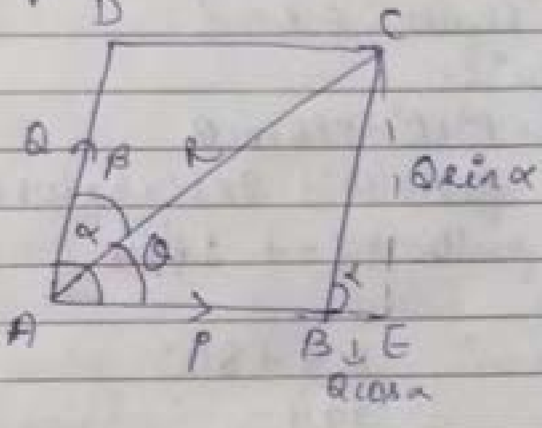
SIR CHHOTU RAM INSTITUTE OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF MECHANICAL ENGINEERING

ENGINEERING MECHANICS (BT-419)

NOTES ON 2-D FORCE SYSTEM

Law of parallelogram of forces:- As per this law if two force acting at a point be represented in magnitude and direction by two adjacent sides of a parallelogram then there resultant is represented in magnitude and direction by the diagnol of the parallelogram.



In Δ ACE

$$AC^2 = AE^2 + CE^2$$

$$R^2 = (P^2 + Q\cos\alpha)^2 + (Q\sin\alpha)^2$$

$$R^2 = P^2 + Q^2\cos^2\alpha + 2PQ\cos\alpha + Q^2\sin^2\alpha$$

$$R^2 = P^2 + 2P\cos\alpha + Q^2(\cos^2\alpha + \sin^2\alpha)$$

$$R^2 = P^2 + 2P\cos\alpha + Q^2$$

Ques:- Two forces of magnitude 10N and 8N are acting at a point. If the angle between the forces is 60°. Determine the magnitude of the resultant force and find the direction of resultant

R from force P and force Q

Given:-
Force P = 10N
Force Q = 8N
Angle b/w P & Q
$\alpha = 60°$

To find
Resultant R = ?

Soln :-

$R^2 = P^2 + Q^2 + 2PQ\cos\alpha$

$= (10)^2 + (8)^2 + 2 \times 10 \times 8 \times \cos 60°$

$= 100 + 64 + \cancel{160}^{80} \times \frac{1}{\cancel{2}}$

$= 164 + 80$

$= 224$

$R = \sqrt{224}$

$R = 11.62\,N$

And Direction

$\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}$

$= \frac{8 \times \sin 60}{10 + 8 \times \cos 60}$

$= \frac{\cancel{8}^{4} \times \frac{\sqrt{3}}{\cancel{2}}}{10 + \cancel{8}^{4} \times \frac{1}{\cancel{2}}}$

$= \frac{\cancel{4}^{2}\sqrt{3}}{\cancel{14}^{7}}$

$= \frac{2\sqrt{3}}{7}$

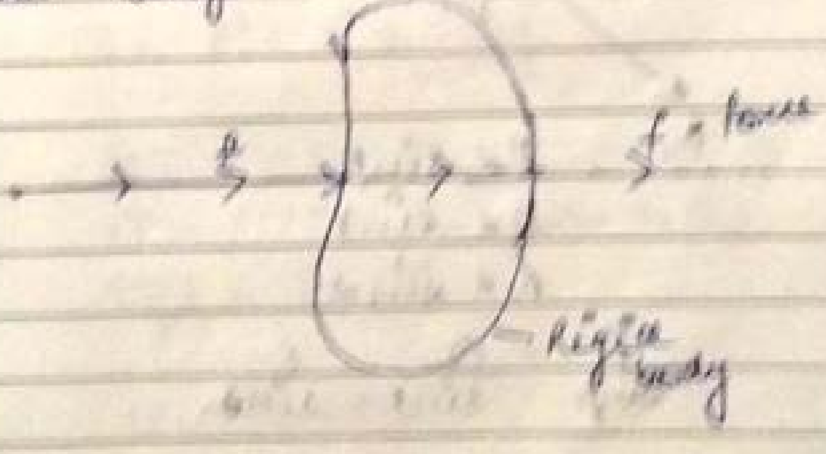
$\theta = 2\sqrt{3}\tan^{-1}$
$\beta = 1 \quad \alpha = 0$

Trigonometry Table

	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

Principle of Transmissibility of forces:

It states "if a force acts any point on a rigid body it may also be considered to act at any other point on its lying of action provided this point is rigidly connected with the body"



Force System: - When two or more forces act on a body they are called to form a system of forces. Some system of forces are:-

1) Coplanar forces
2) Non coplanar forces

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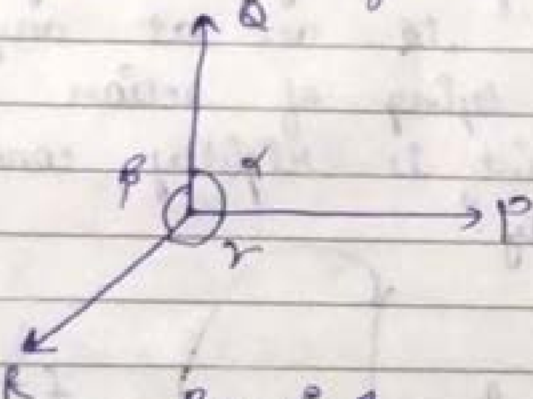
Colinear forces
Concurrent forces.

Coplanar concurrent
Non-coplanar concurrent.

Lami's theorm :- As per this theorm, if three forces acting at a point are an equalitrial then each force will be proportional to sign of angle b/w the other two forces.



