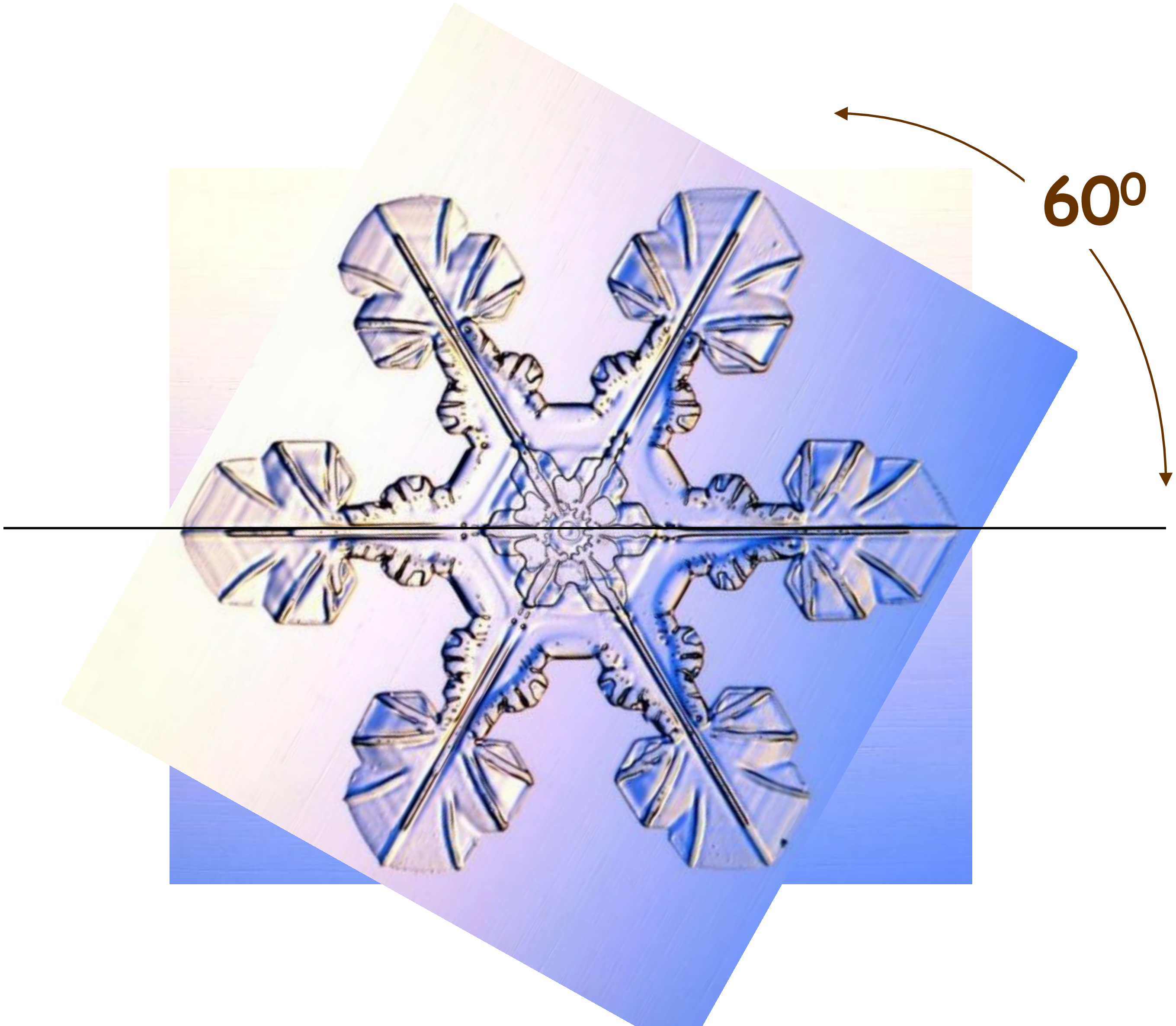
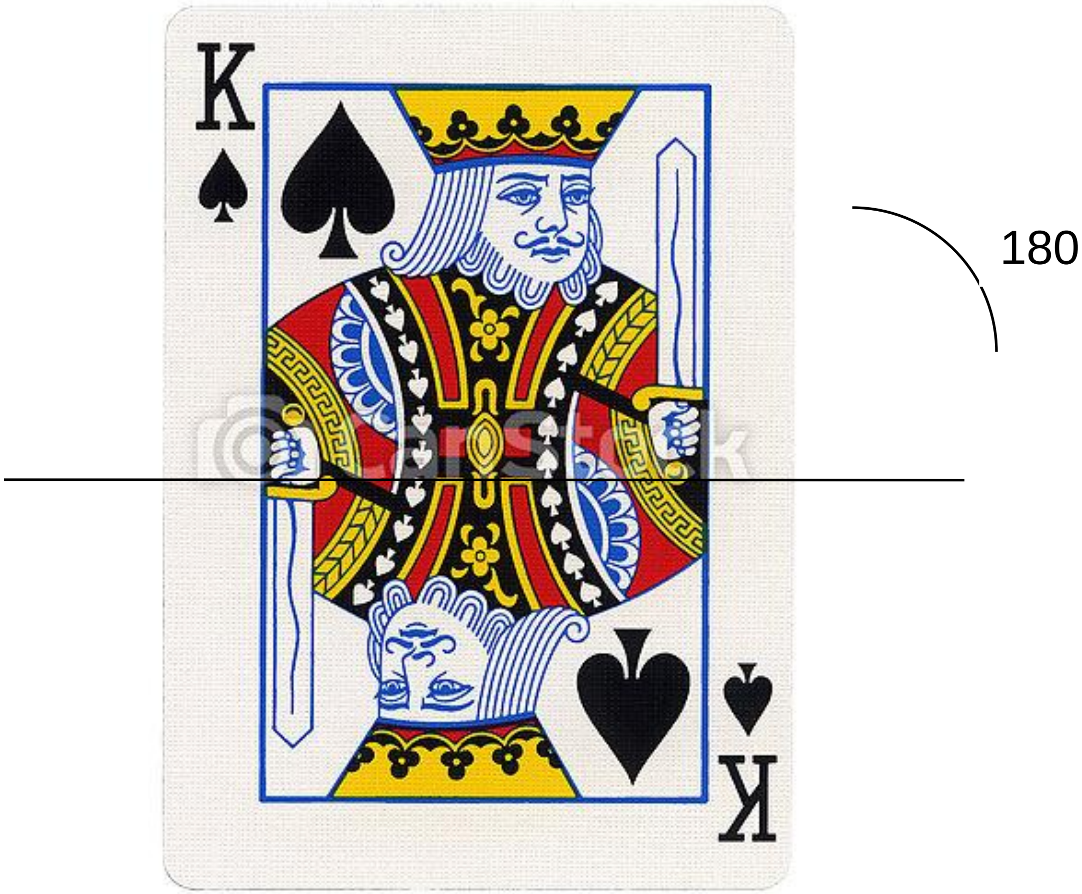
Symmetry and Conservation

Snowflakes

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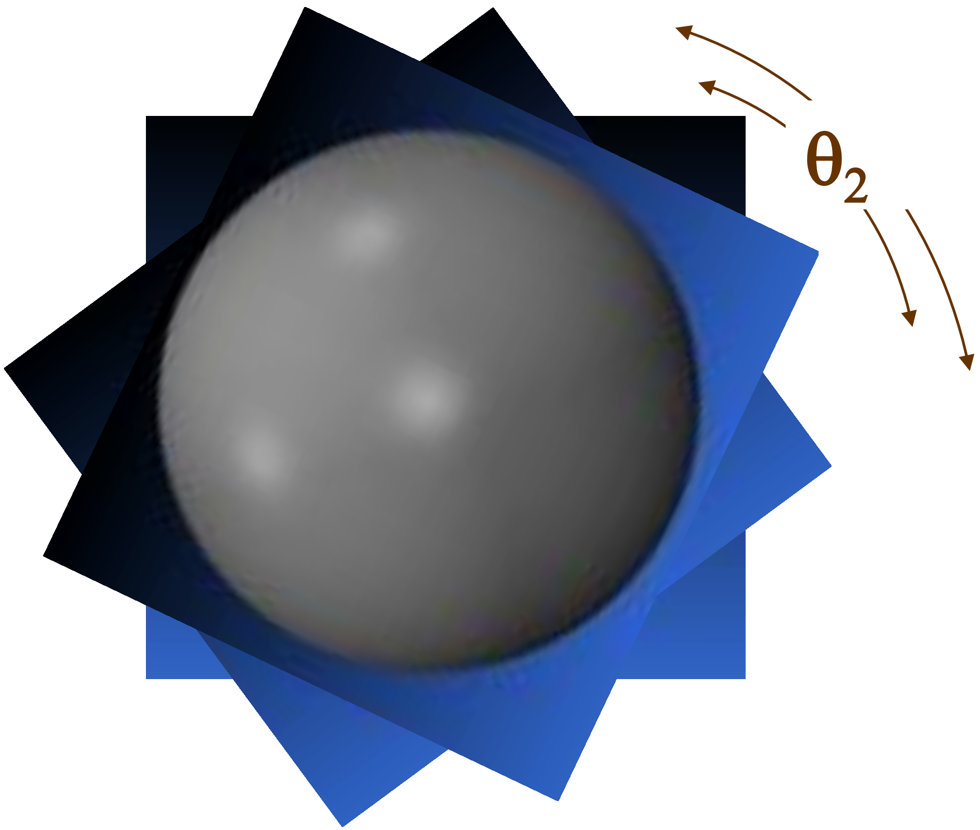
Rotational symmetry



