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Solution of Difference Eq^n Using DTFT →

Question → A Second order Difference Eq^n is given

$$y[n] - \frac{5}{6} y[n-1] + \frac{1}{6} y[n-2] = 3\,\delta[n]$$

Determine $y[n]$ using D.T.F.T

Sol^n → Taken D.T.F.T both Side.

$$DTFT\left\{y[n] - \frac{5}{6} y[n-1] + \frac{1}{6} y[n-2]\right\} = DTFT\left\{3\,\delta[n]\right\}$$

$$Y(e^{j\omega}) - \frac{5}{6} e^{-j\omega} Y(e^{j\omega}) + \frac{1}{6} e^{-j2\omega} Y(e^{j\omega}) = 3$$

$$Y(e^{j\omega})\left\{1 - \frac{5}{6} e^{-j\omega} + \frac{1}{6} e^{-j2\omega}\right\} = 3$$

$$Y(e^{j\omega})\left\{1 - \frac{1}{2} e^{-j\omega} - \frac{1}{3} e^{-j\omega} + \frac{1}{6} e^{-j2\omega}\right\} = 3$$

$$Y(e^{j\omega})\left\{1\left(1 - \frac{1}{2} e^{-j\omega}\right) - \frac{1}{3} e^{-j\omega}\left(1 - \frac{1}{2} e^{-j\omega}\right)\right\} = 3$$

$$Y(e^{j\omega})\left\{\left(1 - \frac{1}{2} e^{-j\omega}\right)\left(1 - \frac{1}{3} e^{-j\omega}\right)\right\} = 3$$

$$Y(e^{j\omega}) = \frac{3}{\left(1 - \frac{1}{2} e^{-j\omega}\right)\left(1 - \frac{1}{3} e^{-j\omega}\right)}$$

By Partial fraction -

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$$Y(e^{jw}) = \frac{A}{(1-\frac{1}{2}e^{-jw})} + \frac{B}{(1-\frac{1}{3}e^{-jw})}$$

$$Y(e^{jw}) = \frac{9}{(1-\frac{1}{2}e^{-jw})} - \frac{6}{(1-\frac{1}{3}e^{-jw})}$$

Taking IDTFT Both side -

$$IDTFT\{Y(e^{jw})\} = IDTFT\frac{9}{(1-\frac{1}{2}e^{-jw})} - IDTFT\frac{6}{(1-\frac{1}{3}e^{-jw})}$$

$$y[n] = 9\left(\frac{1}{2}\right)^n u[n] - 6\left(\frac{1}{3}\right)^n u[n]$$

Question → A second order difference Eq^n is given as -

$$y[n] - \frac{7}{12}y[n-1] + \frac{1}{12}y[n-2] = 5u[n]$$

Determine $y[n]$ using DTFT.

Sol^n → Taking DTFT Both Side -

$$DTFT\{y[n] - \frac{7}{12}y[n-1] + \frac{1}{12}y[n-2]\} = DTFT\{5u[n]\}$$

$$Y(e^{jw}) - \frac{7}{12}e^{-jw}Y(e^{jw}) + \frac{1}{12}e^{-j2w}Y(e^{jw}) = \frac{5}{(1-e^{-jw})}$$

$$Y(e^{jw})\left\{1 - \frac{7}{12}e^{-jw} + \frac{1}{12}e^{-j2w}\right\} = \frac{5}{1-e^{-jw}}$$

$$Y(e^{j\omega})\left\{1-\frac{3}{12}e^{-j\omega}-\frac{4}{12}e^{-j\omega}+\frac{1}{12}e^{-j2\omega}\right\}=\frac{5}{(1-e^{-j\omega})}$$

$$Y(e^{j\omega})\left\{1-\frac{1}{4}e^{-j\omega}-\frac{1}{3}e^{-j\omega}+\frac{1}{12}e^{-j2\omega}\right\}=\frac{5}{(1-e^{-j\omega})}$$

$$Y(e^{j\omega})\left\{1\left(1-\frac{1}{4}e^{-j\omega}\right)-\frac{1}{3}e^{-j\omega}\left(1-\frac{1}{4}e^{-j\omega}\right)\right\}=\frac{5}{(1-e^{-j\omega})}$$

$$Y(e^{j\omega})\left\{\left(1-\frac{1}{4}e^{-j\omega}\right)\left(1-\frac{1}{3}e^{-j\omega}\right)\right\}=\frac{5}{(1-e^{-j\omega})}$$

$$Y(e^{j\omega})=\frac{5}{(1-e^{-j\omega})(1-\frac{1}{4}e^{-j\omega})(1-\frac{1}{3}e^{-j\omega})}$$