$P \propto \sin\beta$
$Q \propto \sin\gamma$
$R \propto \sin\alpha$

$$\frac{P}{\sin\beta} = \frac{Q}{\sin\gamma} = \frac{R}{\sin\alpha}$$

Numerical :-

Ques :- Find the magnitude of two forces such that if they act at right angles there resultant is $\sqrt{10}$ N but if they act at 60° there resultant is $\sqrt{13}$ N.

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Given that

In I^{st} case $\alpha = 90°$

and $R = \sqrt{10}$ N

and In 2^{nd} case

$\alpha = 60°$

and $R = \sqrt{13}$ N

From I case

$R^2 = P^2 + Q^2 + 2PQ\cos\alpha$

$(\sqrt{10})^2 = P^2 + Q^2 + 2PQ\cos 90°$

$10 = P^2 + Q^2 \cancel{+2P}$

$10 = P^2 + Q^2 \longrightarrow (1)$

From 2^{nd} case

$R^2 = P^2 + Q^2 + 2PQ\cos\alpha$

$(\sqrt{13})^2 = P^2 + Q^2 + 2PQ\cos 60°$

$13 = P^2 + Q^2 + 2PQ \times \frac{1}{2}$

$13 = P^2 + Q^2 + PQ$

$\cancel{13 = P(P+Q) + Q^2} \longrightarrow (2)$

$\cancel{\frac{13 \times 1}{P} = P + Q^2}$

$13 = 10 + PQ$

$3 = PQ$

$(P+Q)^2 = P^2 + Q^2 + 2PQ$

$(P-Q)^2 = P^2 + Q^2 - 2PQ$

$(P+Q)^2 = 10 + 2 \times 3$

$= 16$

$P+Q=4$ ——— (3)

$(P-Q)^2 = 10-2\times3$

$= 10-6$

$= 4$

$P-Q = 2$ ——— (4)

from adding 3 and 4

$P+Q=4$

$+P-Q=2$

$2P=6 \qquad P=3$

put in 3

$P+Q=4$

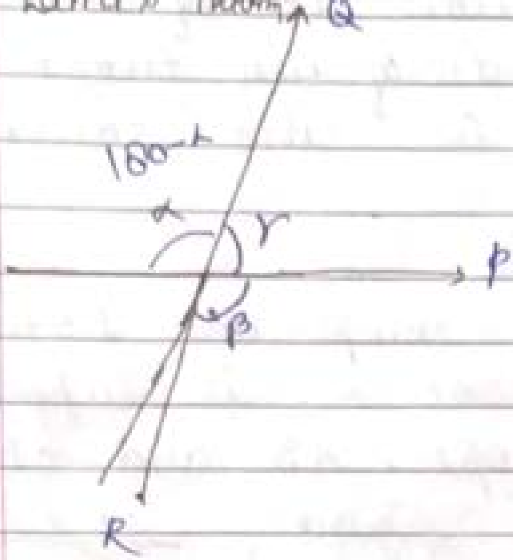
$3+Q=4$

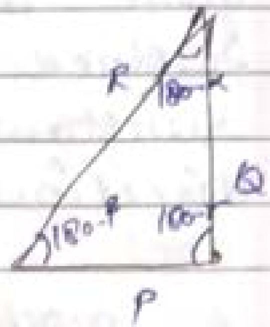
$Q=1$

$P=3$

& $Q=1$

According Lami's theorm :- Proof of Lami's Theorm





Sine Rule:-

$$\frac{P}{\sin(180-\alpha)} = \frac{Q}{\sin(180-\beta)} = \frac{R}{\sin(180-\gamma)}$$

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

Resolution of a force

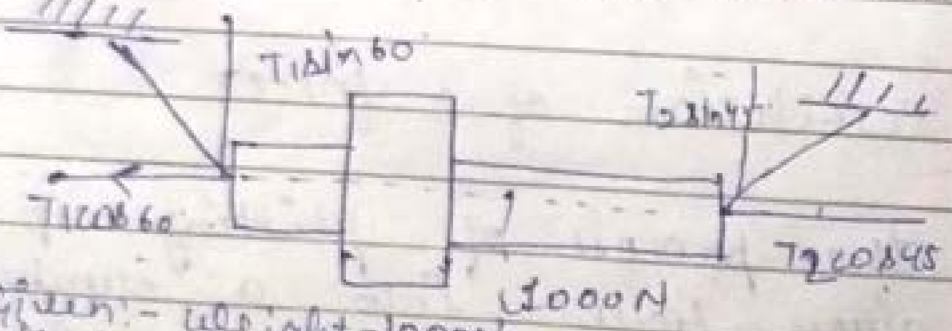
The process of splitting up the given force into a number of components without changing its effect on the body is called resolution of a force

Resolution of princle

Principle of resolution states the algebric sum of the resolved parts of a number of forces in a given direction is equal to the resolved part of uth there resultant in the same direction.

Que 2:- A machine component 1.5m long and weight 1000 N. is supported by two ropes AB and CD as shown in figure.

Calculate the tensions T_1 & T_2 in the ropes AB and CD.



Given:- Weight - 1000N

To find:- Tension in Rope AB $T_1 = ?$

CD $T_2 = ?$

$$\Sigma f x = 0$$

$$-T_1 \cos 60 + T_2 \cos 45 = 0$$

$$T_1 = \frac{T_2 \cos 45}{\cos 60}$$

$$= \frac{T_2 \left(\frac{1}{\sqrt{2}}\right)}{1/2}$$

$$= \sqrt{2}\, t_2$$

Scanned with CamScanner

$$\Sigma f_y = 0$$

$$T_1 \sin 60 - 1000 + T_2 \sin 45 = 0$$

$$T_1\left(\frac{\sqrt{3}}{2}\right) + T_2\left(\frac{1}{\sqrt{2}}\right) = 1000$$

