_\alpha) = \alpha$$

	Level of Significance		
	1% (0.01)	5% (0.05)	10% (0.1)
Two tailed test	$ z_{\alpha}  = 2.58$	$ z_{\alpha}  = 1.96$	$ z_{\alpha}  = 1.645$
Right tailed test	$z_{\alpha} = 2.33$	$z_{\alpha} = 1.645$	$z_{\alpha} = 1.28$
Left tailed test	$z_{\alpha} = -2.33$	$z_{\alpha} = -1.65$	$z_{\alpha} = -1.28$

If  $|z| > z_{\alpha}$ , we reject  $H_0$  and conclude that there is significant difference. If  $|z| < z_{\alpha}$ , we accept  $H_0$  and conclude that there is no significant difference.

### Test of Significance for Large Samples: $\rightarrow$

If the sample size  $n > 30$ , the sample is taken as large sample. For sample we apply distributions, as Binomial, Poisson, which are closely approximated by normal distributions assuming the population as normal.

Under large sample test, the following are the important tests to test the significance:

- 1) Testing of significance for single proportion.
2. Testing of significance for difference of Proportions.
3. Testing of significance for single mean.
4. Testing of significance for difference of means.
5. Testing of significance for difference of standard deviations.