By Partial fraction -

$$Y(e^{j\omega})=\frac{A}{(1-e^{-j\omega})}+\frac{B}{(1-\frac{1}{3}e^{-j\omega})}+\frac{C}{(1-\frac{1}{4}e^{-j\omega})}$$

or

$$5=A\left(1-\frac{1}{3}e^{-j\omega}\right)\left(1-\frac{1}{4}e^{-j\omega}\right)+B\left(1-e^{-j\omega}\right)\left(1-\frac{1}{4}e^{-j\omega}\right)+C\left(1-e^{-j\omega}\right)\left(1-\frac{1}{3}e^{-j\omega}\right)$$

when $e^{-j\omega}=1$	when $e^{-j\omega}=3$	when $e^{-j\omega}=4$
$5=A(1-\frac{1}{3})(1-\frac{1}{4})$	$5=B(1-3)(1-3/4)$	$5=C(1-4)(1-\frac{4}{3})$
$5=A\ \frac{2}{3}\times\frac{3}{4}$	$5=B(-2)(\frac{1}{4})$	$5=C(-3)(-\frac{1}{3})$
$A=10$	$B=-10$	$C=5$

Then

$$Y(e^{j\omega})=\frac{10}{(1-e^{-j\omega})}-\frac{10}{(1-\frac{1}{3}e^{-j\omega})}+\frac{5}{(1-\frac{1}{4}e^{-j\omega})}$$

Now Taking IDTFT Both side -

$y[n] = 10u[n] - 10\left(\frac{1}{3}\right)^n u[n] + 5\left(\frac{1}{4}\right)^n u[n]$ R

Question $\Rightarrow$ $y[n] - y[n-1] + \frac{1}{4}y[n-2] = \left(\frac{1}{3}\right)^n u[n]$

A second order difference Eqn is given above. Determine $y[n]$ using DTFT

Soln $\Rightarrow$ Taking DTFT Both Side -

$$\text{DTFT}\left\{y[n] - y[n-1] + \frac{1}{4}y[n-2]\right\} = \text{DTFT}\left\{\left(\frac{1}{3}\right)^n u[n]\right\}$$

$$Y(e^{j\omega}) - e^{-j\omega}Y(e^{j\omega}) + \frac{1}{4}e^{-j2\omega}Y(e^{j\omega}) = \frac{1}{(1-\frac{1}{3}e^{-j\omega})}$$

$$Y(e^{j\omega})\left\{1 - e^{-j\omega} + \frac{1}{4}e^{-j2\omega}\right\} = \frac{1}{(1-\frac{1}{3}e^{-j\omega})}$$

$$Y(e^{j\omega})\left\{(1-\frac{1}{2}e^{-j\omega})^2\right\} = \frac{1}{(1-\frac{1}{3}e^{-j\omega})}$$

$$Y(e^{j\omega}) = \frac{1}{(1-\frac{1}{3}e^{-j\omega})(1-\frac{1}{2}e^{-j\omega})^2}$$

By Partial Fraction -

$$Y(e^{j\omega}) = \frac{A}{(1-\frac{1}{3}e^{-j\omega})} + \frac{B}{(1-\frac{1}{2}e^{-j\omega})} + \frac{C}{(1-\frac{1}{2}e^{-j\omega})^2}$$

or $1 = A(1-\frac{1}{2}e^{-j\omega})^2 + B(1-\frac{1}{3}e^{-j\omega})(1-\frac{1}{2}e^{-j\omega}) + C(1-\frac{1}{3}e^{-j\omega})$

when $e^{-j\omega} = 2$	when $e^{-j\omega} = 3$	By Comparison method -
$1 = C(1-\frac{1}{3}\times 2)$	$1 = A(1-\frac{1}{2}\times 3)^2$	$1 = A + B + C$
$1 = C(\frac{1}{3})$	$1 = A(-\frac{1}{2})^2$	$B = 1 - A - C$
$C = 3$	$A = 4$	$1 - 4 - 3 = -6$

then $$Y(e^{j\omega}) = \frac{4}{\left(1-\frac{1}{3}e^{-j\omega}\right)} - \frac{6}{\left(1-\frac{1}{2}e^{-j\omega}\right)} + \frac{3}{\left(1-\frac{1}{2}e^{-j\omega}\right)^2}$$

