

Alternative hypothesis,  $H_1: \mu \neq 56$  [Two tailed Test]

Test statistic under  $H_0$ , test statistic is  $t = \frac{\bar{x} - \mu}{S/\sqrt{n}}$

Given:  $\bar{x} = 53$ ,  $\mu = 56$ ;  $n = 16$ ,  $\sum (x - \bar{x})^2 = 135$

$$S = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}} = \sqrt{\frac{135}{15}} = \sqrt{9} = 3$$

$$t = \frac{\bar{x} - \mu}{S/\sqrt{n}} = \frac{53 - 56}{3/\sqrt{16}} = -4$$

$$|t| = 4, \quad \text{d.f.} = 16 - 1 = 15$$

Conclusion:  $\rightarrow$  Since  $|t| = 4 > t_{0.05} = 2.13$ , i.e.; the calculated value of  $t$  is more than the tabulated value, the null hypothesis is rejected. Hence, the sample mean has not come from a Population having 56 as mean.

### F-Test or Snedecor's Variance Ratio Test.

To test if two independent samples have been drawn from the same Population we have to test (i) equality of the means by applying the  $t$ -test and (ii) equality of Population variance by applying  $F$ -test.

Let  $n_1$  and  $n_2$  be the sizes of two samples with variance  $s_1^2$  and  $s_2^2$ . The estimate of the Population variance based on these samples  $S_1^2 = \frac{n_1 s_1^2}{n_1 - 1}$  and

$S_2^2 = \frac{n_2 s_2^2}{n_2 - 1}$ . The degree of freedom of these estimates are  $V_1 = n_1 - 1$ ,  $V_2 = n_2 - 1$ .

We setup the null hypothesis  $H_0: \sigma_1^2 = \sigma_2^2 = \sigma^2$

i.e., the independent estimates of the common Population do not differ significantly.

To carry out the test of significance of the difference of the variances we calculate the test statistic  $F = \frac{S_1^2}{S_2^2}$ , the Numerator is greater than the denominator. i.e.  $S_1^2 > S_2^2$ .

Conclusion: If the calculated value of  $F$  exceeds  $F_{0.05}$  for  $(n_1-1)$ ,  $(n_2-1)$  degree of freedom given in table we conclude that the ratio is significant at 5% Level. i.e. we conclude that the sample could have come from two normal Population with same variance.

The assumptions on which  $F$ -test is based

1. The Populations for each sample must be normally distributed.
2. The samples must be random and independent.
3. The ratio of  $\sigma_1^2$  to  $\sigma_2^2$  should be equal to 1 or greater than 1. That is why we take the larger variance in the numerator of the ratio.

Applications:  $F$ -test is used to test

- (i) whether two independent samples have been drawn from the normal Populations with the same Variance  $\sigma^2$ .
- (ii) whether the two independent estimates of the Population variance are homogeneous or not.

Question: → The random samples revealed the following data:

Sample No.	Size	Mean	Variance
I	16	440	40
II	25	460	42

Test whether the samples come from the same normal Population.