7.4C The Two-winding Transformers

If we place a second winding on the core of the reactor of Fig. 7.11, we obtain the simplest form of a transformer. This is depicted in Fig. 7.15. The rms value of the induced emf E_1 appearing in the primary windding as follows from Eqn. (7.12) as

$$E_1 = \frac{N_1 \Phi_m \omega}{\sqrt{2}} = \frac{2\pi f}{\sqrt{2}} \Phi_m N_1 = 4.44 f \Phi_m N_1 \qquad \text{...(7.17)}$$

The mutual flux is

$$\Phi = A_i . B_m$$

where B_m = Maximum flux density

A_i = Effective iron section = K_i . cross-section of the core

K_i = Stacking factor (usually varies from 0.9 to 0.95)

Therefore, $E_1 = 4.44 f N_1 A_i B_m$

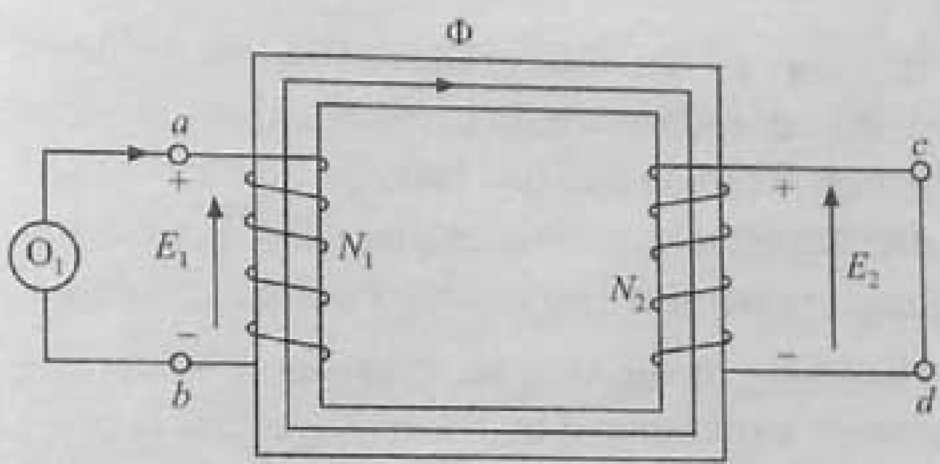


Fig. 7.15 *The two winding transformer*

Note that for transformers having relatively small winding resistance ($E_1 \cong V_1$) and the value of the maximum flux is determined by the applied voltage.

The induced emf appearing across the secondary winding terminals is produced by the same flux that causes E_1. Hence, the only difference in the rms values is due to the difference in the number of turns. That is

$$e_2 = -N_2 \frac{d\Phi_m}{dt}$$

which gives the rms value of secondary winding voltage

$$E_2 = 4.44 f \Phi_m N_2 \qquad \text{...(7.18)}$$

Dividing Eqn. (7.17) by (7.18), we have

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} = a = \text{ratio of transformation} \qquad \text{...(7.19)}$$

Note that, phasors $\overline{E}_1$ and $\overline{E}_2$ are in the same phase because they are induced by the same changing flux Φ which is also termed as the mutual flux.

- If $N_2 > N_1$, the secondary voltage will be greater than the primary voltage and the transformer is termed as step-up transformer.
- If $N_2 < N_1$, the secondary voltage will be less than the primary voltage and the transformer is termed as step-down transformer.

- For an ideal transformer:

$$V_1 I_1 = V_2 I_2$$

or

$$\frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2} = a$$

7.4D Transformer Phasor Diagram at No-Load

Figure 7.16 depicts the situation when a transformer is at no load. The development of the phasor diagram for the present case follows from Fig. 7.14 with additional two modifications as given below:

1. To identify the secondary induced emf $\bar{E}_2$ in the phasor diagram which as explained above must be in phase with $\bar{E}_1$. The amplitude of $\bar{E}_2$ will be E_1/a.
2. To a count for the fact that all the flux produced by the primary winding does not link the secondary winding. That is there is some leakage flux at the primary winding and this leakage flux is denoted by Φ_{11}. This is actually the difference between the total flux linking the primary winding and mutual flux (Φ) linking both the windings.

