(vii) Inverse Z Transform → [redacted]

(a) By Partial fraction method.
(b) By Infinite Series method.
(c) By Residue method.

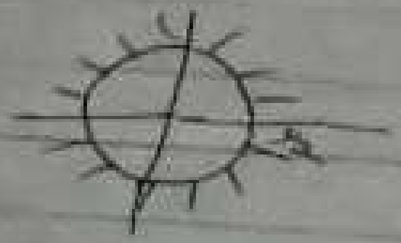
(a) By Partial fraction method →

Basics —

Case (i) → $x[n] = \left(\frac{1}{2}\right)^n u[n]$ {This is +ve sided Infinite signal

then $X(z) = \frac{1}{1-\frac{1}{2}z^{-1}}$

then ROC ⇒



Case (ii) → $x[n] = -\left(\frac{1}{2}\right)^n u[-n-1]$ {This is -ve sided signal

then $X(z) = \frac{1}{1-\frac{1}{2}z^{-1}}$

then ROC ⇒

We Saw That:

$$Z^{-1}\left[\frac{1}{1-\frac{1}{3}Z^{-1}}\right]$$ has two Sol^n

$x[n] = \left(\frac{1}{3}\right)^n u[n]$ when $Z > \frac{1}{3}$

$x[n] = -\left(\frac{1}{3}\right)^n u[-n-1]$ when $Z < \frac{1}{3}$

In These Problem we Solve the Equality Carefully.

(i) बड़े से बड़ा सही है

$Z > 2$
$Z > 3$ ✓

(ii) छोटे से छोटा सही है

$Z < 2$
$Z < 1$ ✓

<u>EXAMPLE 9</u>

$$X(z) = \frac{5}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}}$$

Determine The Value of $x[n]$ by Partial fraction Method.

Case (i) $|Z| > \frac{1}{2}$

Case (ii) $|Z| < \frac{1}{3}$

Case (iii) $\frac{1}{2} > |Z| > \frac{1}{3}$

ROC

<u>Sol^n</u> ⇒ $$X(z) = \frac{5}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}}$$

DATE

$$X(z) = \frac{5}{\left(1-\frac{3}{6}z^{-1}\right)\left(1-\frac{2}{6}z^{-1}\right)}$$

$$= \frac{5}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)}$$

$$= \frac{A}{\left(1-\frac{1}{2}z^{-1}\right)} + \frac{B}{\left(1-\frac{1}{3}z^{-1}\right)}$$

$$X(z) = \frac{15}{\left(1-\frac{1}{2}z^{-1}\right)} - \frac{10}{\left(1-\frac{1}{3}z^{-1}\right)} \quad \text{——— ①}$$

<u>Case (i) →</u> $|z| > \frac{1}{2}$ [redacted]

Then we write that, $|z| > \frac{1}{3}$

* <u>Since ROC is Exterior of the Circle so the Inverse Z Transform is +ve sided Infinite sequence or $u[n]$ but Not $u[-n-1]$.</u>

Taken Inverse Z Transform both side Eq. ①

$$x[n] = 15\left(\frac{1}{2}\right)^n u[n] - 10\left(\frac{1}{3}\right)^n u[n]$$

Case (ii) → $|z| < \frac{1}{3}$

or $|z| < \frac{1}{2}$

* Since ROC is Interior of the Circle so the Inverse Z Transform is -ve sides Infinite sequence or $u[-n-1]$

Taking Inverse Z Transform both side -

$$x[n] = -15\left(\frac{1}{2}\right)^n u[-n-1] + 10\left(\frac{1}{3}\right)^n u[-n-1]$$

Case (iii) $\frac{1}{2} > |z| > \frac{1}{3}$

-ve sided +ve sided

* Since ROC is Exterior of the circle for $|z| > \frac{1}{3}$ and Interior of the circle for $\frac{1}{2} > |z|$

$$\therefore x[n] = -15\left(\frac{1}{2}\right)^n u[-n-1] - 10\left(\frac{1}{3}\right)^n u[n]$$ R

