6.9. Discrete-time Processing of Continuous-time Signals

Discrete-time processing of continuous-time signals forms an important part of the signal analysis. Discrete-time processing of continuous-time signal can be achieved by first covering a continuous-time signal into a discrete-time signal and then after discrete-time processing of a discrete-time signal, converting it back to a continuous-time signal. Practically the discrete-time signal processing may be implemented with a general, or special-purpose computer *or* with microprocessors etc. Figure 6.17 shows the block-diagram for discrete-time processing of continuous-time signals.

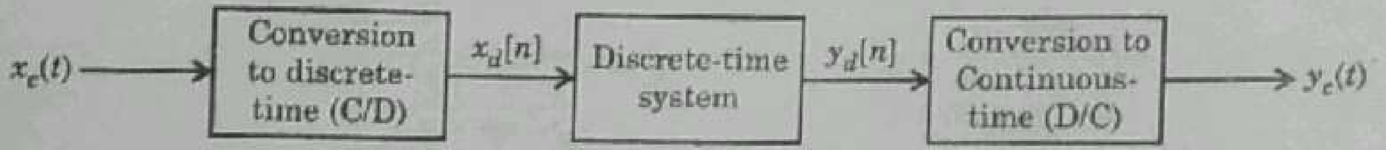

Fig. 6.20. Block-diagram representation for discrete-time processing of continuous-time signals.

From block-diagram, it may be observed that discrete-time processing of continuous-time signals is cascade of three operations. First, a continuous-time signal $x_c(t)$ is converted to a discrete-time signal $x_d(n)$. This discrete-time signal $x_d(n)$ is processed by a discrete-time system to produce another discrete-time signal $y_d(n)$. This discrete-time signal $y_d(n)$ is again converted to the continuous-time signal $y_c(t)$.

Hence, the overall system is a continuous-time system because its input and output are both continuous-time signals.

With the help of sampling theorem, the continuous-time signal $x_c(t)$ is completely represented by a sequence of instantaneous samples denoted by $x_c(nT)$.

Therefore,

$$x_d(n) = x_c(nT) \qquad T \rightarrow \text{sampling period.} \qquad \text{...(6.25)}$$

This process *i.e.* continuous-to-discrete-time conversion is performed by first block and is denoted as *C/D*.

The reverse process *i.e.* discrete-to-continuous-time conversion, denoted by *D/C*, is performed by third block.

This means that D/C operation produces a continuous-time signal $y_c(t)$ which is related to the discrete-time signal $y_d(n)$ as

$$y_d(n) = y_c(nT) \qquad \text{...(6.26)}$$

In view of the above two operations, the block diagram in figure 6.21 may be modified as

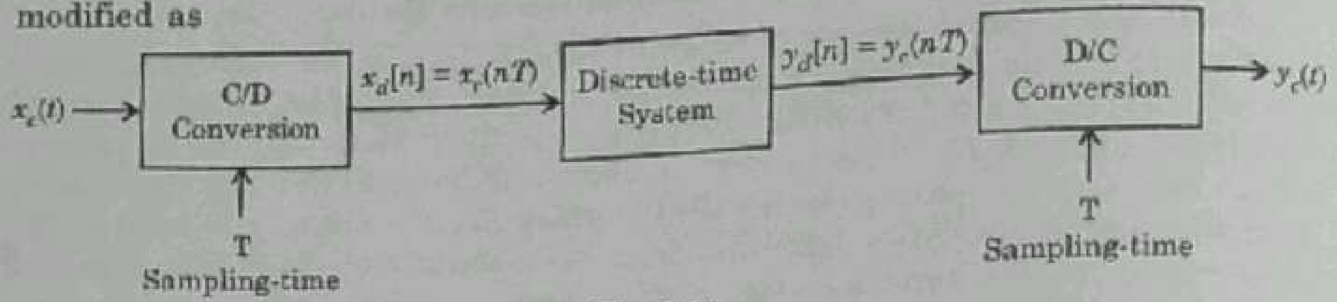

Fig. 6.21.

The *C/D* process may further be explained with the help of figure 6.21.