Fig. 7.16 Phasor diagram of transformer at no-load

The mutual flux exists only in the iron so involves a hysteresis loop of finite area. The reluctance experienced by the leakage flux is practically that of air. Thus its cyclic variation involves no hysteresis (or lagging effect). Hence in the phasor diagram the primary leakage flux must be placed in phase with the primary winding current. This leakage flux induces reaction voltage $(-\bar{E}_{l1})$ which lags Φ_{11} by 90 degrees. Therefore, the primary applied voltage must then provide a component equal and opposite to $(-\bar{E}_{l1})$ which is called the primary leakage reactance drop and is given by

$$\bar{E}_{l1} = jX_1 \bar{I}_0 \qquad (7.20)$$

This equation shows that phasor $\bar{E}_{l1}$ must lead $\bar{I}_0$ by 90°.

The phasor addition of the above drop and primary winding resistance drop as well as the voltage drop E_1 associate with the mutual flux yields the primary voltage V_1 as depicted in Fig. 7.16. Thus

$$\bar{V}_1 = R_1 \bar{I}_0 + jX_1 \bar{I}_0 + \bar{E}_1 \qquad (7.21)$$

The quantity X_1 of Eqn. (7.21) is called the primary winding leakage reactance; it is a fictitious quantity introduced as a convenience in representing the facts of the primary leakage flux.

7.4E Transformer Phasor Diagram at Load

In order to arrive at the phasor diagram of a transformer at full-load condition, consider a pure resistive load connected through a switch in the secondary circuit as shown in Fig. 7.17. The direction of the flux is as indicated in the figure with the flux increasing. By Lenz's law there is an emf induced in the secondary winding which instantaneously makes the terminal c positive with respect to terminal

9.8 An Ideal Transformer :

An ideal transformer is the transformer having the following characteristics :

- The losses are zero (No iron loss, no copper loss).
- The primary and secondary winding resistances are zero.
- The leakage flux is zero. Therefore all the flux produced by the primary winding is coupled to the secondary.
- A small current is required to develop flux inside the core. This happens because the permeability of the core is very large.
- The external voltage applied to the primary, V_1 is same as the primary induced voltage E_1. This is because the primary winding resistance is zero and so there is no voltage drop across it.

$$\therefore \quad E_1 = V_1$$

- Similarly the voltage induced in the secondary winding (E_2) will be equal to the load voltage V_2, because the secondary resistance is zero.

