DISCRETE TIME FOURIER TRANSFORM (DTFT)

$$x[n] \longrightarrow \boxed{\text{DTFT}} \longrightarrow X(e^{j\omega})$$

Where,

$x[n] \rightarrow$ Time domain Discrete Signal

$X(e^{j\omega}) \rightarrow$ Freqⁿ domain Signal.

Symbolically

$$\text{DTFT}\{x[n]\} = X(e^{j\omega})$$

Mathematically,

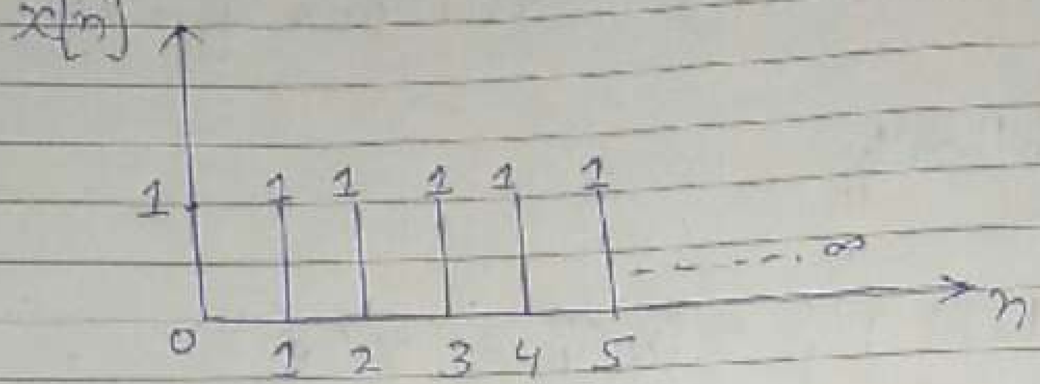
$$\text{DTFT}\{x[n]\} = X(e^{j\omega})$$

$$= \sum_{n=-\infty}^{\infty} x[n] \cdot e^{-j\omega n}$$

DTFT of Some Important Signals

① Unit Step Sequence

$$x[n] = u[n] = \begin{cases} 0, & n<0 \\ 1, & n \geq 0 \end{cases}$$



Soln. –

We know that

$$DTFT\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

So $DTFT\{x[n]\} = \sum_{n=-\infty}^{-1} x[n] \cdot e^{-j\omega n}$

$$+ \sum_{n=0}^{\infty} x[n] \cdot e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{-1} 0 \cdot e^{-j\omega n} + \sum_{n=0}^{\infty} 1 \cdot e^{-j\omega n}$$

[because $u(n) = 0$ for $n < 0$
& $1, n \geq 0$]

$$= e^{-0} + e^{-j\omega} + e^{-2j\omega} + \cdots$$

$$= 1 + e^{-j\omega} + e^{-2j\omega} + \cdots$$

Sum of Infinite terms of G.P

$$= \frac{a}{1-r}$$

$$= \frac{1}{1-e^{-j\omega}}$$

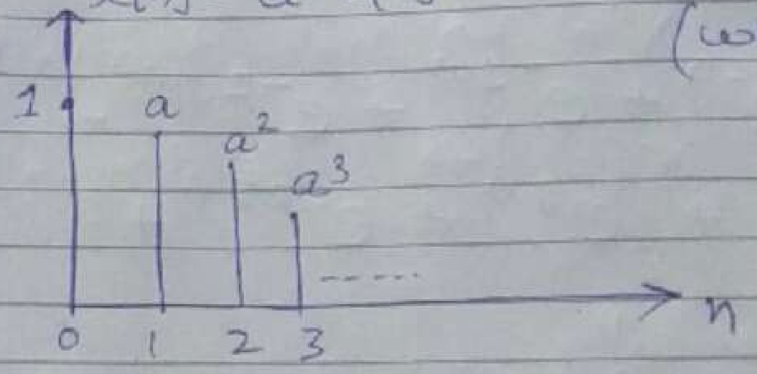
(2) $x[n] = a^n u[n]$ $(a<1)$

(i) Draw the signal

(ii) Determine DTFT of Given Signal

Sol^n:- (i) $x[n] = a^n u[n]$

(where, $a<1$)



(ii)

$$DTFT\{x[n]\} = \sum_{n=-\infty}^{\infty} a^n u[n] \cdot e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{-1} 0 + \sum_{n=0}^{\infty} a^n u[n] \cdot e^{-j\omega n}$$

$$= 1 + a \cdot e^{-j\omega} + a^2 e^{-2j\omega} + \ldots$$

Sum of G.P $\Rightarrow$ $S_\infty = \dfrac{a}{1-r} = \dfrac{1}{1-ae^{-j\omega}}$

Solve the following Questions

(1) $x[n] = n \cdot u[n]$

(2) $x[n] = n \cdot a^n \, u[n]$

Sketch the signals & find DTFT of both the signals.