Space translation symmetry

Time-translation symmetry in music

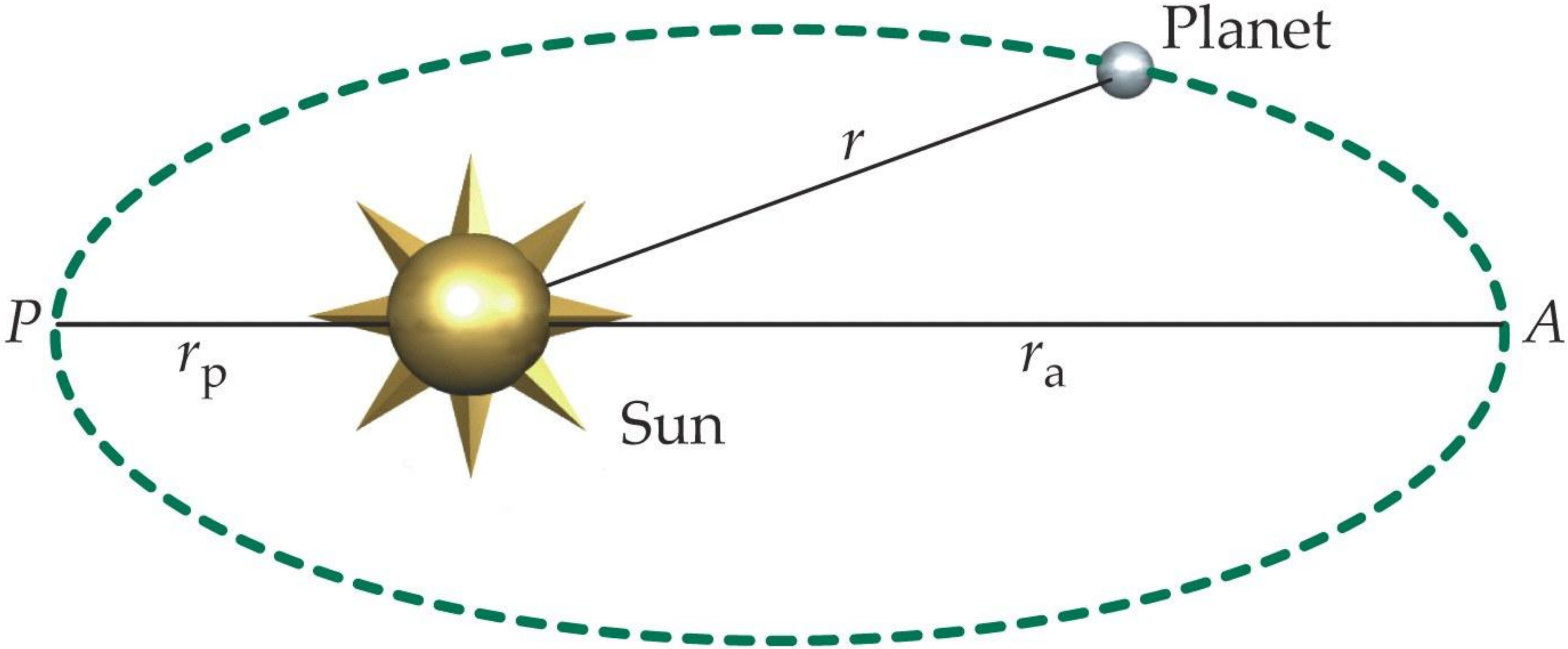
Prior to Kepler, Galileo, etc

God is perfect, therefore nature must be perfectly symmetric:

Planetary orbits must be perfect circles

Celestial objects must be perfect spheres

Kepler: planetary orbits are ellipses; not perfect circles



Galileo:There are mountains on the Moon; it is not a perfect sphere!

Catch in Newton's Laws

Law of Inertia (1st Law):

only works in inertial reference frames.

What is an inertial reference frame?:

a frame where the law of inertia works.

Newton's 2nd Law

$$\vec{F} = m\vec{a}$$

But what is $\vec{F}$?

whatever gives you the correct value for $m\vec{a}$

Is this a law of nature?
or a definition of force?

But Newton’s laws led us to discover Conservation Laws!

- Conservation of Momentum

- Conservation of Energy

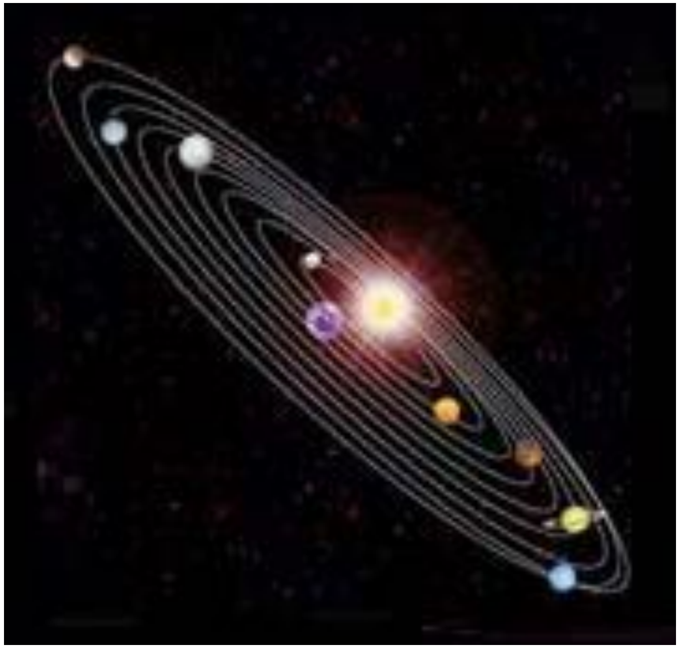
- Conservation of Angular Momentum

Newton's laws implicitly assume that they are valid for all times in the past, present & future

Processes that we see occurring in these distant Galaxies actually happened billions of years ago

Newton's laws have time-translation symmetry

Newton's laws are supposed to apply equally well everywhere in the Universe



Newton realized that the same laws that cause apples to fall from trees here on Earth, apply to planets billions of miles away from Earth.

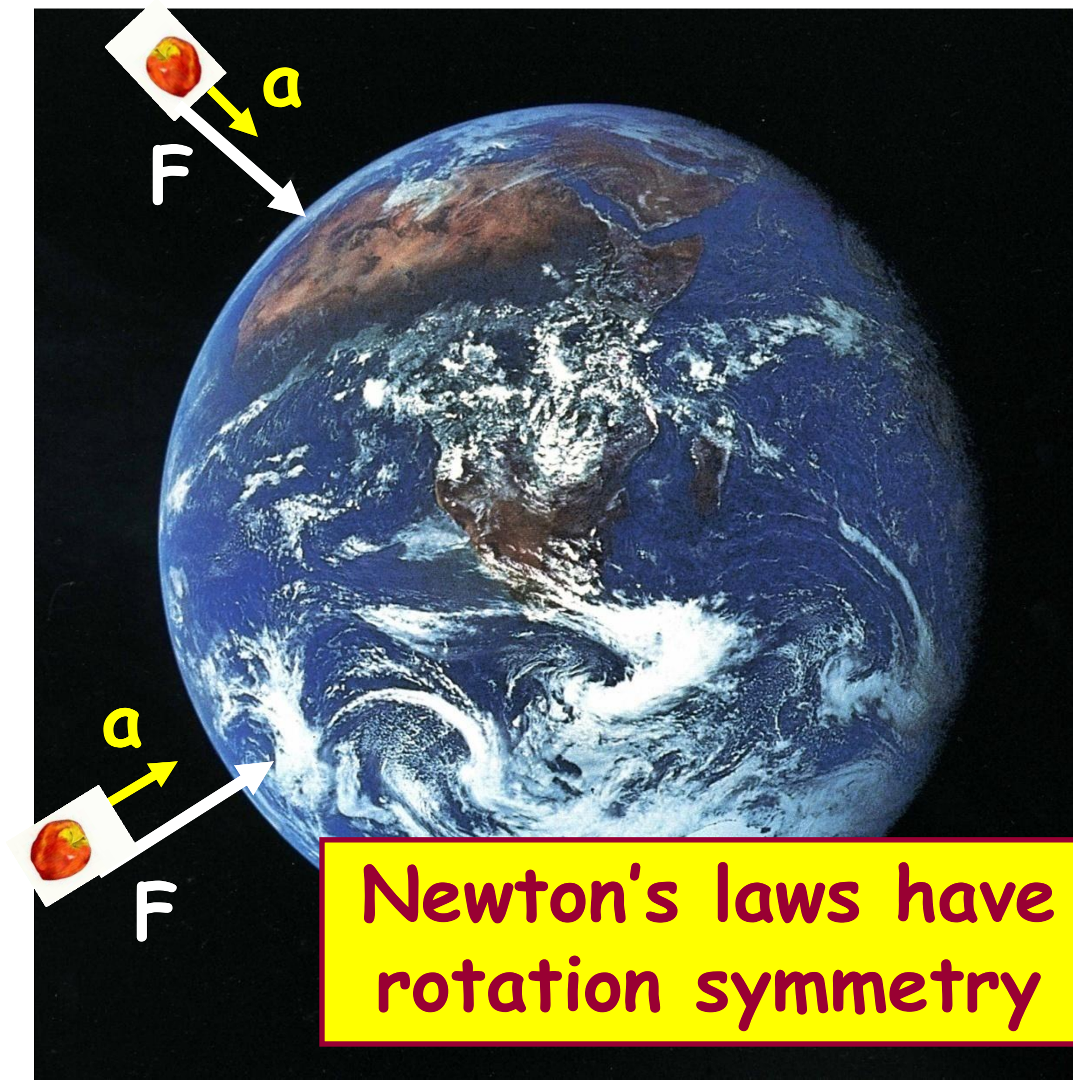
Newton's laws have space-translation symmetry