$$\therefore \quad E_2 = V_2$$

- The transformation ratio for an ideal transformer is given by,

$$K = \frac{E_2}{E_1} = \frac{V_2}{V_1}$$

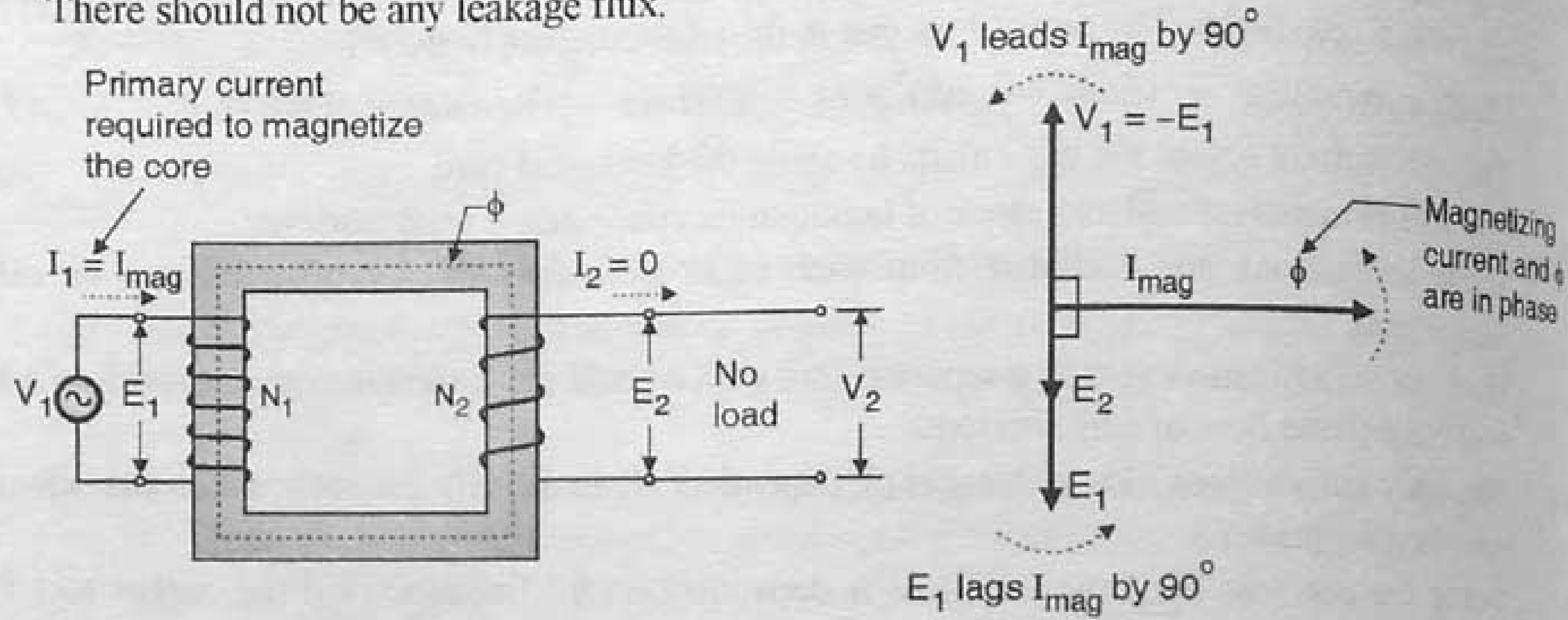
- Efficiency of an ideal transformer is 100%. This is because there are no losses taking place.
- The voltage regulation is 0%. That means the secondary voltage will remain constant irrespective of the load current.

9.8.1 Ideal Transformer on No Load :

An ideal transformer is as shown in Fig. 9.8.1(a). An ideal transformer is the one which has no power losses. In order to have a transformer with zero loss, the following conditions should be satisfied

Conditions for an ideal or lossfree transformer :

1. The primary and secondary windings do not have any resistance. i.e. winding resistance of primary secondary should be zero.
2. The losses taking place in the core i.e. hysteresis loss and eddy current loss should be zero.
3. There should not be any leakage flux.



(a) Ideal transformer on load (b) Phasor diagram

Fig. 9.8.1

Operation and phasor diagram :

- An ac voltage V_1 is applied across the primary winding of the transformer. As the load on the transformer is zero i.e. $R_L = \infty$, ideally the primary current $I_1 = 0$. But practically a small current called the magnetizing current I_{mag} flows through the primary winding.
- The magnetizing current is used for magnetizing the transformer core. As the primary winding is assumed to be purely reactive (R = 0) the magnetizing current lags behind the primary induced voltage by 90°, as shown in Fig. 9.8.1(b).
- Due to the sinusoidal magnetizing current, a sinusoidally varying magnetic flux is produced in the iron core. The flux is in time phase with I_{mag} as shown in Fig. 9.8.1(b).
- Due to this varying flux, emfs are induced in the primary (self induced voltage) E_1 and secondary (mutually induced voltage) E_2 respectively.

$$E_1 = -V_1 \text{ and } E_2 = V_2 \quad \text{...(9.8.1)}$$

- The magnitudes of E_1 and E_2 are proportional to the number of turns N_1 and N_2 respectively.
- The secondary induced voltage E_2 will also oppose V_1. So E_2 also appears in phase opposition with V_1. The magnitude of E_2 however is dependent on the turns ratio N_2 / N_1.
- E_1 and E_2 appear in phase with each other and in phase opposition with V_1.