$$\sqrt{2}\, T_2\left(\frac{\sqrt{3}}{2}\right) + \frac{T_2}{\sqrt{2}} = 1000$$

$$\frac{\sqrt{3}}{\sqrt{2}} T_2 + \frac{T_2}{\sqrt{2}} = 1000$$

$$T_2\left(\frac{\sqrt{3}+1}{\sqrt{2}}\right) = 1000$$

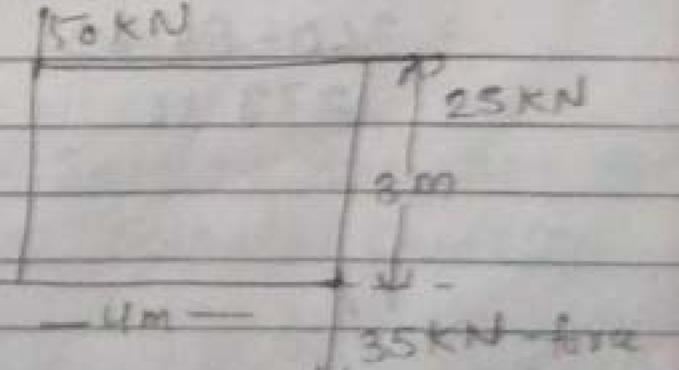
$$T_2 = \frac{1000\sqrt{2}}{\sqrt{3}+1}$$

$$T_2 = \cancel{518}\ 518.1\ N$$

$$T_1 = \sqrt{2} \times 518.1$$

$$= \cancel{732.6}\ 732.6\ N$$

Ques:- A system of forces that are acting at the corners of the rectangular block as shown in figure.



Return in the magnitude and direction of the resultant force.

$$\Sigma fx = 25 - 20 = 5$$

$$\Sigma fy = -50 - 35$$

$$= -85$$

$$R^2 = Fx^2 + Fy^2$$

$$= (5)^2 + (-85)^2$$

$$= 25 + 7125$$

$$= 7150$$

$$R = \sqrt{7150}$$

$$= 85.15 \text{ kN}$$

$$\text{Tan}\,\theta = \frac{Fy}{Fx}$$

$$= \frac{-85}{5}$$

$$= \cancel{-85}\ -17$$

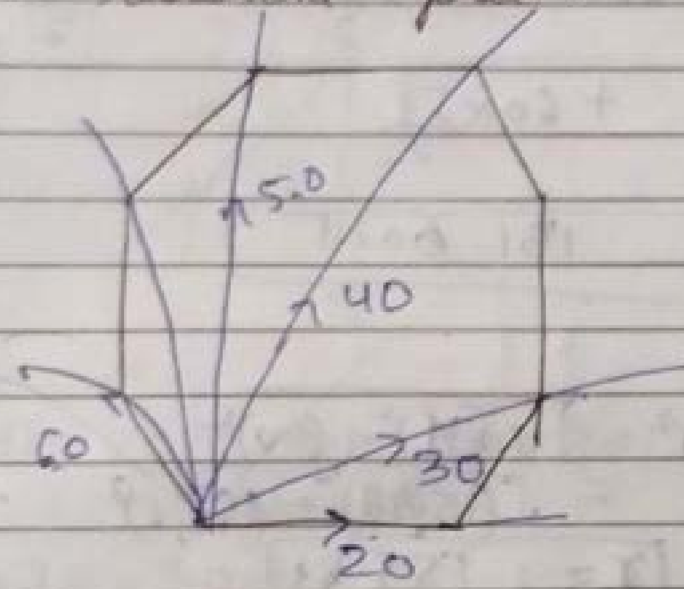
$$\theta = \tan^{-1}(-17)$$

$$= -86.63$$

$$= 360 - 86.63$$

$$= 273.36$$

Ques:- The forces 20 N, 30N, 40N, 50 and 60N are acting at one of the angular points of a regular hexagon towards the other five angular points taken in order. Find the magnitude and direction of the resultant force.



$\Sigma f(x) = 20$

$\Sigma f(y) = \cancel{60+50+}\ 40+30$

$= \cancel{180}$

R

$\Sigma(H) = 20\cos 0 + 30\cos 30 + 40\cos 60 + 50\cos 90 + 60\cos 120$

$\Sigma(H) = 20 + 30 \times \frac{\sqrt{3}}{2} + 40 \times \frac{1}{2} + 50 \times 0 + 60 \times -\frac{1}{2}$

$= 20 + 15\sqrt{3} + 20 - 30$

$= 45.98 + 20 - 30$

$= 65.98 - 30$

$= 35.98$

$\Sigma V = 20\sin 0 + 30\sin 30 + 40\sin 60 + 50\sin 90 + 60\sin 120$

$= 20\times 0 + 30\times \frac{1}{2} + 40\times \frac{\sqrt{3}}{2} + 50\times 1 + 60\times \frac{\sqrt{3}}{2}$

$\boxed{\Sigma V = 151.6025}$

$r = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$= \sqrt{(35.98)^2 + (151.6)^2}$

$\boxed{r = 155.81}$

$\tan\theta = \frac{\Sigma V}{\Sigma H}$

$= \frac{151.6}{36}$

$\tan\theta = \Sigma V / \Sigma H$

$\theta = \tan^{-1}(4.211)$

$\theta = 76.6^\circ$

Ques 6- The following forces act at a point

1 20 N inclined at 30° towards north of east.

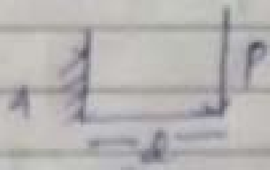
2 25N towards north.

3 30N toward north-west

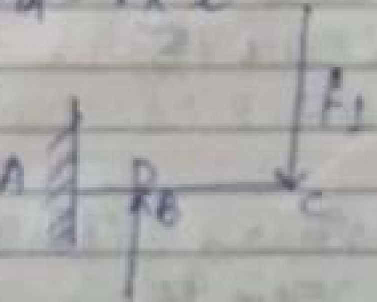
4 35N inclined at 40° towards south of west.