rotational symmetry

$$\vec{F} = m\vec{a}$$

Same rule for all directions

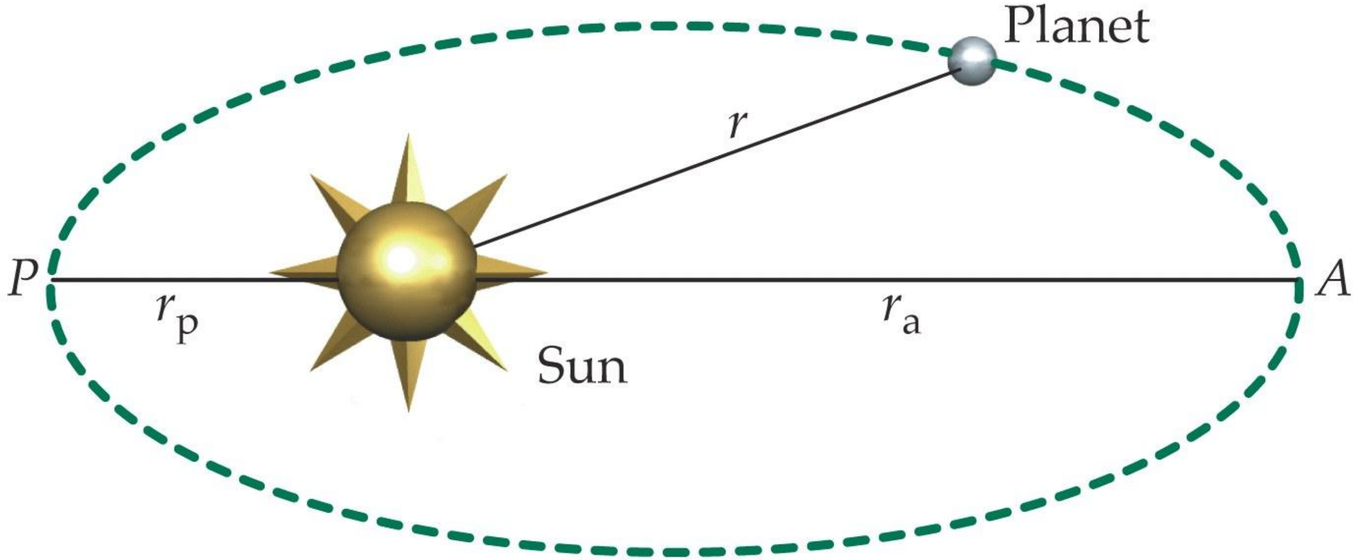
(no "preferred" directions in space.)



Newton's laws have rotation symmetry

Symmetry recovered

Symmetry resides in the laws of nature, not necessarily in the solutions to these laws.



Emmy Noether

Symmetries ←→ Conservation laws

Symmetry		Conservation law
Rotation	←→	Angular momentum
Space translation	←→	Momentum
Time translation	←→	Energy

Noether's discovery:

Conservation laws are a consequence of the simple and elegant *properties of space and time!*

Content of Newton's laws is in their *symmetry properties*

Symmetries

- Assume that the Lagrangian is symmetric under some transformation of variables
  - That is, all of the q's change according to some rule:

    $$q \rightarrow q(s)$$

    but the Lagrangian doesn't change, no matter what value of s is used:

    $$\frac{d}{ds} L\{q(s), \dot{q}(s); t\} = 0$$

- Noether claimed that for any such symmetry, the quantity

  $$C = p_q \frac{dq(s)}{ds}$$

  must be conserved

Proof

- Let's see what happens when we take the derivative of C with respect to time (here we assume that s has no time dependence):

$$\frac{dC}{dt} = \dot{p}_q \frac{dq(s)}{ds} + p_q \frac{d}{dt}\frac{dq(s)}{ds}$$

- Using the definition of p_q, this becomes:

$$\frac{dC}{dt} = \left(\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} \right) \frac{dq(s)}{ds} + \frac{\partial L}{\partial \dot{q}_i}\frac{d}{dt}\frac{dq(s)}{ds}$$

- We now use Lagrange's Equation of Motion:

$$\frac{dC}{dt} = \left(\frac{\partial L}{\partial q_i} \right) \frac{dq(s)}{ds} + \frac{\partial L}{\partial \dot{q}_i}\frac{d\dot{q}(s)}{ds}$$

- The right-hand side of the previous equation is equivalent to

$$\frac{d}{ds} L\{q(s), \dot{q}(s); t\}$$

- But we required this derivative to be 0!
- So we've shown that:

$$\frac{dC}{dt} = 0$$

- In other words, C is conserved

This result is one of the most important theorems in physics. It holds not only for classical mechanics, but also for quantum mechanics, relativity, and quantum field theories!

Examples of Conserved Quantities

- If we express the Lagrangian in rectangular coordinates, and find that it is invariant under the transformation

$$x_i \rightarrow x_i + s_i$$

  what quantity is conserved?

- We know that the generalized momentum associated with x_i is just the linear momentum
- So the conserved quantity is:

$$C = p_i \frac{dx_i}{ds_i} = p_i(1) = p_i$$

For any Lagrangian symmetric under spatial translations, linear momentum is conserved

- Now suppose we write the Lagrangian in terms of angular variables:

$$L = \frac{1}{2} I\dot{\theta}_i^2 - U(\theta_i)$$

- Now let $\theta_i \to \theta_i + s_i$
- In this case, the generalized momentum associated with θ is:

$$p_{\theta_i} = \frac{\partial L}{\partial \dot{\theta}_i} = I\dot{\theta}_i$$

which we recognize as $I\omega$, the angular momentum

For any Lagrangian symmetric under rotations, angular momentum is conserved

What About Symmetries in Time?

- This is outside the scope of our proof of Noether's Theorem (since we assumed *s* was time-independent)
  - though a more general and thorough proof does include it!
- Consider a Lagrangian that has no *explicit* time dependence:

$$\frac{\partial L}{\partial t} = 0$$

then the total time derivative is:

$$\frac{dL\{q_i, \dot{q}_i\}}{dt} = \sum_i \frac{\partial L}{\partial q_i}\dot{q}_i + \sum_i \frac{\partial L}{\partial \dot{q}_i}\ddot{q}_i$$

- But Lagrange's Equation of Motion requires that:

$$\frac{\partial L}{\partial q_i} = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i}$$

- So we have:

$$\frac{dL}{dt} = \sum_i \left(\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} \right) \dot{q}_i + \sum_i \frac{\partial L}{\partial \dot{q}_i} \ddot{q}_i$$

$$= \sum_i \left[\left(\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} \right) \dot{q}_i + \frac{\partial L}{\partial \dot{q}_i} \ddot{q}_i \right]$$

$$= \sum_i \frac{d}{dt} \left(\dot{q}_i \frac{\partial L}{\partial \dot{q}_i} \right) = \frac{d}{dt} \sum_i \left(\dot{q}_i \frac{\partial L}{\partial \dot{q}_i} \right)$$

- Which can be rearranged to show:

$$\frac{d}{dt}\left[\sum_j \left(\dot{q}_i \frac{\partial L}{\partial \dot{q}_i} \right) - L \right] = 0$$

We call this quantity the *Hamiltonian* of the system (H)

Conservation Laws

All particles will decay to lighter particles unless prevented from doing so by some conservation law.

The timescales for these such decays are

10^{-23} s	for the strong interaction
10^{-16} s	for the electromagnetic interaction
10^{-13} s – 15 minutes	for the weak interaction

Note that some conservation laws are absolute (conservation of energy and momentum for instance) but there are others (parity, which we will get to later) which are conserved by some of the three fundamental interactions but not all.

Note that these conservation laws apply equally well to scattering processes, not simply decays. They are properties of the interactions.