Power input on no load :

- The input power to the ideal transformer on no load is given by,

$$P_0 = V_1 I_m \cos \phi_0 \quad \text{where } \phi_0 = \text{Angle between } V_1 \text{ and } I_m \text{ which is } 90^\circ \quad \text{...(9.8.2)}$$

$$\therefore \; P_0 = V_1 I_m \cos 90^\circ = 0 \text{ W} \quad ...(9.8.3)$$

- Thus the input power to an ideal transformer on no load is zero. The output power is zero and there are no losses taking place in the ideal transformer.

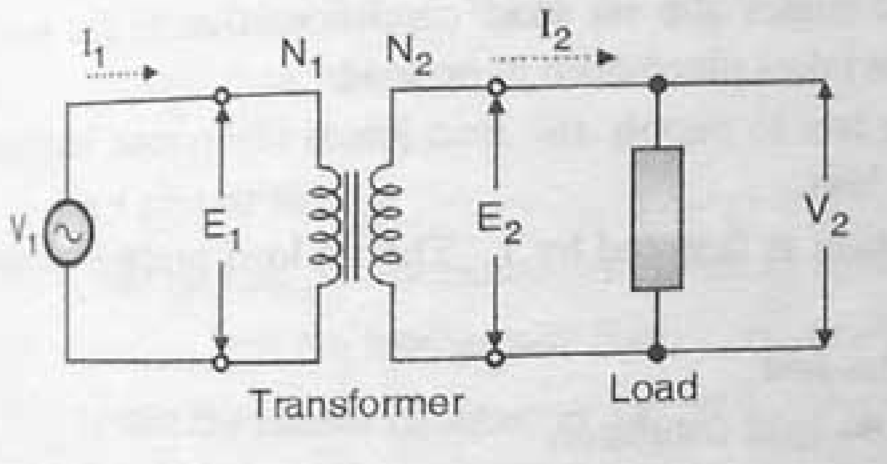
9.8.2 Ideal Transformer on Load :

- When some load is connected between the secondary terminals of the transformer, the transformer is said to be loaded or on load.
- Due to the load on the secondary, a finite secondary current starts flowing. If the load is (R + L) type then I_2 will lag behind V_2 by an angle ϕ_2 as shown in Fig. 9.8.2(b).
- As per the Lenz's law, the secondary current I_2 will oppose the cause producing it. Hence it opposes the magnetic flux. This is called as demagnetizing effect of I_2.
- Due to demagnetizing, the flux is weakened and it reduces the amount of self induced voltage E_1. Due to reduction in E_1, the difference between V_1 and E_1 will increase and the additional primary current I_2' called as load component starts flowing as shown in Fig. 9.8.2(b).

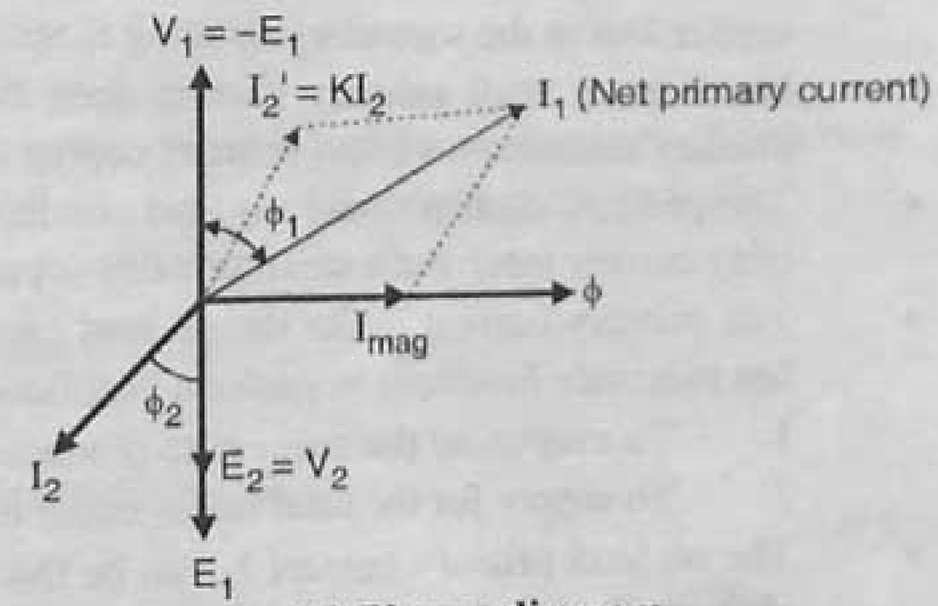
$$I_2' = I_2 \times \frac{N_2}{N_1} = K I_2 \quad ...(9.8.4)$$