Moment

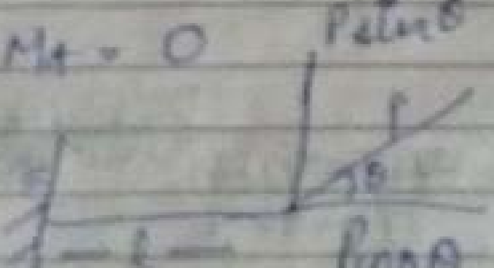
It is a turning effect produced by a force on the body on which it acts. The moment of the force is equal to the product of the force at the ⊥ distance of the point about which the moment is required and the line of action of the force.



$M_A = P \times l$



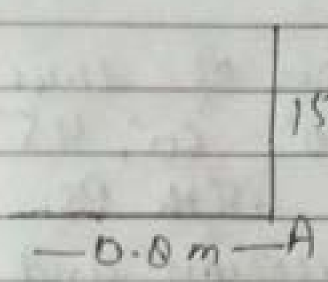
$M_A = 0$



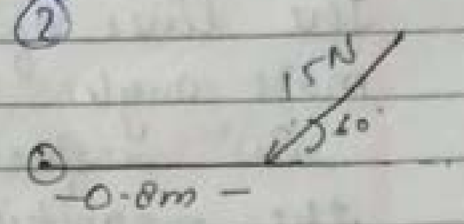
$M_A = P\sin\theta \times l$

A force of 15 N is applied ⊥ to the h of a door 0.8 m by as shown in figure.

(1)



(2)



Find the moment about the force about the he each.

According to I^{st} Figure

$$MA = 15 \times 0.8$$
$$= 12 \text{ Nm}$$

$$MA = P\sin\theta \times l$$
$$= 15 \times \sin 60^\circ \times 0.8$$
$$= 15 \times \frac{\sqrt{3}}{2} \times 0.8$$
$$= 15 \times \frac{1.7}{2} \times 0.8$$
$$= \frac{20.4}{2}$$
$$= 10.2 \text{ Nm}$$

Ques - A horizontal line P, Q, R, S is 12 m long where PQ = QR = RS = 4

m. Forces of 1000N, 1500N, 1000N and 500N act at PQR and S respectively with ⊥ direction.

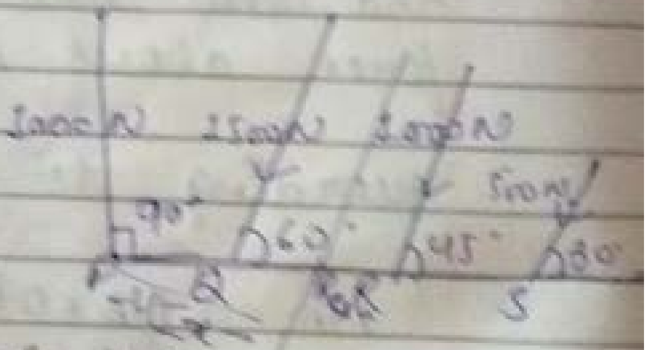
The lines of action of these forces make angles of 90°, 60°, 45° and 30° respectively with PS. Find the magnitude, direction and position of resultance force.

$\Sigma H = ?$

$\Sigma V = ?$

$R = ?$

$\theta = ?$



$\Sigma H = 1000 \cos 90° + 1500 \cos 60° + 1000 \cos 45° + 500 \cos 30°$

$= 0 + 1500 \times \frac{1}{2} + 1000 \times \frac{1}{\sqrt{2}} + 500 \times \frac{\sqrt{3}}{2}$

$= 750 + \frac{1000}{\sqrt{2}} + 250\sqrt{3}$

$= 750 + 707.10 + 250\sqrt{3}$

$= 1890.12$

$\Sigma V = 1000 \sin 90° + 1500 \sin 60° + 1000\ 45 + 500 \sin 30°$

$= 1000 + 1500 \times \frac{\sqrt{3}}{2} + 1000 \times \frac{1}{\sqrt{2}} + 500 \times \frac{1}{2}$

$$= 1000 + 750\sqrt{3} + \frac{1000}{\sqrt{2}} + 250$$

$$= 3256.144$$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= \sqrt{(1890.11)^2 + (3256.14)^2}$$

$$= \sqrt{3575 88 + 10606160}$$

$$= 3765.9$$

$$\theta = \tan^{-1} \frac{\Sigma V}{\Sigma H}$$

$$= \tan^{-1} 1.7227$$

$$= 0.0103$$

$$Rx x = 1500 \sin 60 \times 4 + 1000 \sin 45 \times 8 + 500 \sin 30 \times 12$$

$$= 1500 \times \frac{\sqrt{3}}{2} \times 4 + 1000 \times \frac{1}{\sqrt{2}} \times 8 + 500 \times \frac{1}{2} \times 12$$