Now taken IDTFT Both Side -

$$IDTFT\{Y(e^{j\omega})\} = IDTFT\left\{\frac{4}{\left(1-\frac{1}{3}e^{-j\omega}\right)}\right\} - \frac{6}{1-\frac{1}{2}e^{-j\omega}} + \frac{3}{\left(1-\frac{1}{2}e^{-j\omega}\right)^2}$$

$$= IDTFT\left\{\frac{4}{1-\frac{1}{3}e^{-j\omega}} - \frac{6}{1-\frac{1}{2}e^{-j\omega}} + \frac{3\left(\frac{1}{2}e^{-j\omega}\right)\left(2e^{j\omega}\right)}{\left(1-\frac{1}{2}e^{-j\omega}\right)^2}\right.$$

$$y[n] = 4\left(\frac{1}{3}\right)^n u[n] - 6\left(\frac{1}{2}\right)^n u[n] + 6(n+1)\left(\frac{1}{2}\right)^{n+1} u[n]$$

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Differentiation Property →

(a) Time Diffⁿ Property
(b) Frequence Diffⁿ Property.

(a) Time Differentiation Property →

Its Not Possible.

(b) Frequence Differentiation Property →

If $x[n]$ is Discrete Time Signal.

$$DTFT\{x[n]\} = X(e^{j\omega})$$

then

$$\boxed{\begin{aligned} DTFT\{n\,x[n]\} &= j\frac{d}{d\omega}X(e^{j\omega}) \\ DTFT\{n^2 x[n]\} &= j^2\frac{d^2}{d\omega^2}X(e^{j\omega}) \end{aligned}}$$

Proof → We Know that $DTFT\{x[n]\} = X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$

Differentiate the eqⁿ (i) w.r.t. ω

$$\frac{d}{d\omega}X(e^{j\omega}) = \frac{d}{d\omega}\sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$\frac{d}{d\omega} X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] \left\{ \frac{d}{d\omega} e^{-j\omega n} \right\}$$

$$= \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n} (-jn)$$

$$\frac{d}{d\omega} X(e^{j\omega}) = -j \sum_{n=-\infty}^{\infty} n\, x[n]\, e^{-j\omega n}$$

$$-\frac{1}{j} \frac{d}{d\omega} X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} n\, x[n]\, e^{-j\omega n}$$

So

$$j \frac{d}{d\omega} X(e^{j\omega}) = DTFT\left[n\, x[n] \right]$$

or $$\boxed{DTFT\ n\, x[n] = j \frac{d}{d\omega} X(e^{j\omega})}$$

Question → $DTFT\{n(\frac{1}{2})^n u[n]\} = ?$

Sol^n → we know that, $DTFT\{(\frac{1}{2})^n u[n]\} = \frac{1}{(1-\frac{1}{2}e^{-j\omega})}$

$$\therefore DTFT\{n(\tfrac{1}{2})^n u[n]\} = j\frac{d}{d\omega}\left\{\frac{1}{1-\frac{1}{2}e^{-j\omega}}\right\}$$

$$= j\left\{\frac{0-(0-\frac{1}{2}e^{-j\omega}\times -j)}{1-\frac{1}{2}e^{-j\omega}}\right\}$$

$$= \frac{j \times -\frac{j}{2}e^{-j\omega}}{1-\frac{1}{2}e^{-j\omega}}$$

$$= \frac{\frac{1}{2}e^{-j\omega}}{(1-\frac{1}{2}e^{-j\omega})^2} \quad \text{Ans} \qquad (\because j^2=-1)$$