- The current $I_2' = K I_2$ and it is 180° out of phase with the current I_2. The net primary current I_1 is the phasor sum of I_2' and I_{mag} as shown in Fig. 9.8.2(b).

$$\therefore \; \bar{I}_1 = \bar{I}_2' + \bar{I}_{mag} \quad ...(9.8.5)$$



(a) Ideal transformer on load



(b) Phasor diagram

Fig. 9.8.2

- Thus due to load on secondary side, the primary current of the transformer increases to supply the additional power to the load.
- The angle between V_1 and I_1 is ϕ_1 as shown in Fig. 9.8.2(b). Hence the primary power factor is $\cos \phi_1$.

Why does primary current increase when the load current is increased ?

- When the transformer is loaded, the load current I_2 will start flowing. Due to increase in load current I_2 the secondary ampere turns $N_2 I_2$ will also increase.
- This increased secondary mmf ($N_2 I_2$) will increase the flux ϕ_2 set up by the secondary current.
- This flux opposes the main flux ϕ_1 set up in the core by the current flowing through the primary winding. Hence the secondary mmf $N_2 I_2$ is called as the demagnetizing ampere turns.

impedance ... vice versa.

9.12 Equivalent Circuit of Transformer :

UPTU : 06-07, 08-09, GBTU : 11-12, MTU : 12-13

University Questions

Q. 1 Draw exact equivalent circuit and corresponding phasor diagram of a single phase transformer on load and explain them. Why no load current is kept small and how it is reduced ?

(Sem.-I : 06-07)

Q. 2 Develop the equivalent circuit of a single phase transformation on no-load and on-load conditions.

(Sem.-II : 08-09)

Q. 3 Draw equivalent circuit of transformer. (GBTU : 11-12, MTU : 12-13)

- The equivalent circuit of a transformer is as shown in Fig. 9.12.1. It consists of fixed and variable resistances, fixed and variable reactances for primary and secondary sides of the transformer as well as the no load components.