$$= 5196 + 5656 + 3000$$

$$13852$$

$$x = \frac{13852}{3256.14}$$

$$= 4.25 \text{ cm}$$

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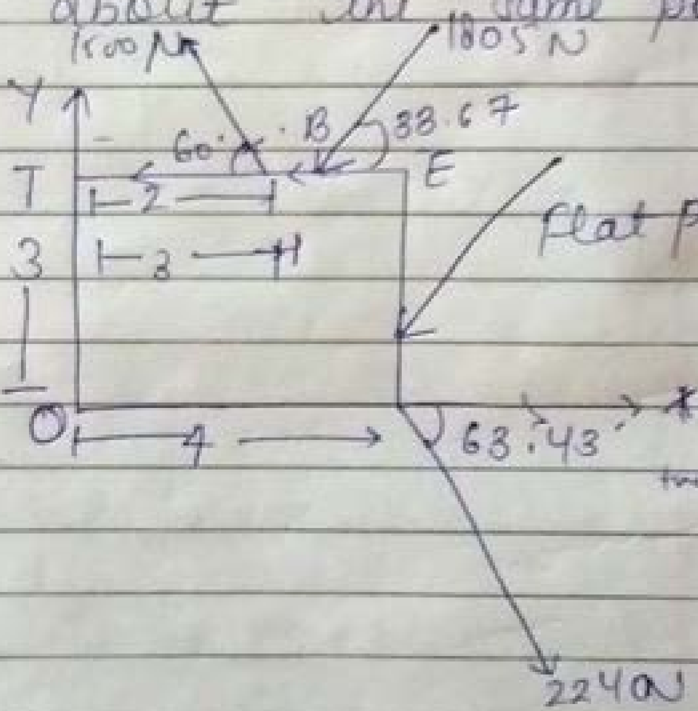
Principle of Moments and Varignon's Theorm :-

Principle of moments states that the moment of the resultant of the number of forces about any point is equal to the algebric sum of the moments of all the force about of the system about the same point.

ues:-



Figures shows the coplanar forces acting on a flat plate. Determine:-

1) The resultant
2) The X and Y intersects of the resultant.

Solⁿ:-

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$\Sigma H = -1500 \cos 60° + 1805 \cos 33.67$

Given

$OD = 3, \quad OA = 4, \quad AE = 3, DB = 3$

$DC = 2$

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2 + 2 \Sigma H \Sigma V \cos\theta}$

$\Sigma H = 2240 \cos 63.43 + 1805 \cos 33.67$
$-1500 \cos 60°$

$= 2240 \times 0.44 + 1805 \times 0.83 -$
1500×0.5

$= \cancel{985.6} + \cancel{1498.15} - 750$

$\Sigma H = \cancel{1733.75N} \quad -1250.3N$

$\Sigma V = -2240 \sin 63.43 - 1805 \sin 33.67$
$+1500 \sin 60°$

$= -2240 \times 0.89 - 1805 \times 0.55$
$+1500 \times 0.86$

$= -1993.6 - 992.75 +$
1290

$\Sigma V = \cancel{-1696.35} \; -1705.1N$

$\cancel{R = \sqrt{(1733.75)^2 + (-1696.35)^2}}$
$\cancel{= \sqrt{3005889.068 + 2877603.32}}$
$\cancel{= \sqrt{5883492.388}}$
$\cancel{R = 2425.59 N}$

$R = \sqrt{R_x^2 + R_y^2}$
$= \sqrt{(1250.3)^2 + (-1705.1)^2}$
$= \sqrt{1563250 + 2907366}$

$\boxed{R = 2114.4 N}$

$\theta = \tan^{-1} \frac{\Sigma x}{\Sigma y}$

$= 58.74180$

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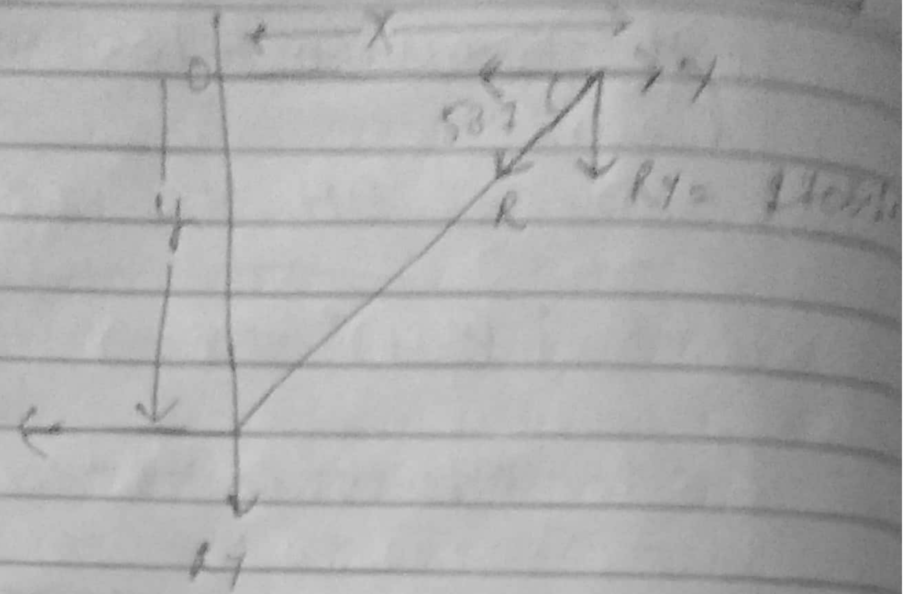

R: Moment of R about O point.

$M = -1500\cos 60 \times 3 - 1500\sin 60$
$-1805\cos 33.67 \times 3 + 1805\sin$
$33.67 \times 3 + 2240\sin 63.43$
$+ 2240\sin 63.43 \times 4$

$M = 1659.55$ Nm

Moment of R about O
= sum of moments of
Rx and Ry at O

But moment of R about O =
1659.66

$1659.66 = R_x \times 0 + R_y \times x$

$1659.66 = 1705.1 \times x$

$x = \frac{1705.1}{1659.66} \quad \frac{1659.66}{1705.1}$

$x = 0.97$ m

$[x = 0.97\text{m}]$ right of O

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Moment of R about O = Sum of moments of Rx and Ry at O.