(b) Infinite Series Method ⇒

EXAMPLE → $$X(z) = \frac{1}{(1-\frac{1}{3}z^{-1})}$$

Determine Inverse z transform using Infinite series method - Case (i) $|z| > \frac{1}{3}$

Case (ii) $|z| < \frac{1}{3}$

Sol → Case (i) $|z| > \frac{1}{3}$

[redacted]

$$1-\frac{1}{3}z^{-1} \overline{)\,1\,}\; 1+\frac{1}{3}z^{-1}+\frac{1}{3^2}z^{-2}+\dots$$

$$\frac{1-\frac{1}{3}z^{-1}}{\frac{-}{}\;\frac{+}{}}$$

$$\cancel{\frac{1}{3}z^{-1}}$$

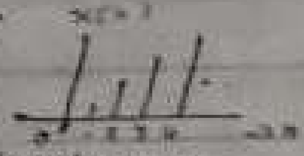
$$\frac{-\frac{1}{3}z^{-1}-\frac{1}{3^2}z^{-2}}{\frac{}{}\;+}$$

$$\cancel{\frac{1}{3^2}z^{-2}}$$

$$\frac{1}{3^2}z^{-2}-\frac{1}{3^3}z^{-3}$$

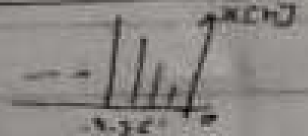
$$-\qquad +$$

$$\frac{1}{3^3}z^{-3}$$



+ve side sequence

$X(z) = 1 + z^{-1} + 2z^{-2} + 3z^{-3} + \dots$



-ve sided sequence

$X(z) = 1 + z + 2z^{2} + 3z^{3} + \dots$

$$X(z) = 1 + \frac{1}{3}z^{-1} + \frac{1}{9}z^{-2} + \frac{1}{27}z^{-3} + \dots$$

$$\boxed{a^n u[n] = \frac{1}{1-\frac{1}{3}z^{-1}}}$$

By Time Shifting Property -

$$\therefore x[n] = \delta[n] + \frac{1}{3}\delta[n-1] + \frac{1}{9}\delta[n-2] + \frac{1}{27}\delta[n-3] + \dots$$

or $$x[n] = \left(\frac{1}{3}\right)^n u[n] \quad R_2$$

This is +ve sided Infinite sequence signal.

Case(iii) $|z| < \frac{1}{3}$

$$-\frac{1}{3}z^{-1}+1 \overline{\big)\ 1\ } \big(-3z - 9z^2 - 27z^3 \ldots$$

$$\cancel{1} - 3z$$

$$\cancel{3z}$$

$$\cancel{3z} - 9z^2$$

$$\cancel{9z^2}$$

$$\cancel{9z^2} - 27z^3$$

$$27z^3$$

$$x(z) = -3z - 9z^2 - 27z^3 - - -$$

By time shifting property –

$$x[n] = -3\delta[n+1] - 9\delta[n+2] - 27\delta[n+3] - - -$$

or $x[n] = -\left(\frac{1}{3}\right)^n u[-n-1]$

This is -ve sided Infinite sequence

(c) Residue Method Or {Counter Integration Method}

If $X(z)$ is Z domain signal

then $$\boxed{Z^{-1}[X(z)] = x[n] = \left[\sum_i (z-z_i)\,X(z)\,z^{n-1}\right]_{z=z_i}}$$

EXAMPLE→ $$X(z) = \frac{5z}{(z-1)(z-2)}$$

Determine $x[n]$ by Residue method ?

Sol^n → Given Problem has two Poles $z = 1$ & 2

Then $$x[n] = \left[\cancel{(z-1)}\left[\frac{5z}{\cancel{(z-1)}(z-2)}\right]z^{n-1}\right]_{z=1} + \left[\cancel{(z-2)}\left[\frac{5z}{(z-1)\cancel{(z-2)}}\right]z^{n-1}\right]_{z=2}$$

$$= \left[\frac{5z^n}{z-2}\right]_{z=1} + \left[\frac{5z^n}{z-1}\right]_{z=2}$$

$$x[n] = -5[1]^n + 5[2]^n$$