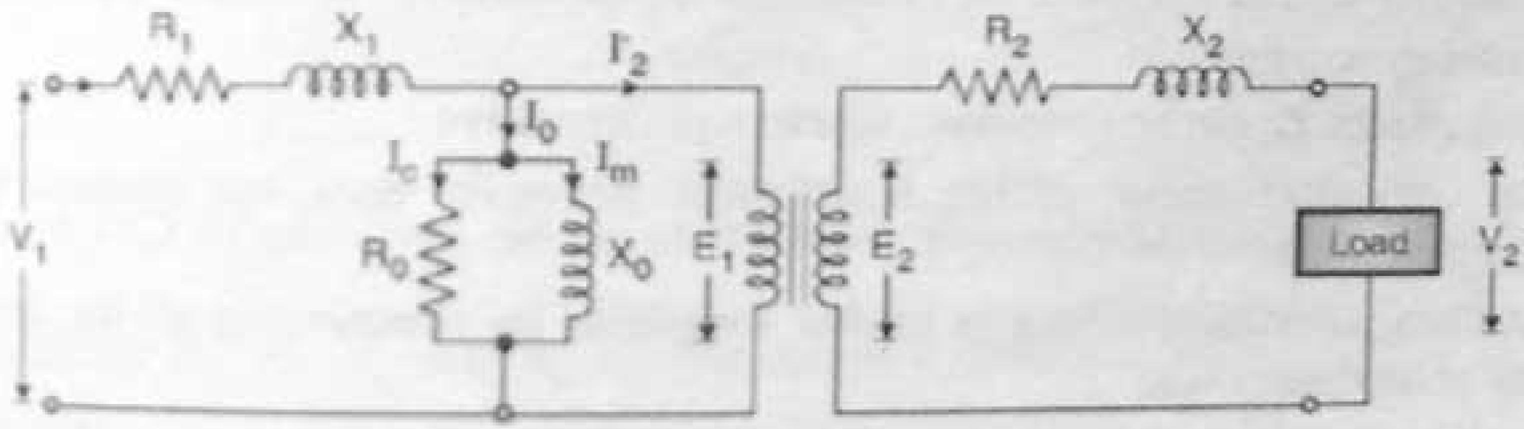


Fig. 9.12.1 : Equivalent circuit of transformer

9.12.1 Equivalent Circuit Referred to Primary Side :

- All the components on the secondary side of the transformer are transferred to the primary side as shown in Fig. 9.12.2.
- This circuit shows the equivalent circuit referred to the primary side.

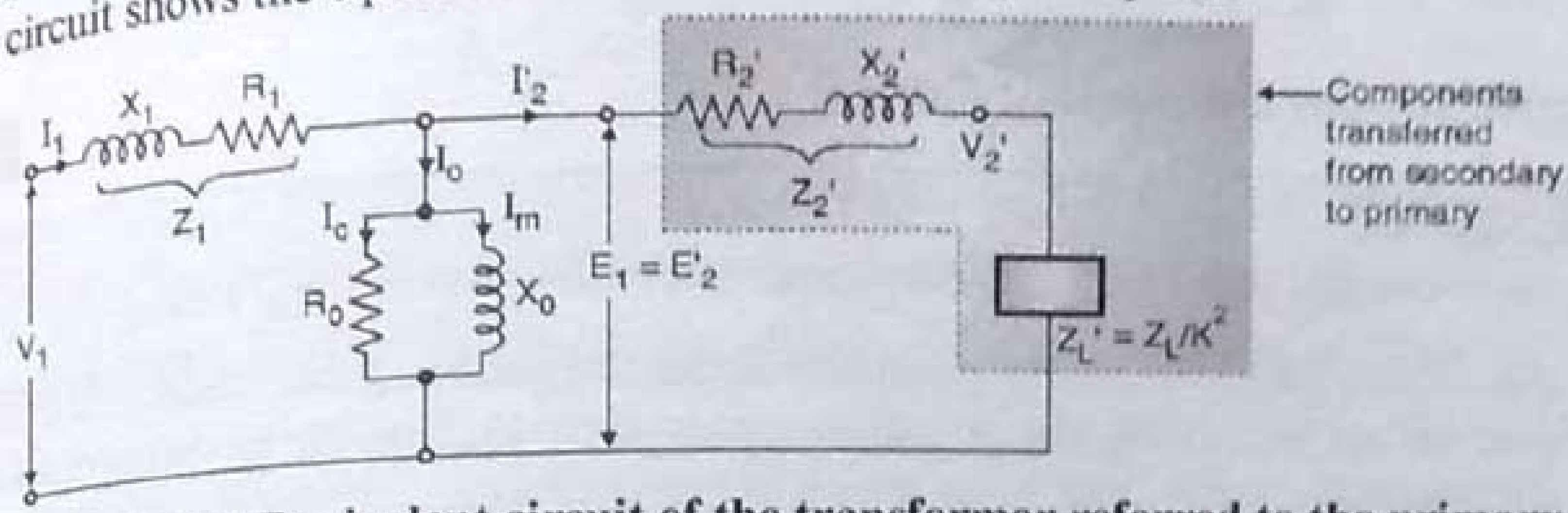


Fig. 9.12.2 : Equivalent circuit of the transformer referred to the primary

- R_2', X_2' and Z_L' are the values of R_2, X_2 and Z_L respectively when transferred to the primary side.
- The values of these components are obtained as follows :

