Numericals on Sampling Theorem (Important)

Q1. An analog signal is ~~extracted~~ expressed by the equation, $x(t) = 3\cos 50\pi t + 10\sin 300\pi t - \cos 100\pi t$. Calculate the Nyquist rate for this signal.

Sol:- $x(t) = 3\cos 50\pi t + 10\sin 300\pi t - \cos 100\pi t$ — (1)

There are three frequencies ω_1, ω_2 & ω_3,

So, that the new equation for signal,

$x(t) = 3\cos\omega_1 t + 10\sin\omega_2 t - \cos\omega_3 t$ — (2).

Comparing equations:- (1) & (2).

$\omega_1 t = 50\pi t, \quad \omega_2 t = 300\pi t; \quad \omega_3 t = 100\pi t$

or, $2\cancel{\pi} f_1 t = 50\cancel{\pi} t, \quad 2\cancel{\pi} f_2 t = 300\cancel{\pi} t; \quad 2\cancel{\pi} f_3 t = 100\cancel{\pi} t$

$\Rightarrow f_1 = 25\,Hz, \quad f_2 = 150\,Hz; \quad f_3 = 50\,Hz$

$\because f_2 = 150\,Hz$ (max. frequency) in $x(t)$

Hence Nyquist rate – $f_s = 2f_m$, f_m = max. freq. of the signal

Nyquist rate $\Rightarrow f_s = 2f_2$

$f_s = 2 \times 150 = \underline{\underline{300\,Hz}}$ Ans.

Q2. Find the Nyquist rate & Nyquist interval for the signal – $x(t) = \frac{1}{2\pi}\cos(4000\pi t)\cos(1000\pi t)$

Sol: $x(t) = \frac{1}{2\pi}\cos(4000\pi t)\cos(1000\pi t)$

$= \frac{1}{2\times 2\pi}\left[2\cos(4000\pi t)\cos(1000\pi t)\right]$

$[\because 2\cos A\cos B = \cos(A+B) + \cos(A-B)]$

$\therefore x(t) = \frac{1}{4\pi}\left[\cos 5000\pi t + \cos 3000\pi t\right]$ — (1)

or, $x(t) = \frac{1}{4\pi}\left[\cos\omega_1 t + \cos\omega_2 t\right]$ — (2)

Comparing equations ① & ②.

$$\omega_1 t = 2\pi f_1 t = 5000\pi t$$

$$\Rightarrow f_1 = 2500\,Hz.$$

&

$$\omega_2 t = 2\pi f_2 t = 3000\pi t$$

$$f_2 = 1500\,Hz.$$

Max. freq. present in $x(t)$ is

$$f_1 = 2500\,Hz = f_m.$$

$\therefore$ Nyquist rate, $f_s = 2f_1 = 5000\,Hz = \underline{5\,KHz}$. Ans.

Nyquist interval is,

$$T_s = \frac{1}{2f_m} = \frac{1}{2f_1}$$

$$T_s = \frac{1}{5000} = \underline{0.2\,msec.}\ \text{Ans.}$$

Example 2.3. *A continuous-time signal is given below :*

$$x(t) = 8 \cos 200\,\pi t$$

Determine:

(i) Minimum sampling rate i.e., Nyquist rate required to avoid aliasing.

(ii) If sampling frequency $f_s = 400$ Hz. What is the discrete-time sigal $x[n]$ or $x[nT_s]$ obtained after sampling ?

(iii) If sampling frequency f_s = ~~400~~ Hz. What is the discrete-time signal $x[n]$ or $x[nT_s]$ obtained after sampling ? (150 Hz) not 400 Hz

(iv) What is the frequeny $0 < f < f_s/2$ of sinusoidal that yields samples identical to those obtained in part (iii) ?

Solution :

(i) The highest frequency component of continuous-time signal is $f = 100$ Hz. Hence minimum sampling rate required to avoid aliasing is called Nyquist rate and is given as

$$\text{Nyquist rate} = 2f = 2 \times 100 = 200 \text{ Hz.} \quad \textbf{Ans.}$$

(ii) The continuous-time signal $x(t)$ is sampled at $f_s = 400$ Hz. The frequency of the discrete-time signal will be

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$$F = \frac{\text{Frequency of continuous - time signal } f}{\text{Sampling frequency}, f_s}$$

$$= \frac{100}{400} = \frac{1}{4}$$

Then the discrete-time signal will be given as

$$x[n] = 8 \cos 2\pi f n = 8 \cos 2\pi \times \frac{1}{4} n$$

or $$x[n] = 8 \cos \frac{\pi n}{2} \quad \textbf{Ans.}$$

(*iii*) The continuous-time signal $x(t)$ is sampled at $f_s = 150$ Hz. The frequency of discrete-time will be

$$F = \frac{f}{f_s} = \frac{100}{150} = \frac{2}{3}$$

Then, the discrete-time signal will be given as

$$x[n] = 8 \cos 2\pi f n = 5 \cos 2\pi \left(\frac{2}{3}\right) n = 8 \cos \frac{4\pi}{3} n$$

$$= 8 \cos \left(2\pi - \frac{2\pi}{3}\right) n = 8 \cos \frac{2\pi n}{3}$$

or $$x[n] = 8 \cos \frac{2\pi n}{3} \quad \textbf{Ans.}$$

(*iv*) For sampling rate of $f_s = 150$ Hz

$$F = \frac{f}{f_s} \text{ or } f = f_s \times F = \frac{1}{3} \times 150 = 50 \text{ Hz}$$

Then, the sinusoidal signal will be

$$y(t) = 8 \cos 2\pi f t = 8 \cos 2\pi \times 50 \times t$$

$$= 8 \cos 100\pi t$$

Sampling at $f_s = 150$ Hz, yields indentical samples hence $f = 100$ Hz is an alias of $f = 50$ Hz for sampling rate $f_s = 150$ Hz. **Ans.**

Example 2.4. *Determine the Nyquist rate for a continuous-time signal*

$$x(t) = 6 \cos 50\pi t + 20 \sin 300\pi t - 10 \cos 100\pi t$$

Solution : In a general form, any continuous-time signal may be expressed as

$$x(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t + A_3 \cos \omega_3 t \quad (i)$$

And the given signal is

$$x(t) = 6 \cos 50\pi t + 2 \sin 300\pi t - 10 \cos 100\pi t \quad (ii)$$

On comparing given signal (*ii*) with standard form of a signal (*i*), we obtain the frequencies for the given signal as

$$f_1 = \frac{\omega_1}{2\pi} = \frac{50\pi}{2\pi} = 25 \text{ Hz}$$

$$f_2 = \frac{\omega_2}{2\pi}$$

$$= \frac{300\pi}{2\pi} = 150 \text{ Hz}$$

$$f_3 = \frac{\omega_3}{2\pi}$$

$$= \frac{100\pi}{2\pi} = 50 \text{ Hz}$$

Thus, the highest frequency component of the given meassage signal will be

$$f_{max} = 150 \text{ Hz}$$

Therefor, Nyquist rate $= 2 f_{max}$

$= 2 \times 150 = 300$ Hz **Ans.**