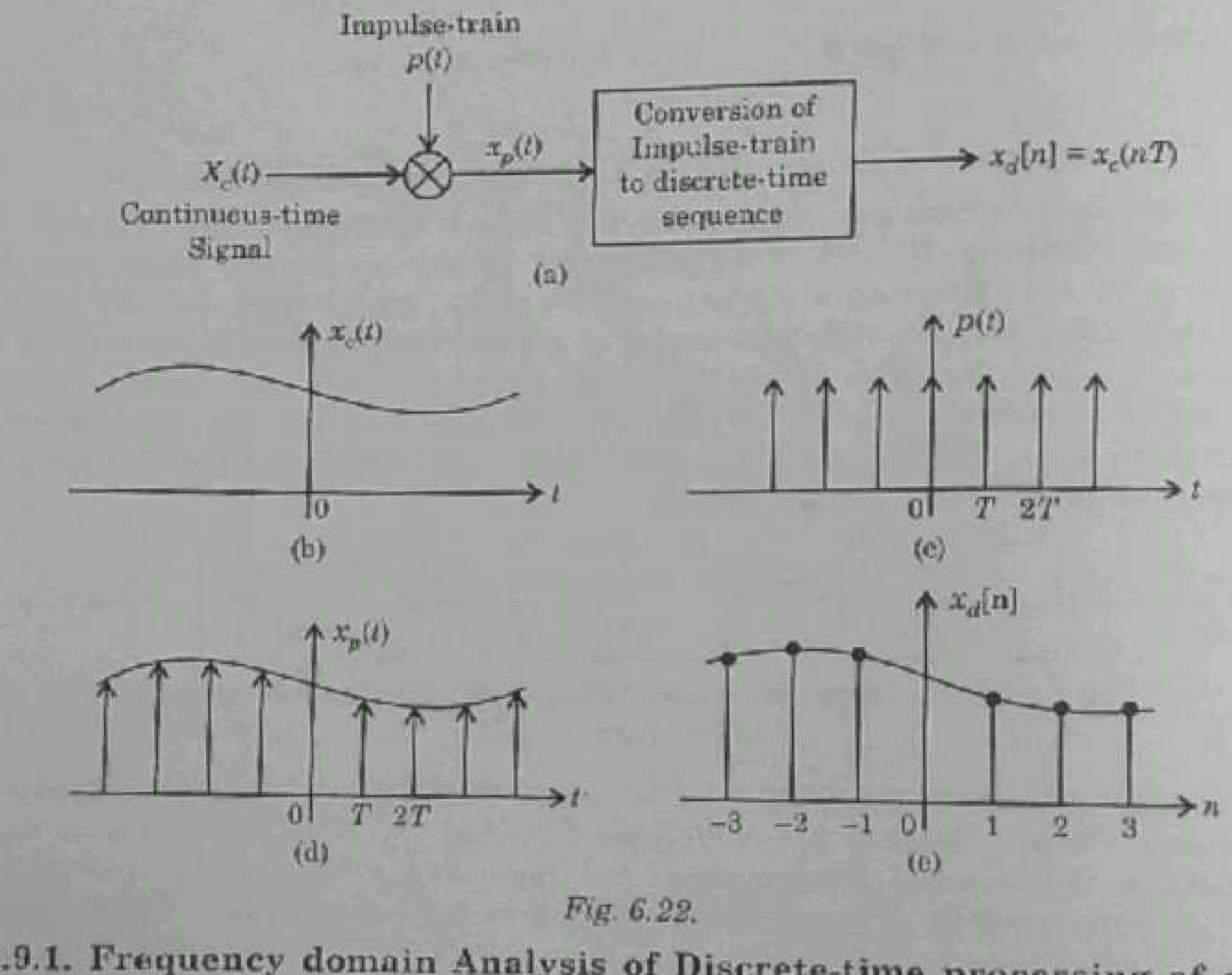

Fig. 6.22.

6.9.1. Frequency domain Analysis of Discrete-time processing of continuous-time signals

The whole process may also be analyzed in frequency domain also. The frequency-domain variables are w and w_0 in continuous-time and discrete-time respectively. Let the continuous-time Fourier transforms of $x_c(t)$ and $y_c(t)$ be $X_c(j\omega)$ and $Y_c(j\omega)$ respectively where as the discrete-time Fourier transforms of $x_d(n)$ and $y_d(n)$ be $X_d(j\omega_0)$ and $Y_d(j\omega_0)$ respectively.