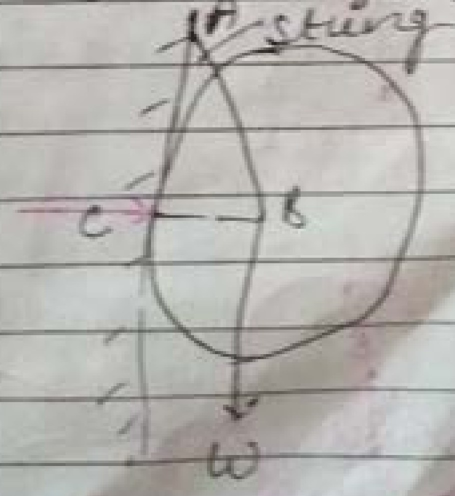
$$1659.66 = R_x \times Y + R_y \times 0$$

$$1659.66 = 1250.3 \times Y$$

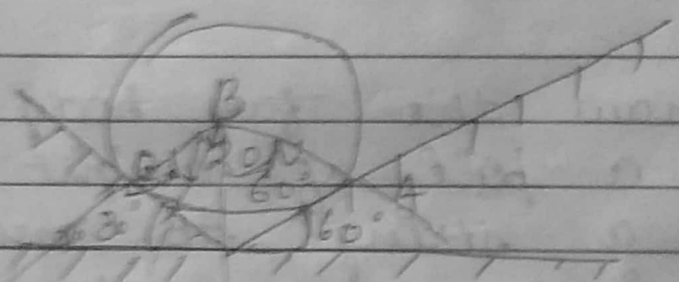
$$Y = \frac{1659.66}{1250.3}$$

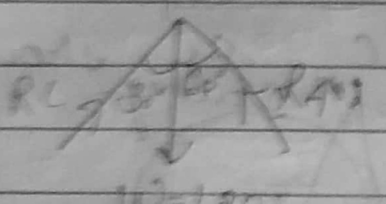
$\boxed{Y = 1.33m}$ below O.

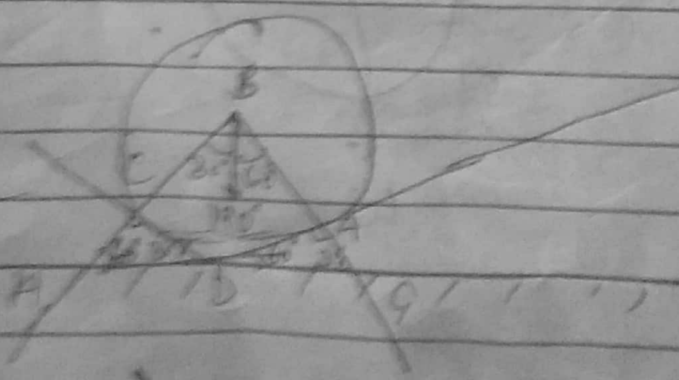
Ques:- Draw the free body diagram of a ball of weight W supported by a string AB and resting against a smooth vertical wall at C as shown in figure.

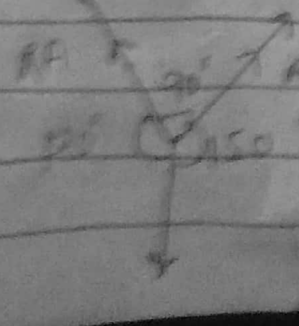


Ques - A ball of weidth 120 N rest in a right angle groorpe as shown in a figure. The sides of the groove are inclined to an angle of 30° and 60° to the horizontal if all the surfaces are smooth then the determines the reaction RA & RC at the point of contact.









120 - 60 => 120°
180 - 30 => 150°

$$\frac{R_A}{\sin 150^\circ} = \frac{R_C}{\sin 120^\circ} = \frac{W}{\sin 90^\circ}$$

$$\frac{R_A}{\sin 30^\circ} = \frac{R_C}{\sin 60^\circ} = W$$

$$R_A = W \sin 30^\circ = 120 \times \frac{1}{2} = 60 \text{ N}$$

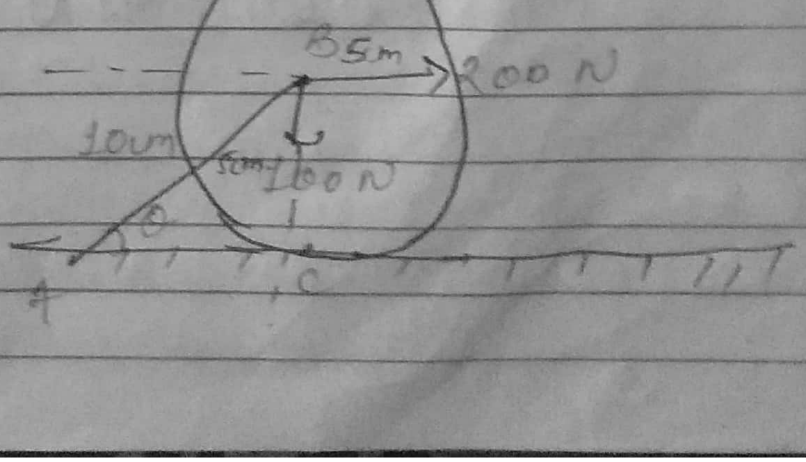
$$R_C = W \sin 60^\circ = 120 \times \frac{\sqrt{3}}{2} = 60 \times \sqrt{3}$$

$$= 60 \times 1.732$$

$$= 17.32 \times 6$$

$$= 103.92 \text{ N}.$$

Ques:- A circular roller of radius 5 cm and weight 100 N rest on a smooth horizontal surface & is held in pollition by an inclined bar AB of length 10 cm as shown in figure. A horizontal force of 200 N is acting at B. Find the tension or force in the bar AB & vertical reaction at C.



