Planning for .....

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Class: B.Tech (E.C.E) 2nd yr.
Subject: Signal & System
Faculty: Mandeep Singh
Date: 14/5/2020

Topic: Numerical on FIR filter.

Q.1 Determine response of FIR filter using DFT if:

$$x(n) = \{1, 2\} \quad \text{and}$$

$$h(n) = \{2, 2\}$$

Sol: Here length of

$$x(n) = L = 2.$$

And length of $h(n) = m = 2$

$N = L + M - 1 = 2 + 2 - 1 = 3$

We will Compute 4 point DFT means N

Step II: We will make lengths of $x(n)$ and $h(n)$ equal to 4 by adding zero at the end.

$\therefore x(n) = \underset{\uparrow}{\{1, 2, 0, 0\}}$ — (1)

$h(n) = \underset{\uparrow}{\{2, 2, 0, 0\}}$ — (2)

Step III: Calculate of $X(k)$:-

We will Calculate DFT of $x(n)$. $X(k)$ using matrix method.

$$X[k] = W_N \cdot x_N \quad \text{—}$$

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Earlier we have obtained the matrix for twiddle factor W_4. It is

$$W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

We have Input Matrix

$$x_N = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

Putting eq. (4) and (1) in eq. (2)

$$\begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2+0+0 \\ 1-2j+0+0 \\ 1-2+0+0 \\ 1+2j+0+0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1-2j \\ -1 \\ 1+2j \end{bmatrix}$$

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$\therefore x(k) = \{3, 1-2j, -1, 1+2j\}$

Step IV: calculations of $H(k)$

$$H[k] = W_N \cdot h_N$$

$$\therefore \begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2+2+0+0 \\ 2-2j+0+0 \\ 2-2+0+0 \\ 2+2j+0+0 \end{bmatrix} = \begin{bmatrix} 4 \\ 2-2j \\ 0 \\ 2+2j \end{bmatrix}$$

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$\therefore H(k) = \{\underset{\uparrow}{4}, 2-2j, 0, 2+2j\}$

Step 5: Now $Y(k) = X(k) \cdot H(k)$

$\therefore Y(k) = \{\underset{\uparrow}{3}, 1-2j, -1, 1+2j\} \cdot \{\underset{\uparrow}{4}, 2-2j, 0, 2+2j\}$

$\therefore Y(0) = X(0) \cdot H(0) = 3 \times 4 = 12$

$Y(1) = X(1) \cdot H(1) = (1-2j)(2-2j)$

$= 2-2j-4j+4j^2$

$= 2-6j-4 = -2-6j$

$Y(2) = X(2) \cdot H(2) = (-1)(0) = 0$

$Y(3) = X(3) \cdot H(3) = (1+2j)(2+2j)$

$= 2+2j+4j+4j^2$

$= 2+6j-4 = -2+6j$

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Here sequence $Y(k)$ is

$Y(k) = \{Y(0), Y(1), Y(2), Y(3)\}$

$\therefore Y(k) = \{12, (-2-6j), 0, (-2+6j)\}$

Step 02 Now we will obtain $y(n)$ by taking IDFT of $Y(k)$

$$y(n) = \frac{1}{N} W_N^{*} \cdot Y(k).$$

Where W_N^* is Complex Conjugate of W_N

$$\therefore W_N^* = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

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$$\therefore y(n) = \begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 12 \\ -2-6j \\ 0 \\ -2+6j \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 12 - 2 - 6j + 0 - 2 + 6j \\ 12 - 2j + 6 + 0 + 2j + 6 \\ 12 + 2 + 6j + 0 + 2 - 6 \\ 12 + 2j - 6 + 0 - 2j - 6 \end{bmatrix}$$

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$$= \frac{1}{4}\begin{bmatrix} 8 \\ 24 \\ 16 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ 4 \\ 0 \end{bmatrix}$$

$$\therefore y(n) = \begin{bmatrix} 2, & 6, & 4, & 0 \\ \uparrow & & & \end{bmatrix}$$