Since $x_p(t)$, the sampled signal is expressed as

$$x_p(t) = \sum_{n=-\infty}^{\infty} x_c(nT)\,\delta(t - nT) \qquad \text{...(6.27)}$$

Therefore, its Fourier Transform will be

$$X_p(j\omega) = \sum_{n=-\infty}^{\infty} x_c(nT)\, e^{-j\omega nT} \qquad \text{...(6.28)}$$

The discrete-time Fourier transform of $x_d(n)$ is given by

$$X_d(j\omega_0) = \sum_{n=-\infty}^{\infty} x_d(n)\, e^{-j\omega_0 n} \qquad [\;x_d(n) = x_c(nT)] \quad \text{...(6.29)}$$

Therefore, $$X_d(j\omega_0) = \sum_{n=-\infty}^{\infty} x_c(n)\, e^{-j\omega_0 n} \qquad \text{...(6.30)}$$

Comparing equations (6.29) and (6.30) we have,

$$X_d(j\omega_0) = X_p(j\omega_0/T) \qquad \text{...(6.31)}$$

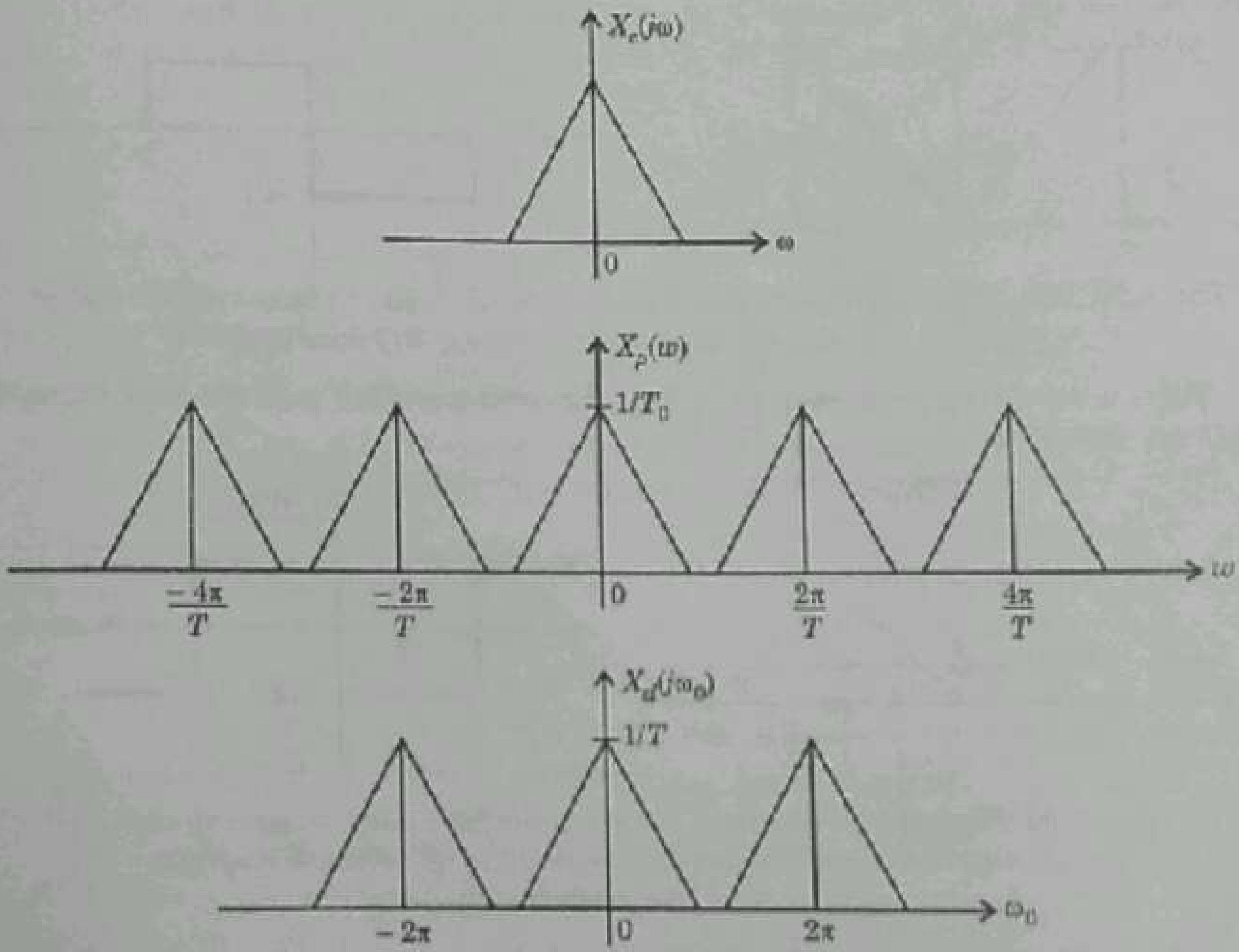

Fig. 6.23.

