Applications of Poisson Distribution

This distribution is applied to Problems concerning.

(i) Arrival Pattern of deffective Vehicles in workshop.

(ii) Patients in a hospitals.

(iii) Telephone calls

(iv) Demand Patterns for certain spare parts.

(v) Number of fragments from a shell hitting a target.

(vi) Emission of radioactive (α) Particles.

Question: M.T.U. 2004, 2013 If the variance of the Poisson distribution is 2, find the Probabilities for $r = 1, 2, 3, 4$ from the recurrence relation of the Poisson distribution. Also, find $P(r \geq 4)$.

Solution: $\because$ m = mean = Variance = 2

Recurrence relation for the Poisson distribution is

$$P(r+1) = \frac{m}{r+1} \cdot P(r) = \frac{2}{r+1} P(r) \quad \text{--- (1)}$$

Now $P(r) = \dfrac{e^{-m} \cdot m^r}{\lfloor r}$ $\Rightarrow$ $P(0) = \dfrac{e^{-2}(2)^0}{\lfloor 0} = e^{-2} = 0.1353$

Putting $r = 0, 1, 2, 3$ in (1), we get

$P(1) = 2P(0) = 2 \times 0.1353 = 0.2706$

$P(2) = \frac{2}{2} P(1) = 0.2706$, $P(3) = \frac{2}{3} P(2) = \frac{2}{3} \times 0.2706 = 0.1804$

$P(4) = \frac{2}{4} P(3) = \frac{1}{2} \times 0.1804 = 0.0902$

Now $P(r \geq 4) = 1 - (P(0) + P(1) + P(2) + P(3))$

$= 1 - [0.1353 + 0.2706 + 0.2706 + 0.1804]$

$= 0.1431$ Ans.

(22)

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Question:→ Using Poisson distribution, find the Probability that the ace of spades will be drawn from a pack of well-shuffled cards at least once in 104 consecutive trials.

Sol:- $p = \frac{1}{52}$, $n = 104$, mean $m = np$

$= 104 \times \frac{1}{52} = 2$

Prob. (at least once) $= P(r \geq 1) = 1 - P(0)$

$= 1 - \frac{m^0 \cdot e^{-m}}{\lfloor 0} = 1 - e^{-2} = 1 - 0.135335$

$= 0.8647.$ Ans

Question:→ An insurance company found that only 0.01% of the Population is involved in a certain type of accident each year. If its 1000 Policy holders were randomly selected from the Population, what is the Probability that not more than two of its clients are involved in such an accident next year? [Given that $e^{-0.1} = 0.9048$]

Solution:→

$p = 0.01\% = \frac{1}{100}\% = \frac{1}{100} \times \frac{1}{100}$

$n = 1000$

$\therefore m = np = 1000 \times \frac{1}{100 \times 100} = 0.1$

$P(r) = \frac{e^{-m} \cdot m^r}{\lfloor r}$

P(not more than 2) = P(0, 1, and 2)

$= P(0) + P(1) + P(2)$

$= \frac{e^{-0.1} \cdot (0.1)^0}{\lfloor 0} + \frac{e^{-0.1} (0.1)^1}{\lfloor 1} + \frac{e^{-0.1} (0.1)^2}{\lfloor 2}$

$= e^{-0.1} \left[1 + 0.1 + \frac{0.01}{2}\right] = 0.9048 \times 1.105$

$= 0.9998$ Ans.