Noether's Theorem

Every symmetry in nature is related to a conservation law and *vice versa*

Invariance under:	leads to
Translations in time	conservation of energy
Translations in space	conservation of momentum
Rotations in space	conservation of angular momentum
Gauge transformations	conservation of charge

One conservation law is well known to us, **conservation of electric charge**

Others that we will discuss today: **conservation of lepton number**

conservation of baryon number

Conservations

- One of the most striking general properties of elementary particles is their tendency to disintegrate.
- Universal principle: Every particle decays into lighter particles, unless prevented from doing so by some conservation law.
- Obvious conservation laws:
  - **Momentum conservation**
  - **Energy conservation**
  - **Charge conservation**
- Stable particles: neutrinos, photon, electron and proton.
  - **Neutrinos and photon are massless, there is nothing to decay for them into**
  - **The electron is lightest charged particle, so conservation of charge prevents its decay.**

BUT

Why proton is stable?

Conservation of Baryon Number

Proton lifetime many orders of magnitude longer than the age of the Universe

Baryon number conservation originally postulated to explain the fact that the proton is not observed to decay.

In the SM, protons are absolutely stable. They are the lightest baryons, so if baryon number is conserved, they cannot decay….ever.

In (high-energy) extensions to the SM (for instance Grand Unified Theories) protons can decay.

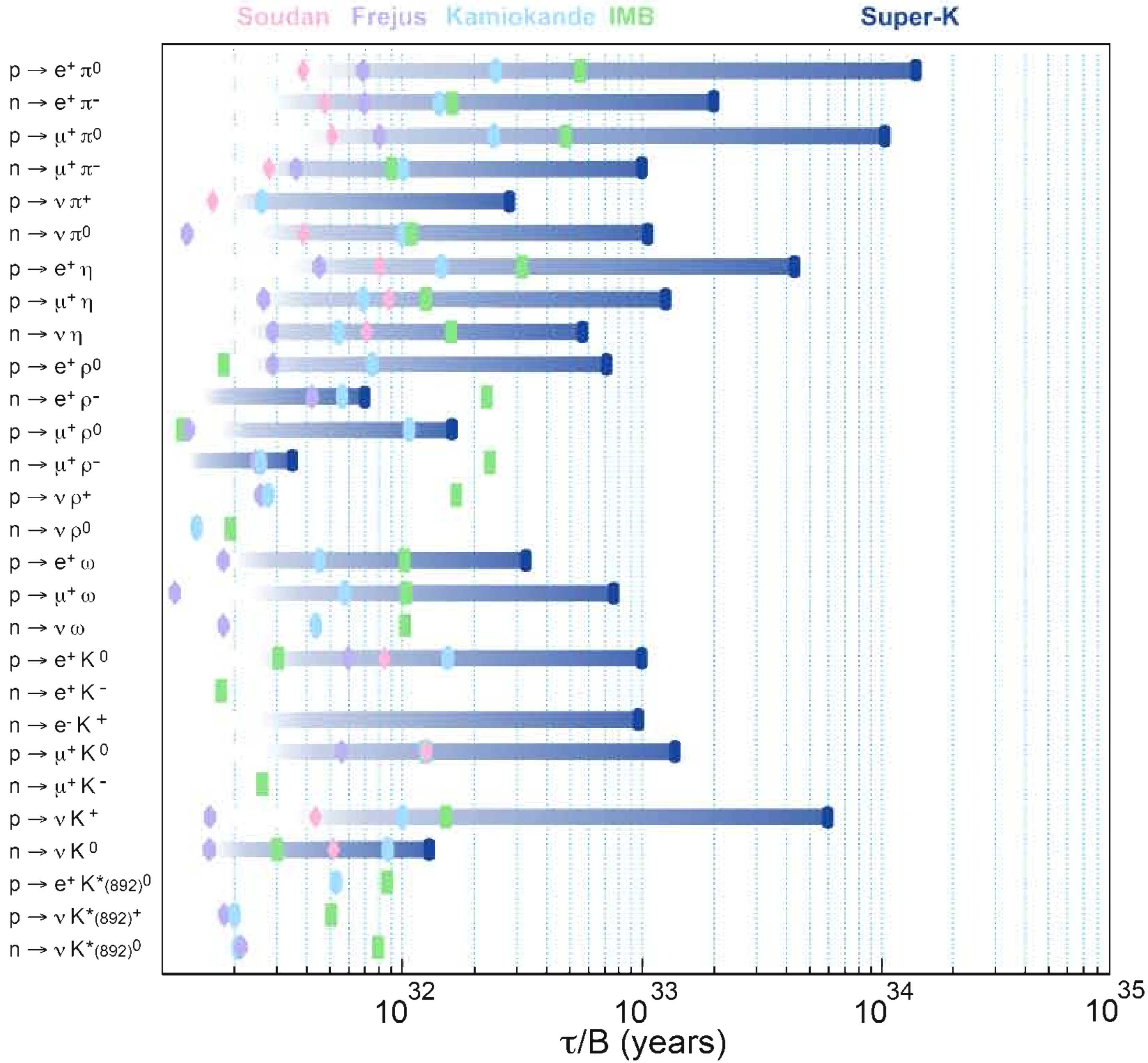
Note that this is effectively conservation of quark number where we assign a baryon number of +1/3 to quarks and -1/3 to antiquarks.

Baryons then have B = +1 and antibaryons have B = -1

Mesons have baryon number 0.

For example,

$$p \rightarrow e^{+} + \gamma$$



Soudan Frejus Kamiokande IMB Super-K
p → e+ π0
n → e+ π-
p → μ+ π0
n → μ+ π-
p → ν π+
n → ν π0
p → e+ η
p → μ+ η
n → ν η
p → e+ ρ0
n → e+ ρ-
p → μ+ ρ0
n → μ+ ρ-
p → ν ρ+
n → ν ρ0
p → e+ ω
p → μ+ ω
n → ν ω
p → e+ K0
n → e+ K-
n → e- K+
p → μ+ K0
n → μ+ K-
p → ν K+
n → ν K0
p → e+ K*(892)0
p → ν K*(892)+
n → ν K*(892)0
10^32
10^33
10^34
10^35
τ/B (years)

Discovery of the anti-proton

B is an additive quantum number. It must balance on either side of any process.

There is no equivalent "meson number" that must balance, no such thing as meson number conservation (can always get a quark-antiquark pair from the vacuum if there is enough energy, but cannot, for instance get the three quarks needed for a baryon without also getting three antiquarks.

Antiprotons first produced experimentally in proton proton collisions:

$$p + p \rightarrow p + p + p + \overline{p}$$

B = 2 before $\quad 1 \;+\; 1 \rightarrow \; 1 \;+\; 1 \;+ 1 \;+ (-1)$ and 2 after

This is the minimal possible final state containing an anti-proton (in pp colisions)

Could also for instance have

$$p + p \rightarrow p + p + p + \overline{p} + \pi^{+} + \pi^{-} + \pi^{0}$$

e.g. no mesons in the initial state but some number in the final state (here 3)

Conservation of Lepton Number

is an empirical fact, based on experiments. It is built into the Standard Model in terms of the allowed interactions (i.e.. allowed vertices) , but it is not a prediction of the theory in the same way that conservation of charge is.

Lepton number defined and conserved separately for electrons, muons and tau leptons (this includes the associated neutrinos)

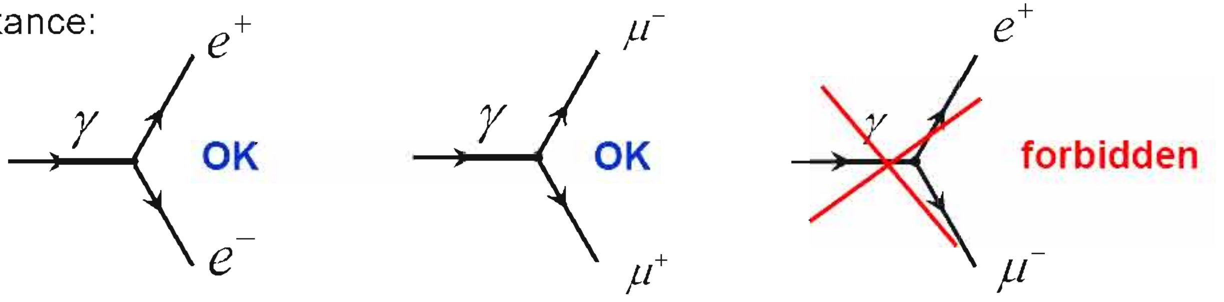
For electrons L_e = +1 for electrons and electon neutrinos

-1 for anti-electrons and anti-electron neutrinos

0 for any other particles

This is an additive quantum number (like charge)

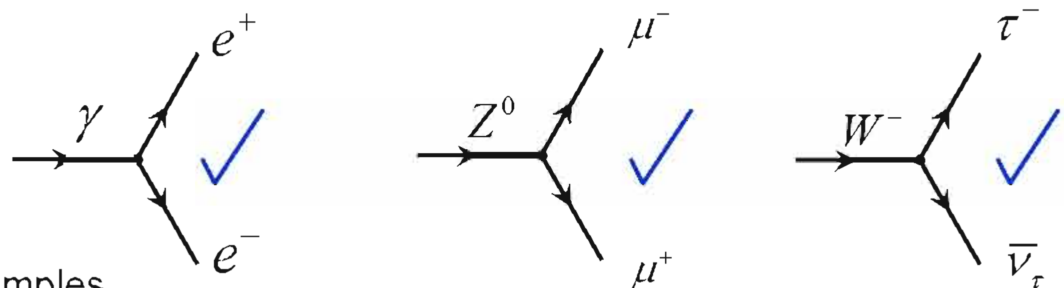
so for instance:



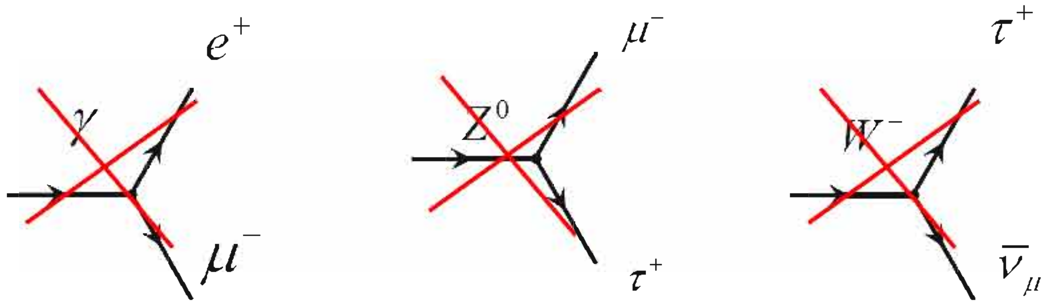
Lepton conservation in EM and Weak Interactions

Strong interaction does not couple to leptons so conservation is trivial there.

Lepton number is an additive quantum number.



Some examples



In the SM, there are no interactions that mix leptons from different generations.

π - Decay

$$\pi \rightarrow \mu + \nu$$

$$\mu \rightarrow e + 2\nu$$

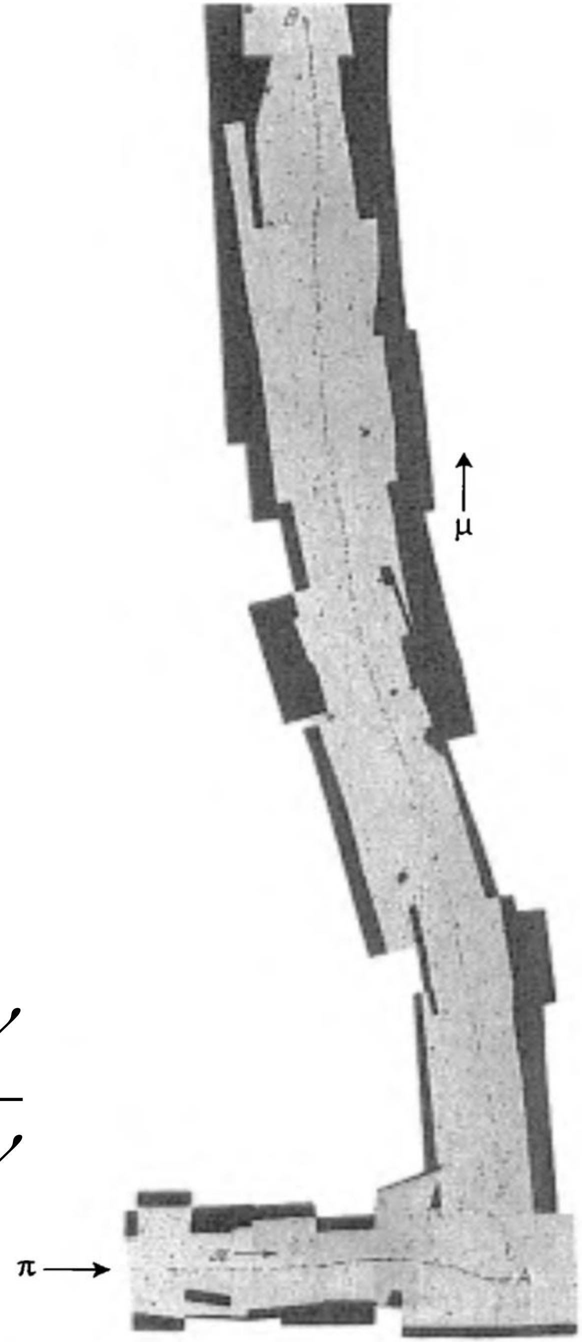
$\overline{\nu} + p^{+} \rightarrow n + e^{+}$ YES

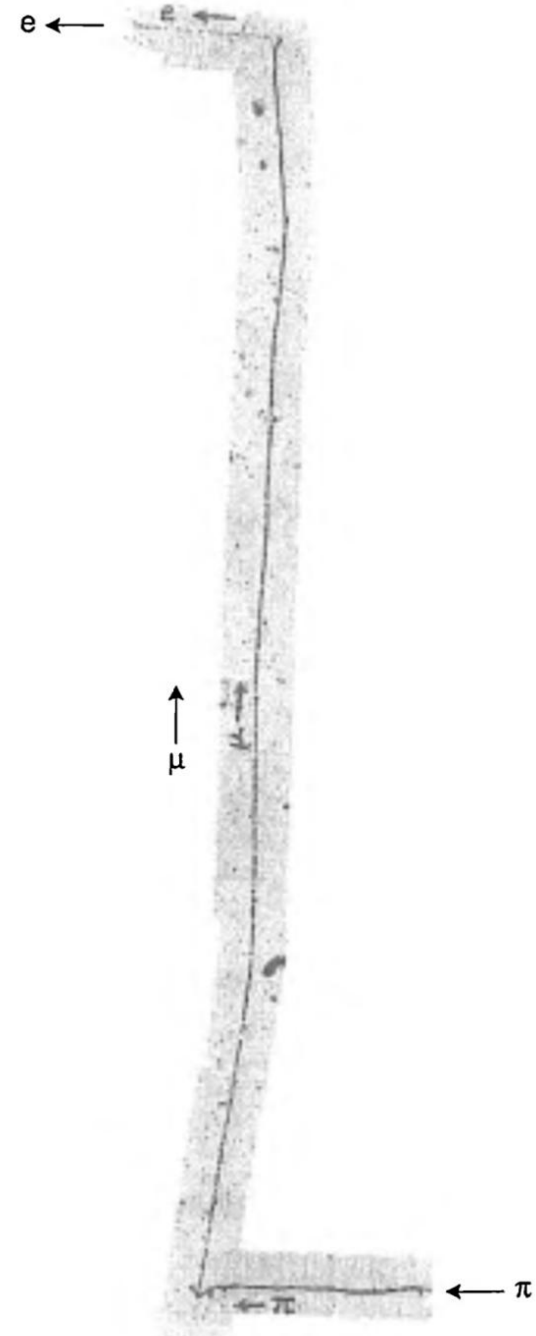
$\nu + n \rightarrow p^{+} + e^{-}$ YES

$\overline{\nu} + n \rightarrow p^{+} + e^{-}$ No

$$L = +1 \;\; for \;\; e^{-}, \mu^{-}, \nu$$

$$L = -1 \;\; for \;\; e^{+}, \mu^{+}, \overline{\nu}$$





$$\pi^- \to \mu^- + \bar{\nu}$$

$$\mu^- \to e^- + \nu + \bar{\nu}$$

$$\pi^+ \to \mu^+ + \nu$$

$$\mu^+ \to e^+ + \nu + \bar{\nu}$$

$$\mu^- \not\to e^- + \gamma$$

$L_e = +1$ for e^-, ν_e ;

$L_e = -1$ for $e^+, \overline{\nu_e}$

While,

$L_\mu = +1$ for μ^-, ν_μ ;

$L_\mu = -1$ for $\mu^+, \overline{\nu_\mu}$

$$\pi^- \to \mu^- + \bar{\nu}_\mu$$

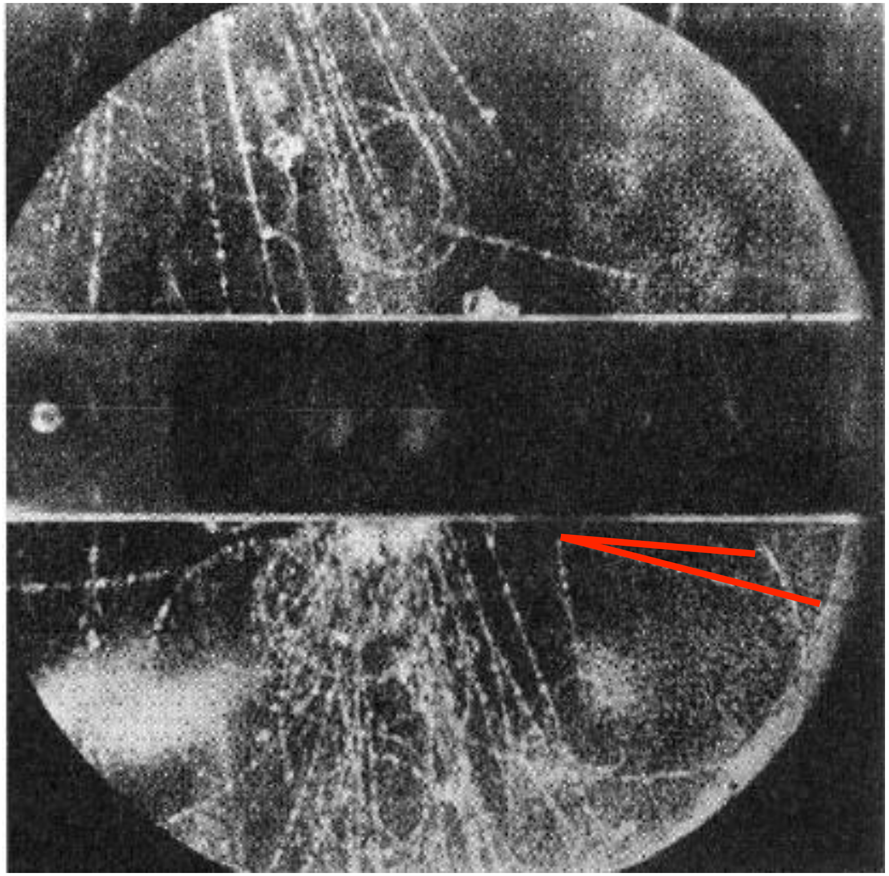
$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$