Figure 6.13. illustrates this relationship among $X_c(j\omega)$, $X_p(j\omega)$ and $X_d(j\omega_0)$. We know that

$$X_p(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c[j(\omega - k\omega_s)] \qquad \text{...(6.32)}$$

So that $$X_d(j\omega_0) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left[j\left(\frac{\omega_0 - 2\pi k}{T}\right)\right] \quad ...(6.33)$$

Now let us discuss discrete-time processing of continuous-time signals with some examples. These are as under

(i) Digital differentiator

(ii) Half-sample delay.

(i) Digital Differentiator

In this article, we shall consider the discrete-time implementation of a continuous-time band-limited differentiating filter.

The frequency response of a continuous-time differentiating filter is expressed as

$$H_c(j\omega) = j\omega \quad ...(6.34)$$

The frequency response of a band-limited differentiator with cut-off frequency ω_c is given by

$$H_c(j\omega) = j\omega,\ |\omega| < \omega_c \quad ...(6.35)$$
$$0,\ |\omega| > \omega_c$$

This frequency response in equation 6.35 is sketched in Figure 6.23.

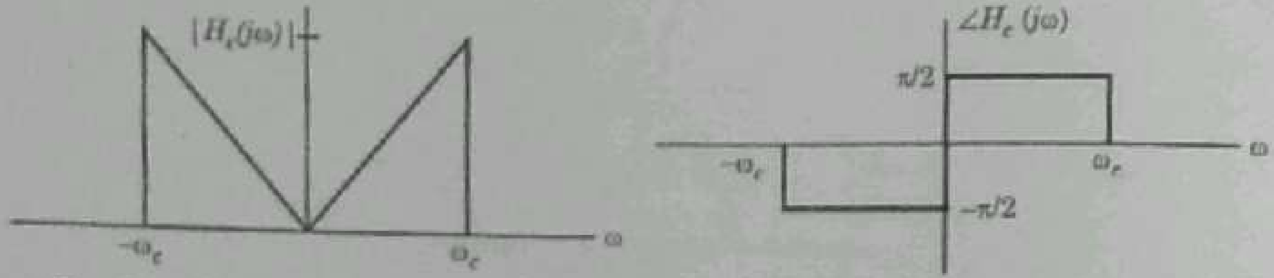

Fig. 6.24. Frequency response of a continuous-time ideal band-limited differentiator $H_c(j\omega) = j\omega$, $|\omega| < \omega_c$ (a) Magnitude response, (b) Phase response.

With a sampling frequency $\omega_s = 2\omega_c$, the corresponding discrete-time transfer function would be

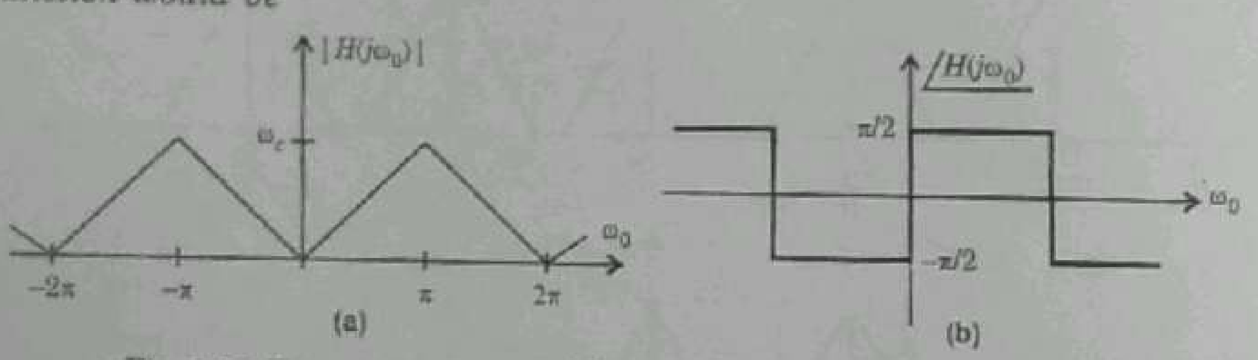

Fig. 6.25. Frequency response of discrete-time filter used to implement a continuous-time band limited differentiator (a) Magnitude response (b) Phase response

$$H_d(e^{j\omega_0}) = j\left(\frac{\omega_0}{T_s}\right), |\omega_0| < \pi \quad ...(6.36)$$

It has been shown in figure 6.25.