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$w = 100N$, $BC = 5\,cm$

$S = 200N$, $AB = 10\,cm$

In ΔABC

$$\sin\theta = \frac{BC}{AB}$$

$$= \frac{5}{10} \Rightarrow 0.5$$

$$\theta = \sin^{-1} 0.5$$

$$\theta = 30^\circ$$

F = tension of AB

$$\Sigma fx = 0$$

$$F\cos\theta - 200 = 0$$

$$F = \frac{200}{\cos\theta}$$

$$F = \frac{200}{\cos 30^\circ}$$

$$= \frac{200}{\frac{\sqrt{3}}{2}} \Rightarrow 230.94N$$

$$R_1 = w + F\sin\theta$$

$$= 100 + 230.94 \times \sin 30^\circ$$

$$= 100 + 230.94 \times \frac{1}{2}$$

$$= 330.94 \times \frac{1}{2}$$

$$= 215.47\,N.$$

Ques 6-

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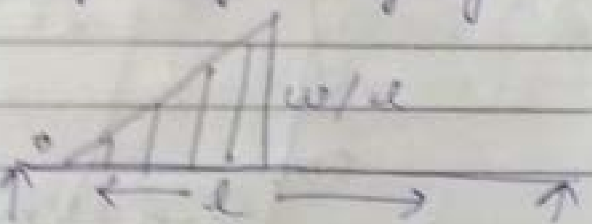
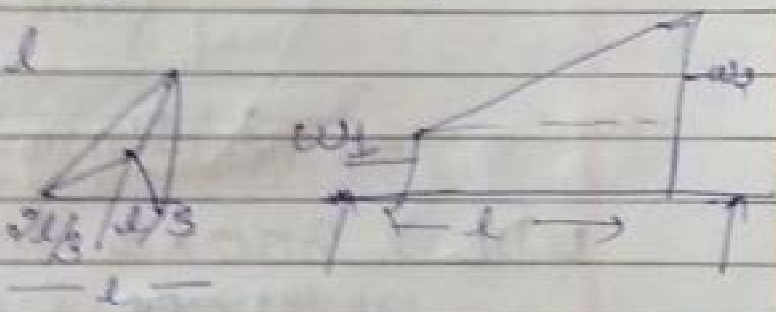
Types of loading
Point loading
UDL (Uniformly Distributed loading)
w intensity per length
$\Rightarrow$ $w \times 2l$

UVL (Uniformly Varying Load)

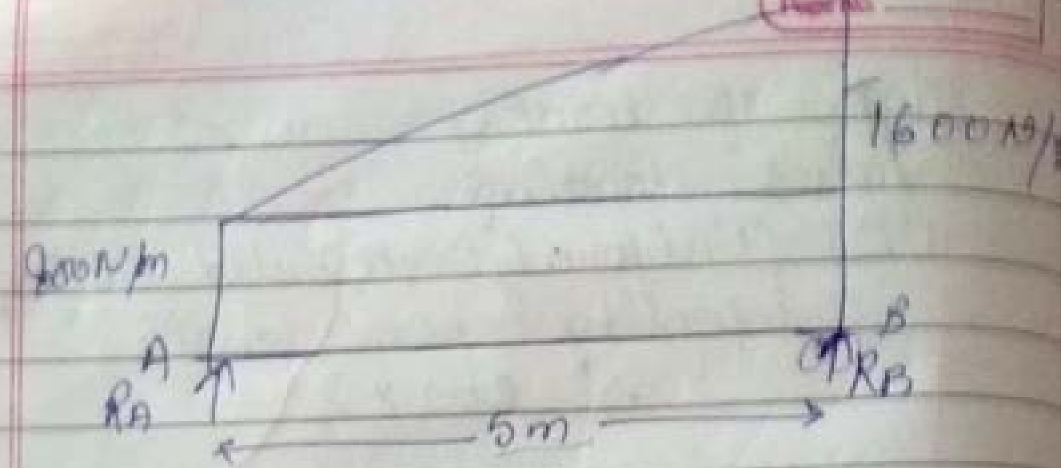


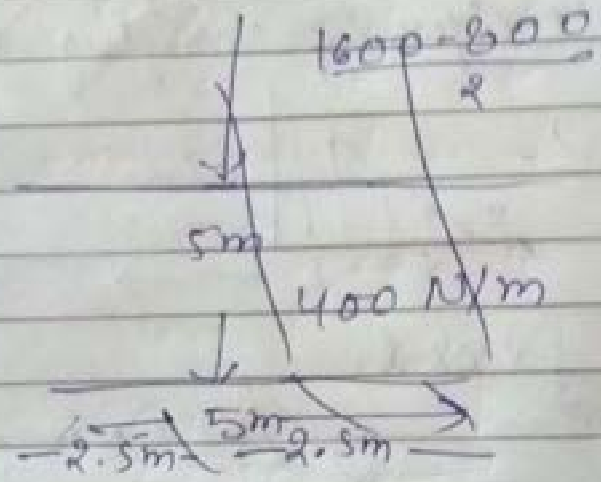
$\Rightarrow \frac{1}{2} \times w \times l$



$\frac{1}{2}(w_2 - w_1) \times l$

Ques:- A simply supported beam of length 5m carries a uniformly increasing load (UVL) of 800 N/m at one end to 1600 N/m at the other end. Calculate the reactions at both the ends.

