Agricultural Engg. IInd SEM, Thermodynamics, Refrigeration and Air Conditioning - by - Yogesh Kumar

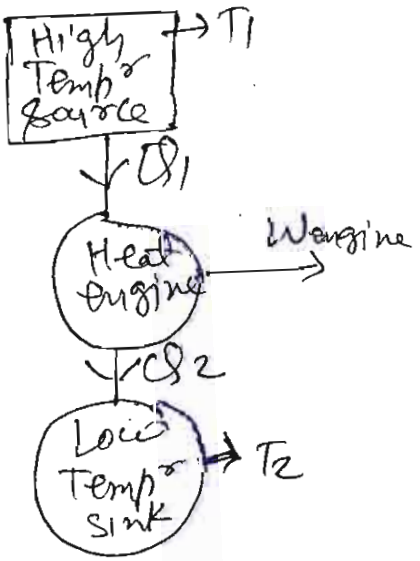
Topic → Carnot's Theorem → It is impossible for any heat engine to be more efficient than a carnot engine when operating between two given temperatures :-

$$\eta_{max} = \eta_{carnot}$$

Proof - Consider a heat engine drawing heat Q_1 from a heat reservior at temperature T_1, delivering work W and dumping (rejecting) heat Q_2 into a heat sink at Temperature T_2.

The heat engine operates in cycle, that is, it takes in heat Q_1, does work W, rejects heat Q_2, and in the end returns back to its original unchanged state.

Heat Engine and efficiency -



Let T_1 = Temp^r of the source (High)

T_2 = Temp^r of the sink (Low)

Q_1 = Heat supplied by source to Heat engine

Q_2 = Heat rejected to sink

W = work output from Heat engine

From First Law of thermodynamics -

$$\Rightarrow Q_1 - Q_2 = W$$

$$\text{Efficiency of Engine, } \eta = \frac{\text{work output}}{\text{Heat supplied}}$$

$$\Rightarrow \eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

Consider, the net change in entropy ΔS of the universe

High Temp^r source releases a heat Q_1 at a constant Temp^r T_1. Thus the change in its entropy is

$$\Delta S_1 = -\frac{Q_1}{T_1}$$

because Heat engine reject heat to the sink (Q_2) at constant Temp^r T_2 Thus the change in its entropy is

$$\Delta S_2 = \frac{Q_2}{T_2}$$

Thus the net change in entropy of the universe is

$$\Delta S = \Delta S_1 + \Delta S_2$$

$$= \frac{Q_2}{T_2} - \frac{Q_1}{T_1}$$

using the second Law of thermodynamics, $\Delta S \geq 0$,

which implies

$$\frac{Q_2}{T_2} - \frac{Q_1}{T_1} \geq 0$$

$$\frac{Q_2}{T_2} \geq \frac{Q_1}{T_1}$$

$$\frac{Q_2}{Q_1} \geq \frac{T_2}{T_1}$$

$$1 - \frac{Q_2}{Q_1} \leq 1 - \frac{T_2}{T_1}$$

Since the left hand side represents the efficiency of the given Heat engine (η) and right hand side is the efficiency of a Carnot engine,

$$\eta \leq \eta_{Carnot}$$

$$\Rightarrow \eta_{max} = \eta_{Carnot}$$

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for any Query - Contact b/n 2-3 P.m