Here, it has been assumed that there is no aliasing, the discrete-time transfer function $H(e^{j\omega_0})$ is shown in figure 6.25.

(ii) Half-Sample Delay

Now let us consider the implementation of a time-shift or delay of a continuous-time signal through the use of a system in the form of Figure 6.25. The input and output of this system are related by

$$y_c(t) = x_c(t - t_0) \quad ...(6.37)$$

When the input $s_c(t)$ is band limited and samplinbg is done at the sampling rate slightly greater than Nyquist rate. The time t_0 is called delay time.

We can determine CTFT of Eq. (6.30) using time-shifting property of CTFT as

$$\text{CTFT }\{y_c(t)\} = \text{CTFT }\{s_c(t - t_0)\}$$

or

$$Y_c(j\omega) = e^{-j\omega t_0} S_c(j\omega) \quad ...(6.38)$$

Eq. 6.31 can be written as

$$H_c(j\omega) = \frac{Y_c(j\omega)}{S_c(j\omega)} = e^{-j\omega t_0} \quad ...(6.39)$$

The transfer function for band-limited signal is given by

$$H_c(j\omega) = \begin{cases} e^{-j\omega t_0}, & |\omega| < \omega_c \\ 0, & \text{otherwise} \end{cases} \quad ...(6.40)$$

where ω_c = the cut-off frequency of the continuous-time filter.

Frequency response $H_c(j\omega)$ corresponds to a time-shift for band-limited signals and rejects all frequencies greater than wc. The magnitude and phase response is shown in figure 7.26.

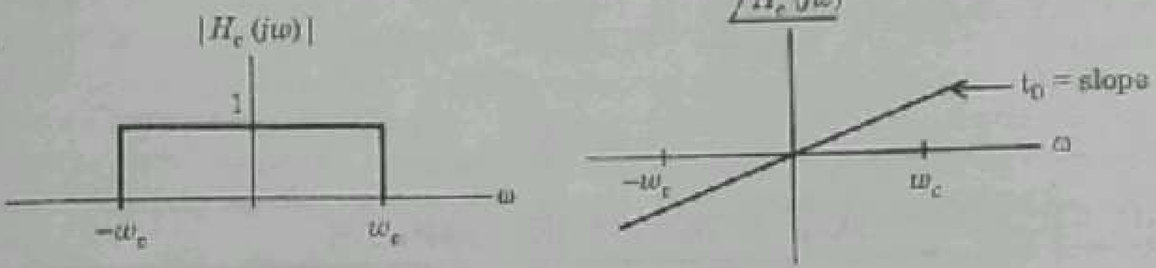

Fig. 6.26. Frequency response for a continuous-time delay (a) Magnitude response (b) Phase response.

The corresponding discrete-time frequency response for $\omega_s = 2\omega_c$ is expressed as

$$H_d(e^{j\omega_0}) = e^{-j\omega_0/T_s} \text{ for } |\omega_0| < \pi \quad ...(6.41)$$

This frequency response is shown in figure 6.27.

For band-limited inputs, the output is a delayed replica of the input.

For $\frac{t_0}{T_s}$ an integer, the sequence $y_d(n)$ is a delayed replica of $x_d(n)$. It isexpressed as

$$y_d(n) = x_d\left[n - \frac{t_0}{T_s}\right] \quad ...(6.42)$$

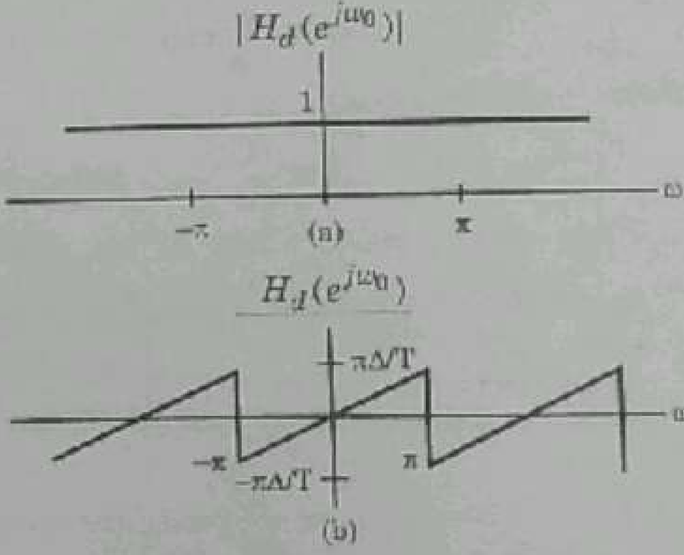

Fig. 6.27. Frequency response for the corresponding discrete-time delay (a) Magnitude response (b) Phase response.