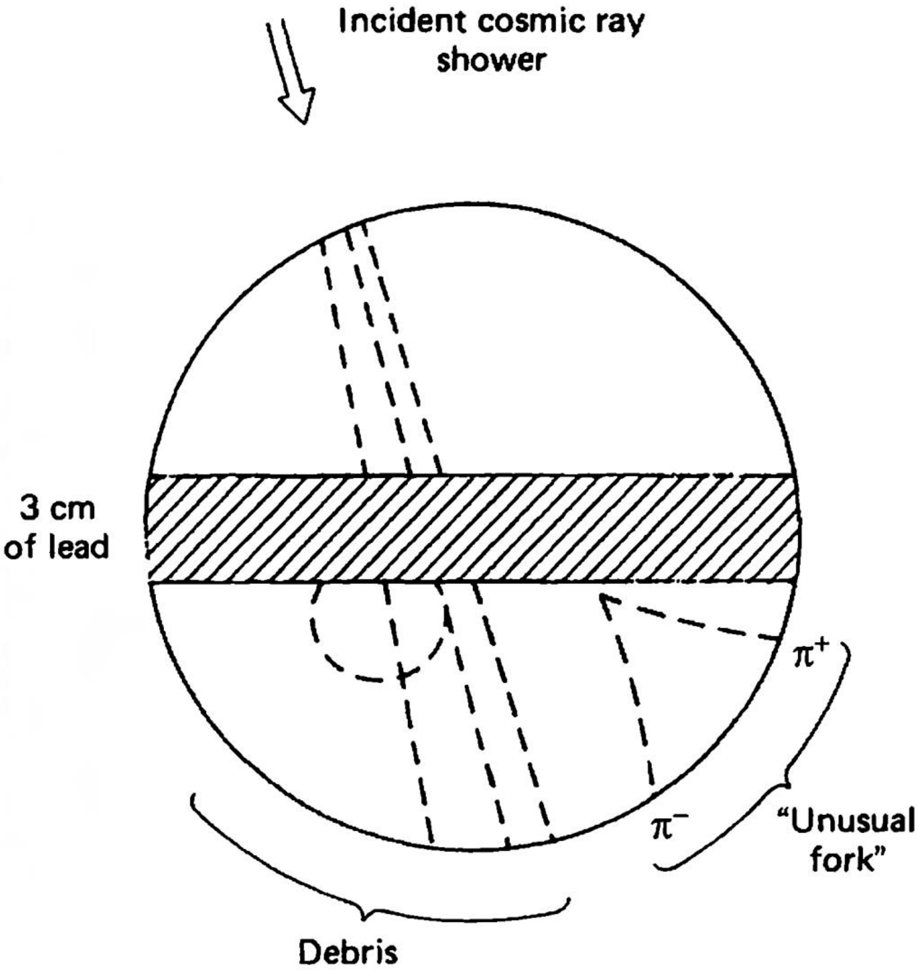
$$\pi^+ \to \mu^+ + \nu_\mu$$

$$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$$

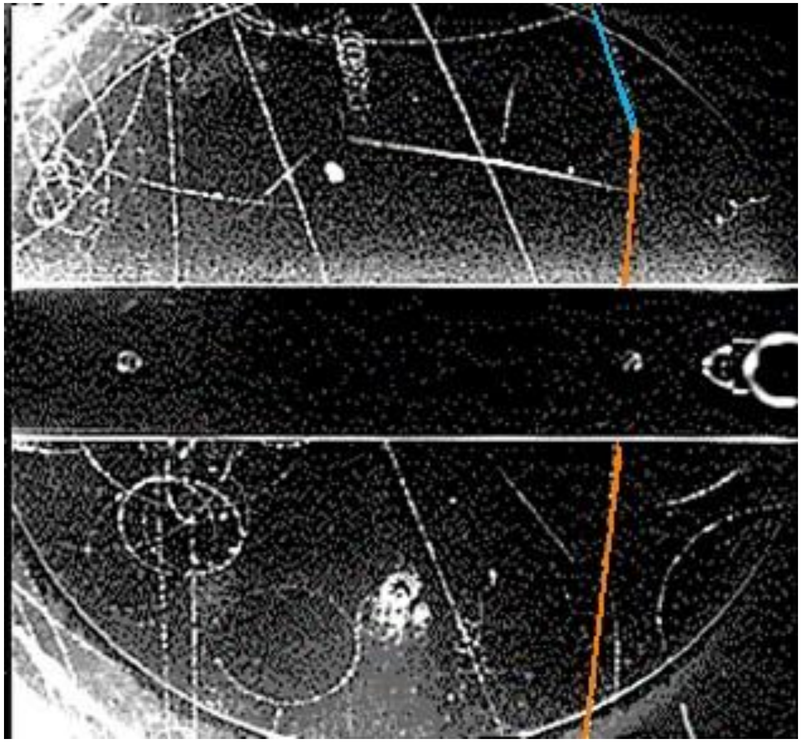
	Lepton number	Electron number	Muon number
Leptons			
e^-	1	1	0
ν_e	1	1	0
μ^-	1	0	1
ν_μ	1	0	1
Antileptons			
e^+	−1	−1	0
$\bar{\nu}_e$	−1	−1	0
μ^+	−1	0	−1
$\bar{\nu}_\mu$	−1	0	−1

- 1943 Leprince-Ringuet: identified a particle whose mass was 506±61 MeV.
- 1947 Rochester e Butler identified very clearly some neutral V particles in the data taken for one year with a cloud chamber at sea level.





$$K^0 \rightarrow \pi^+ + \pi^-$$



$$K^+ \to \mu^+ \nu$$

- associate production:

in 1947 was evident that the new particles were always pair produced; one had mass around 500 MeV (K) and the other one has a higher mass, higher than the mass of the nucleon (it was called hiperon);

- The hyperon decayed into nucleon plus pion.

$$K^+ \rightarrow \pi^+ + \pi^+ + \pi^-$$

$$\Lambda \rightarrow p^+ + \pi^-$$

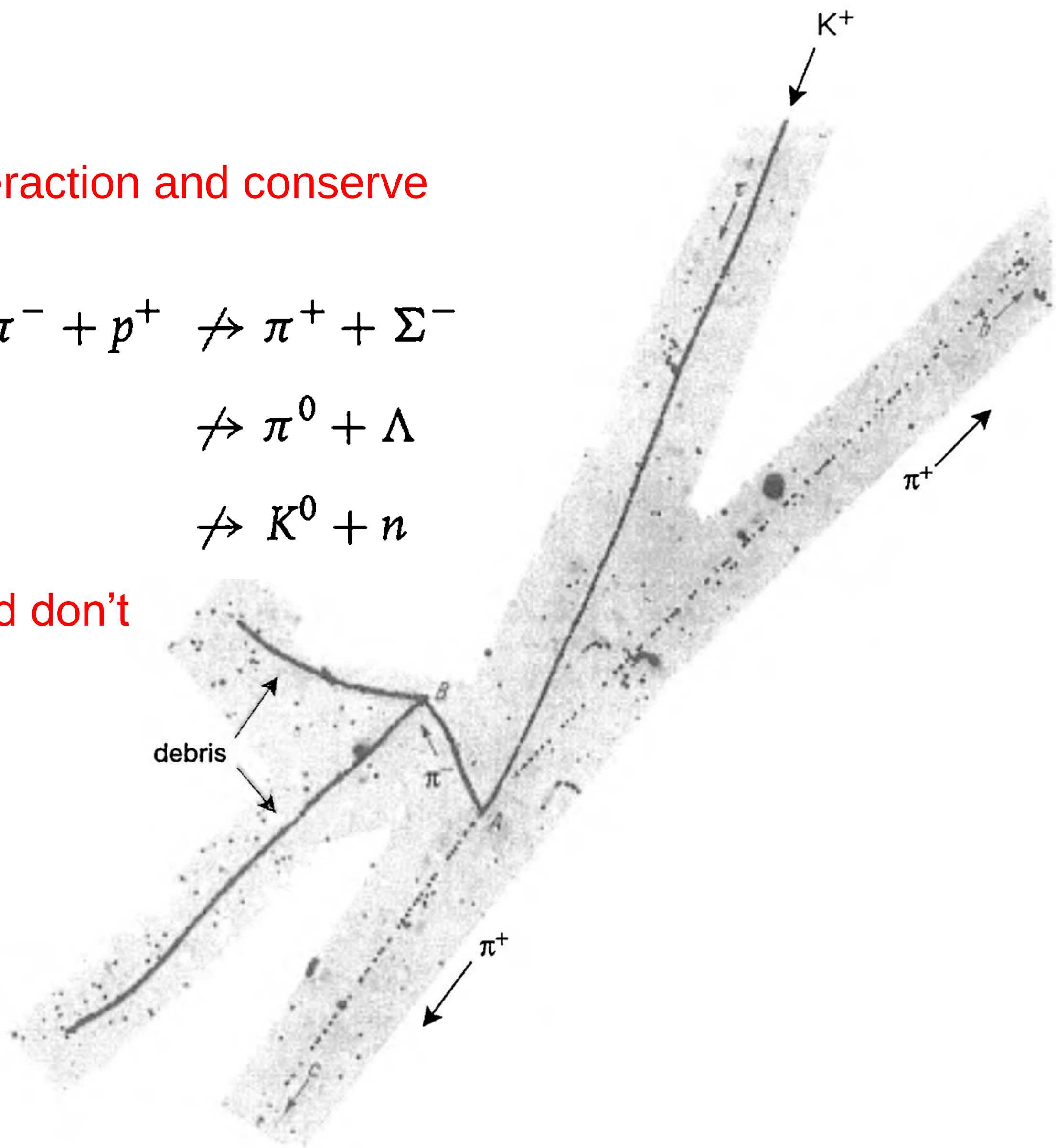
Produce in pairs via strong interaction and conserve strangeness

