

Module - III (Statistical Techniques-I)

(1) Measures of central tendency: - Do your self from
YouTube: Link Bhagwan Singh Vishwakarma.

Moments: Moments are statistical tools, used in statistical investigations. The moments of a distribution are the arithmetic means of the various powers of the deviations of items from some given number.

Moments About Mean (Central Moments)

(i) For individual Series: \rightarrow Let $x_1, x_2, x_3, \dots, x_n$ be the data. Then, we know that,

$$\text{Mean } \bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

\therefore r^{th} Moment μ_r about mean \bar{x} is defined as

$$\mu_r = \frac{\sum_{i=1}^n (x_i - \bar{x})^r}{n}; \quad r = 0, 1, 2, \dots$$

EX: Find first four moments for the following

x	3	6	8	10	18
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Sol: - Here $n = 5$, $\sum x = 45$, $\bar{x} = 9$, $\bar{x} = \frac{\sum x_i}{n} = \frac{45}{5} = 9$

$$\begin{aligned} \sum (x_i - \bar{x}) &= 0 \\ \sum (x_i - \bar{x})^2 &= 128 \\ \sum (x_i - \bar{x})^3 &= 486 \\ \sum (x_i - \bar{x})^4 &= 7940 \end{aligned}$$

$$\mu_r = \frac{\sum (x_i - \bar{x})^r}{n}$$

$$\therefore \mu_1 = \frac{\sum (x_i - \bar{x})^1}{n} = \frac{0}{5} = 0$$

$$\mu_2 = \frac{\sum (x_i - \bar{x})^2}{n} = \frac{128}{5} = 25.6$$

$$\mu_3 = \frac{\sum (x_i - \bar{x})^3}{n} = \frac{486}{5} = 97.2$$

$$\mu_4 = \frac{\sum (x_i - \bar{x})^4}{n} = \frac{7940}{5} = 1588$$

moments about mean for a frequency distribution

$$\mu_r = \frac{1}{N} \sum_{i=1}^n f_i (x_i - \bar{x})^r, \text{ where } r = 0, 1, 2, \dots$$

$$\text{and } N = \sum_{i=1}^n f_i$$

For ① $r=0$,
$$\mu_0 = \frac{1}{N} \sum_{i=1}^n f_i (x_i - \bar{x})^0 = \frac{1}{N} \sum_{i=1}^n f_i = \frac{1}{N} \times N = 1$$

$\Rightarrow \boxed{\mu_0 = 1}$

For ② $r=1$,
$$\mu_1 = \frac{1}{N} \sum f_i (x_i - \bar{x}) = \frac{1}{N} \cdot \sum f_i x_i - \bar{x} \cdot \frac{\sum f_i}{N} = \bar{x} - \bar{x} = 0$$

$\therefore \boxed{\mu_1 = 0}$

For $r=2$,
$$\mu_2 = \frac{1}{N} \sum_{i=1}^n f_i (x_i - \bar{x})^2 = (\text{S.D.})^2$$

where S.D = Standard Deviation = Variance

Similarly
$$\mu_3 = \frac{1}{N} \sum_{i=1}^n f_i (x_i - \bar{x})^3$$

$$\mu_4 = \frac{1}{N} \sum_{i=1}^n f_i (x_i - \bar{x})^4$$

and so on

Question: \rightarrow calculate $\mu_1, \mu_2, \mu_3, \mu_4$ for the following frequency distribution

Marks	5-15	15-25	25-35	35-45	45-55	55-65
No. of Students	10	20	25	20	15	15

Hint: \rightarrow See Page No. 3.

$$\bar{x} \text{ (Mean)} = \frac{\sum f_i x_i}{N} = \frac{3400}{100} = 34$$

$$\mu_r = \frac{\sum f_i (x_i - \bar{x})^r}{N}$$

$$r=0, \mu_0 = \frac{\sum f_i (x_i - \bar{x})^0}{N} = \frac{\sum f_i}{N} = 1$$

$$r=1 \Rightarrow \mu_1 = \frac{\sum f_i (x_i - \bar{x})^1}{N} = \frac{0}{100} = 0$$

$$r=2 \Rightarrow \mu_2 = \frac{\sum f_i (x_i - \bar{x})^2}{N} = \frac{21400}{100} = 214$$

$$r=3 \Rightarrow \mu_3 = \frac{\sum f_i (x_i - \bar{x})^3}{N} = \frac{46800}{100} = 468$$

$$r=4 \Rightarrow \mu_4 = \frac{\sum f_i (x_i - \bar{x})^4}{N} = \frac{9671200}{100} = 96712$$

Moments About an Arbitrary Number (Raw Moments)
 (किसी एक संख्या) = मोड़ों की संख्या]

If $x_1, x_2, x_3, \dots, x_n$ are the value of Variables x with corresponding frequencies $f_1, f_2, f_3, \dots, f_n$ respectively, Then the r th Moment μ'_r about the Number A defined as

$$\mu'_r = \frac{1}{N} \sum_{i=1}^n f_i (x_i - A)^r, \quad r=0, 1, 2, 3, \dots$$

$$\text{Where } N = \sum_{i=1}^n f_i$$

$$\text{For } r=0 \Rightarrow \mu'_0 = \frac{1}{N} \sum_{i=1}^n f_i (x_i - A)^0 = 1$$

$$\begin{aligned} r=1 \Rightarrow \mu'_1 &= \frac{1}{N} \sum_{i=1}^n f_i (x_i - A)^1 \\ &= \frac{1}{N} \sum f_i x_i - \frac{A}{N} \sum f_i = \bar{x} - A \end{aligned}$$