If $\frac{t_0}{T_s}$ is not an integer, then equation 7.42 has no meaning, since sequences are defined only at integer values of the index.

However, we can interpret the relationship between $x_d(n)$ and $y_d(n)$ in these cases in terms of band-limited interpolation. The signals $x_c(t)$ and $s_d(n)$ are related through sampling and band-limited interpolation.

With transfer function $H_d(e^{j\omega})$ in equatioin 6.41, $y_d(n)$ is equal to samples of a shifted version of the band-limited interpolation of the sequence $x_d(n)$.

This is shown in figure 7.25 for $\frac{t_0}{T_s} = \frac{1}{2}$, which is sometimes referred to as a half-sample delay.

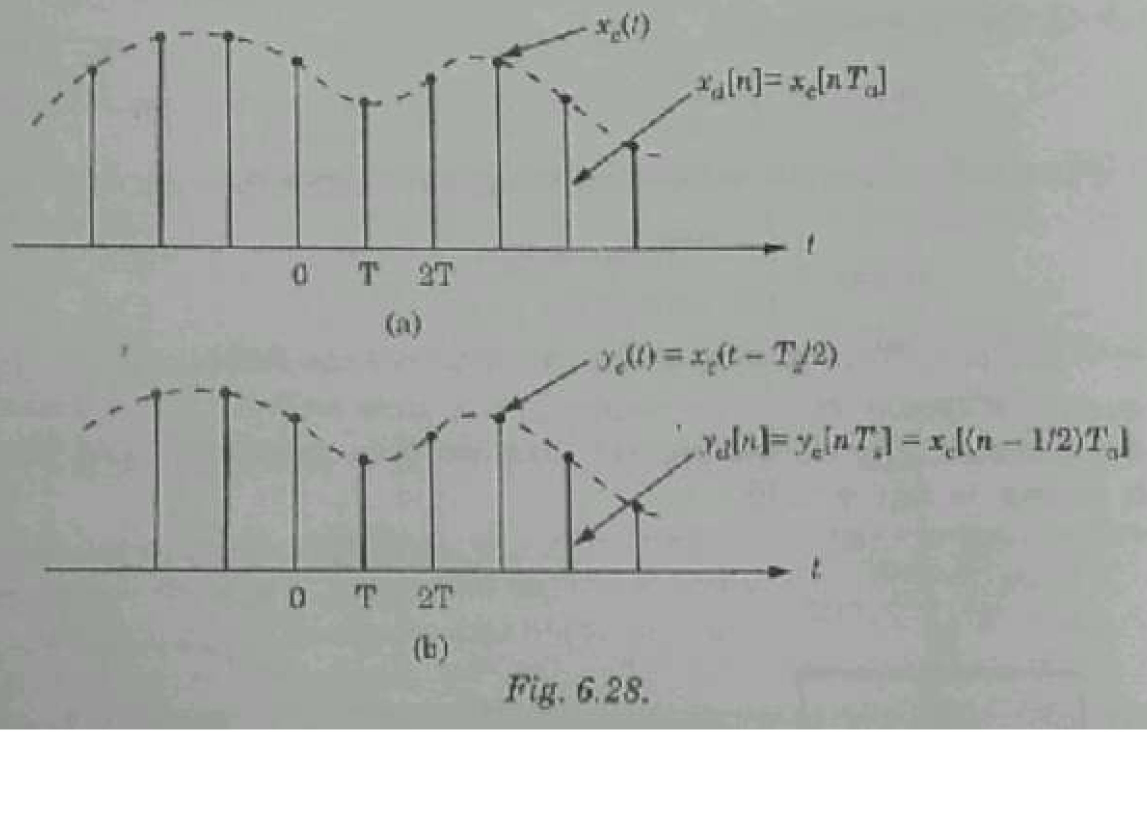

Fig. 6.28.