$$R_2' = \frac{R_2}{K^2},\ X_2' = \frac{X_2}{K^2} \text{ and } Z_L' = \frac{Z_L}{K^2} \qquad \text{where } K = N_2 / N_1 \qquad ...(9.12.3)$$

- The voltage E_2 and current I_2 also are transferred to the primary side as E_2' and I_2' respectively.

The expressions for E_2' and I_2' are as follows :

$$E_2' = \frac{E_2}{K} \text{ and } I_2' = KI_2 \qquad ...(9.12.4)$$

9.12.2 Equivalent Circuit Referred to the Secondary :

- The other way of drawing the equivalent circuit is to transfer all the quantities from primary to secondary.
- The equivalent circuit of the transformer referred to the secondary side is as shown in Fig. 9.12.3.

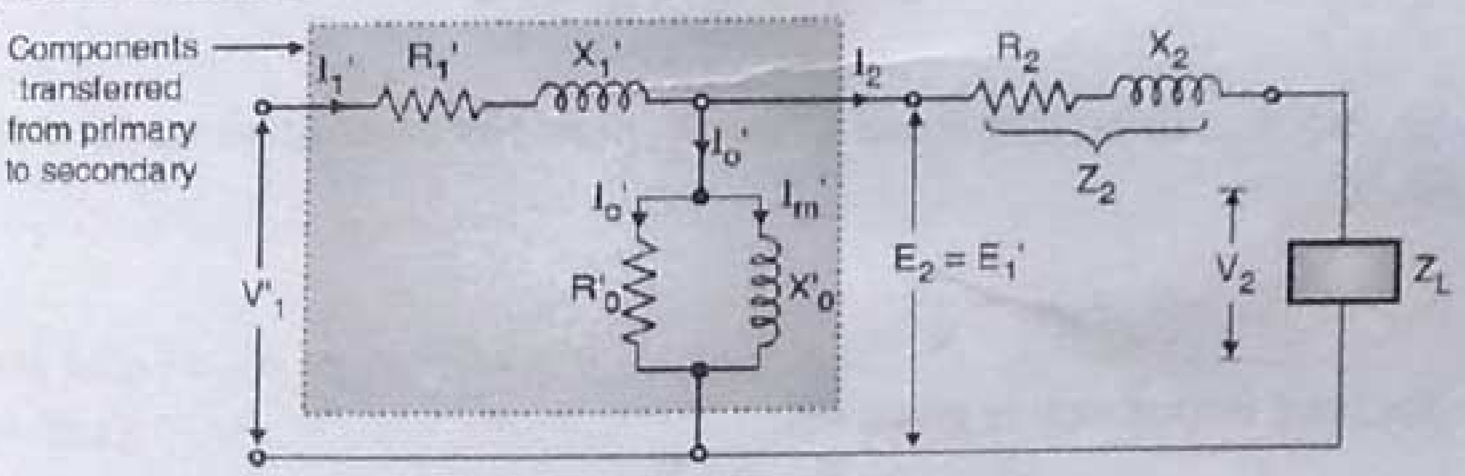


Fig. 9.12.3 : Equivalent circuit of transformer referred to the secondary

- The components R_1', X_1', R_0' and X_0' are the components which are transferred from primary to secondary side.
- The transfer expressions for these components are as follows :

$$R_1' = K^2 R_1, X_1' = K^2 X_1, Z_1' = K^2 Z_1 \qquad ...(9.12.5)$$

$$R_0' = K^2 R_0, X_0' = K^2 X_0' \qquad ...(9.12.6)$$