$$\pi^- + p^+ \rightarrow K^+ + \Sigma^-$$
$$\rightarrow K^0 + \Sigma^0$$
$$\rightarrow K^0 + \Lambda$$

$$\pi^- + p^+ \not\rightarrow \pi^+ + \Sigma^-$$
$$\not\rightarrow \pi^0 + \Lambda$$
$$\not\rightarrow K^0 + n$$

Decay via Weak interaction and don't conserve strangeness

$$\Lambda \rightarrow p^+ + \pi^-$$
$$\Sigma^+ \rightarrow p^+ + \pi^0$$
$$\rightarrow n + \pi^+$$



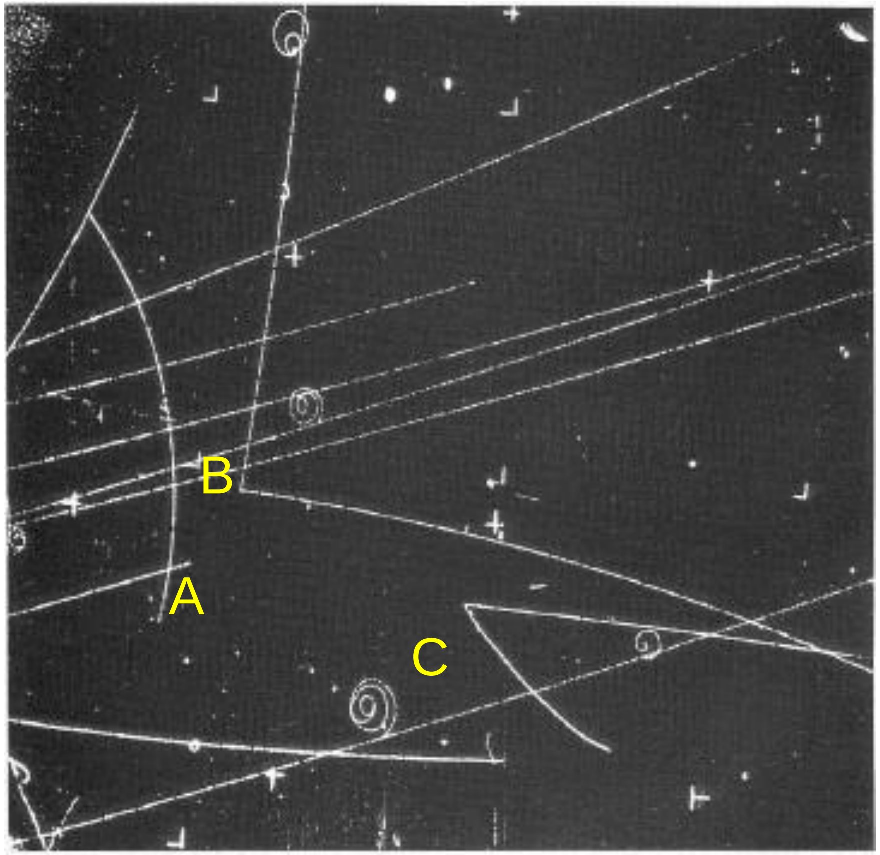
Associate production: $\pi^- + p \rightarrow \Lambda + K$

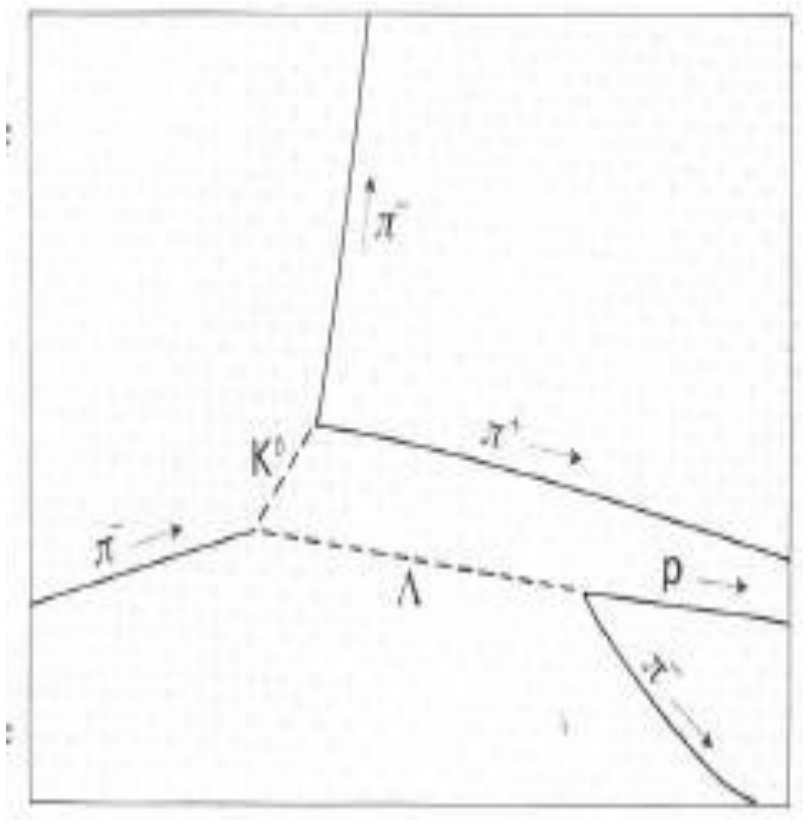
1 GeV/c π^- in a bubble chamber filled with liquid hidrogen

A) $\pi^- + p \rightarrow K^0 + \Lambda$

B) $K^0 \rightarrow \pi^- + \pi^+$

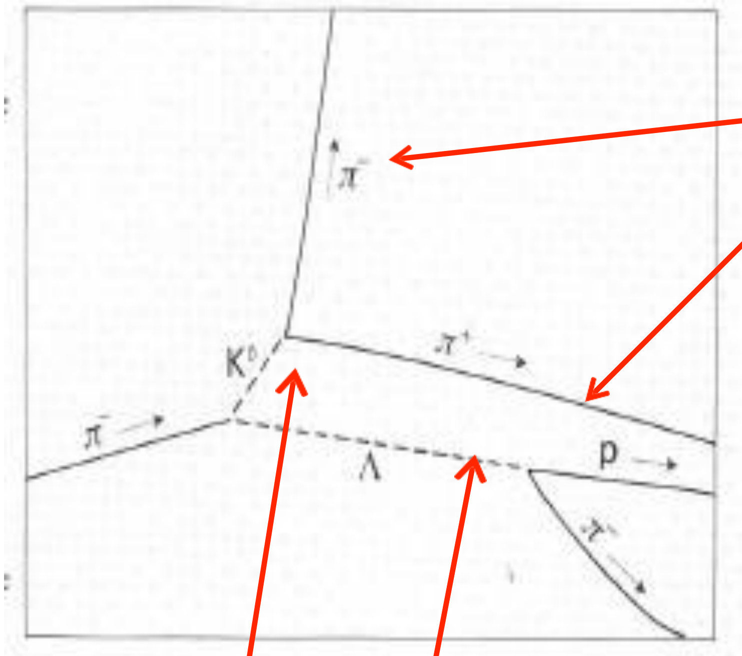
C) $\Lambda \rightarrow p + \pi^-$





N.B. : why K^0 and not anti-K^0?

Measurement of mass and life time



From the curvature radius we get the momentum of the charged particles and, by knowing the kind of particles, their energies.