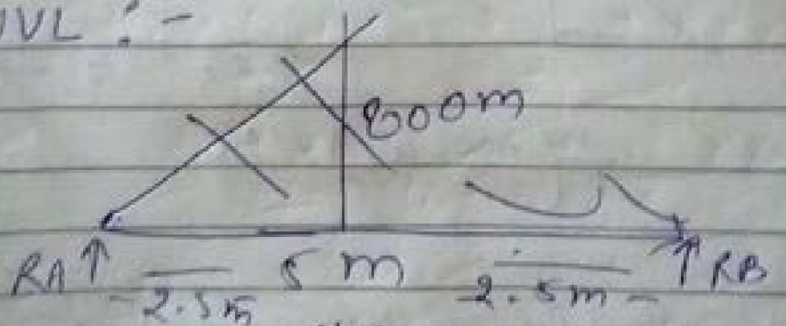




UDL => 800×5

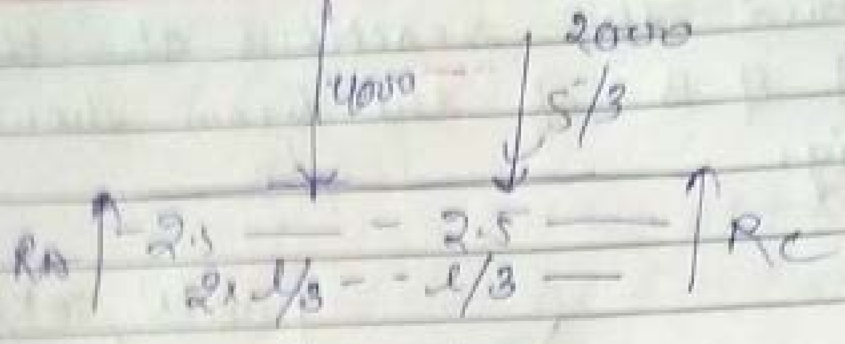
=> $4000\cancel{0}$ N/m^2

UVL :-



=> $\frac{1}{\cancel{2}} \times \overset{400}{\cancel{800}} \times 5$

=> 2000 N/m^2



$\Sigma f_x = 0$

$\Sigma f_y = 0$

$R_A + R_B = \cancel{6000}\ (4000 + 2000)$

$= 6000$ —— (1)

$\Sigma M_A = 0$

(Moment About A)

$R_A \times 0 + 4000 \times 2.5 + 2000 \times \frac{10}{3}$

$- R_B \times 5 = 0$

$R_B = 3333.33\ N.$

$R_A + R_B = 4000 + 2000$

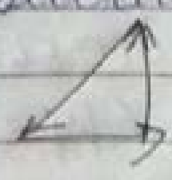
$R_A + 3333.33 = 6000$

$R_A = 6000 - 3333.33$

$R_A = 2666.67\ N.$

Law of Δ :-

It states that if three forces acting at a point be represent in magnitude & direction by 3 sides of a Δ. taken in order then they will be in equilibrium.



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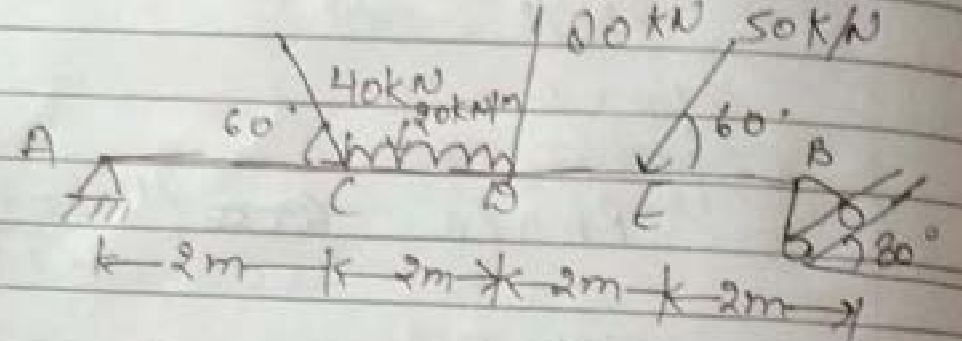
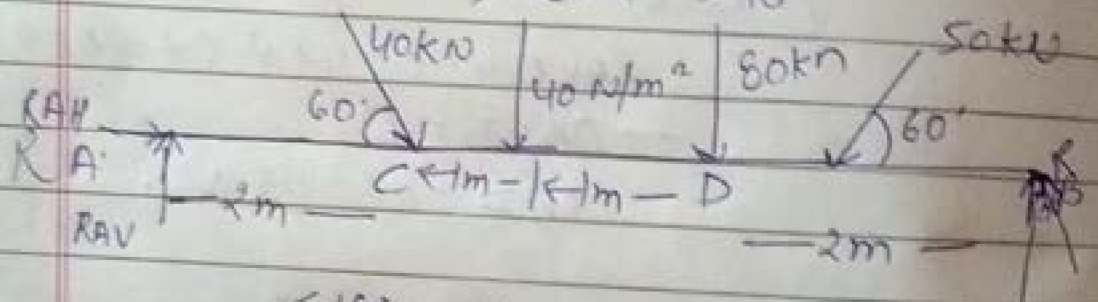
Ques:- Find the reactions at the support A & B of the beam shown in fig.



$\Rightarrow 20\times2 = 40$



$\Sigma f(x) = 0$

$40\cos60° + 50\cos60° + RB\cos30$

$40\times\frac{1}{2} + 50\times\frac{1}{2} + RB\times\frac{\sqrt{3}}{2}$

$20+25 + \frac{\sqrt{3}}{2}RB$

$\frac{90+\sqrt{3}RB}{2}$

$\Sigma f(y) = 40\sin60° + 50\sin60°$

$= 40\times\frac{\sqrt{3}}{2} + 50\times\frac{\sqrt{3}}{2}$

$= 20\sqrt{3} + 25\sqrt{3}$

$= 45\sqrt{3}$

$\Sigma fx = 0$

$RAH + 40\cos60 = 50\cos60 + RB\sin30$

$\Sigma fy = 0$

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$R_{AV} - 40\sin 60 - 40 - 80 - 50\sin 60$
$+ R_B\cos 30 = 0$

$\Sigma M_A = 0$

$40\sin 60 \times 2 + 40 \times 3 + 80 \times 4$
$+ 50\sin 60 \times 6 - R_B\cos 30 \times 6$
$= 0$

$R_B = 111\ kN$
$R_{AV} = 101.8\ kN$
$R_{AH} = 60.5\ kN$
$R_A = \sqrt{R_{AH}^2 + R_{AV}^2} = 118.42\ kN$