Then we get the invariant mass of the mother.

$$m_K = \sqrt{(E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2}$$

From the mass and the energy (E1+E2) we find the γ and then the β of the particle

$$\gamma = \frac{E}{m}$$

From the measure of the mean free path λ we get the life time τ of the particle

$$\lambda = \gamma\beta c\tau$$

Why are they strange particles?

- The production cross section of these particles is the order of mb, typical of the strong interactions;
- the life time are of the order of 10^{-10} s, typical of the weak interactions (e.m. int. ~ 10^{-20} s , strong int. ~ 10^{-23} s)

1. why the decay $\Lambda \to p + \pi^-$ does not happen through strong interactions?
2. Why the new particles are always produced in pairs?
3. (moreover τ-θ puzzle: same mass and same life time but opposed parity)

The strangeness

- An explanation of this anomaly was given in 1954 by Gell-Mann and Pais and, independently, by Nishijima.

They introduced a new quantum number, the strangeness, that was conserved by the strong interaction but it is violated by the weak interaction.

- The strangeness is an additive quantum number. The “old” hadrons, the nucleon and the pion, have S=0, the hyperons S=-1 while the K mesons have S=±1.

- In the production the strange particles must be produced in pairs (associate production) with opposed strangeness. The initial state has total strangeness equal to zero so, since the strong interaction conserves the strangeness, also the final state must have strangeness zero.

Example of associate production

$\pi^- + p \to K^0 + \Lambda$; $\pi^- + p \to K^0 + K^- + p$

$\pi^+ + n \to K^+ + \Lambda$; $\pi^+ + n \to K^+ + K^- + p$

$\pi^- + p \to K^0 + \Sigma^0$; $\pi^- + p \to K^+ + \Sigma^-$

$\pi^+ + n \to K^+ + \Sigma^0$; $\pi^+ + n \to K^0 + \Sigma^+$

$\pi^+ + p \to K^+ + \Sigma^+$

$m(\pi^\pm) = 139.6$ MeV ; $m(p) = 938.3$ MeV ; $m(n) = 939.6$

$m(K^\pm) = 493.68$ MeV

MeV $m(\Lambda)$ = ; $m(K^0) = 497.67$ MeV

1115.7 MeV $m(\Sigma^\pm)$; $m(\Sigma^0) = 1192.6$ MeV

= 1189.4 MeV $m(\Xi^0$; $m(\Xi^{-1}) = 1321.3$

) = 1314.8 MeV MeV

(question: why are not produced the anti-hyperons)

Isospin and strangenes of Σ and Ξ

$$Q = I_3 + \frac{1}{2}(B+S)$$

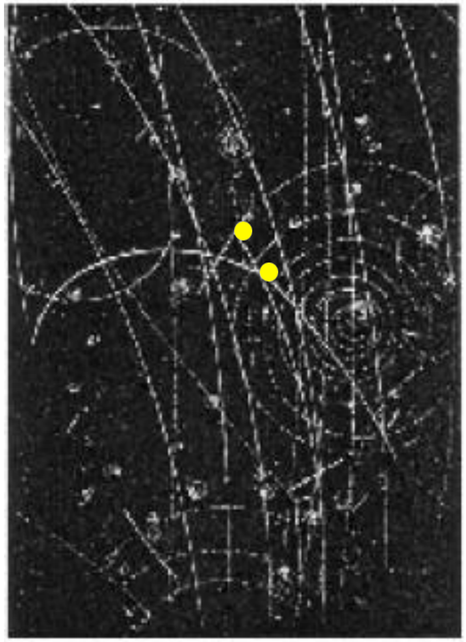
➡ We infere the isospin

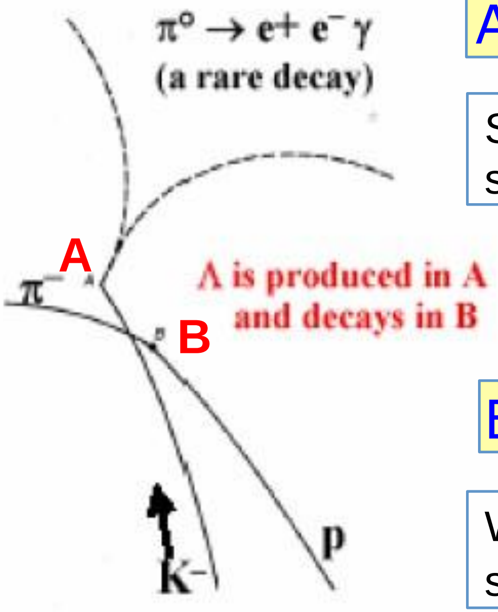
$$\left.\begin{array}{ll} Q(\Sigma^+)=1,\ B(\Sigma^+)=1,\ S(\Sigma^+)=-1 & \Rightarrow I_3(\Sigma^+)=1 \\ Q(\Sigma^0)=0,\ B(\Sigma^0)=1,\ S(\Sigma^0)=-1 & \Rightarrow I_3(\Sigma^0)=0 \\ Q(\Sigma^-)=-1,\ B(\Sigma^-)=1,\ S(\Sigma^-)=-1 & \Rightarrow I_3(\Sigma^-)=-1 \end{array}\right\rangle \Rightarrow I=1$$

$$\left.\begin{array}{ll} Q(\Xi^0)=0,\ B(\Xi^0)=1,\ S(\Xi^0)=-2 & \Rightarrow I_3(\Xi^0)=\frac{1}{2} \\ Q(\Xi^{-1})=-1,\ B(\Xi^{-1})=1,\ S(\Xi^{-1})=-2 & \Rightarrow I_3(\Xi^{-1})=-\frac{1}{2} \end{array}\right\rangle \Rightarrow I=\frac{1}{2}$$

The production of strange particles

There have been used beams of charged K to produce new strange particles. We have an example of a K^- that it is stopped in a bubble chamber of liquid hidrogen.





A) $K^- + p \rightarrow \Lambda + \pi^0$

Strong Int. :
strangeness is conserved.

B) $\Lambda \rightarrow p + \pi^-$

Weak Int.:
strangeness is violated.

Interactions of the K mesons

We start from an initial state with strageness ±1

$S=1$, $B=1$	$K^+p\rightarrow$	K^+p					
	$K^+n\rightarrow$	K^+n	K^0p				
$S=-1$, $B=1$	$K^-p\rightarrow$	K^-p	K^0n	$\pi^0\Lambda^0$	$\pi^+\Sigma^-$	$\pi^0\Sigma^0$	$\pi^-\Sigma^+$
	$K^-p\rightarrow$	$K^0\Xi^0$	$K^+\Xi^-$				
	$K^-n\rightarrow$	K^-n	$\pi^-\Lambda^0$	$\pi^0\Sigma^-$			
	$K^-n\rightarrow$	$K^0\Xi^-$					

With equal energy, the K^- produce more particles than K^+ because the hyperons (B=1) have strangeness = -1

Strange hyperons metastable

In the cosmic rays and in the accelerator experiments they have been found 6 strange hyperons metastable

	Q	S	m (MeV)	τ (ps)	$c\tau$ (mm)	Principal decays (BR in %)
Λ	0	−1	1116	263	79	$p\pi^-(64)$, $n\pi^0(36)$
Σ^+	+1	−1	1189	80	24	$p\pi^0(51.6)$, $n\pi^+(48.3)$
Σ^0	0	−1	1193	7.4×10^{-8}	2.2×10^{-8}	$\Lambda\gamma(100)$
Σ^-	−1	−1	1197	148	44.4	$n\pi^-(99.8)$
Ξ^0	0	−2	1315	290	87	$\Lambda\pi^0(99.5)$
Ξ^-	−1	−2	1321	164	49	$\Lambda\pi^-(99.9)$

To be noticed the life time typical of the e.m. interactions of the Σ^0. Why it is the only one that does not decays weakly?

The barions $(½)^+$ and the mesons 0^-

Let's classify the particles by using their spin and parity

barioni $\frac{1}{2}^+$	B	S	Y	I_3	Q	*mesoni* 0^-	B	S	Y	I_3	Q
p	+1	0	+1	+1/2	+1	K^+	0	+1	+1	+1/2	+1
n	+1	0	+1	−1/2	0	K^0	0	+1	+1	−1/2	0
Λ^0	+1	−1	0	0	0	η^0	0	0	0	0	0
Σ^+	+1	−1	0	+1	+1	π^+	0	0	0	+1	+1
Σ^0	+1	−1	0	0	0	π^0	0	0	0	0	0
Σ^-	+1	−1	0	−1	−1	π^-	0	0	0	−1	−1
Ξ^0	+1	−2	−1	+1/2	0	$\bar{K}^0$	0	−1	−1	+1/2	0
Ξ^-	+1	−2	−1	−1/2	−1	K^-	0	−1	−1	−1/2	−1

Mesonic resonances 1^-

	$m\ (MeV/c^2)$	$\Gamma\ (MeV)$	*decadimento*
K^*	894	51	$K\pi$
ρ	770	150	$\pi\pi$
ω	783	8.4	$\pi^+\pi^0\pi^-$
ϕ	1019	4.4	K^+K^- $K^0\bar{K}^0$ $\pi^+\pi^0\pi^-$

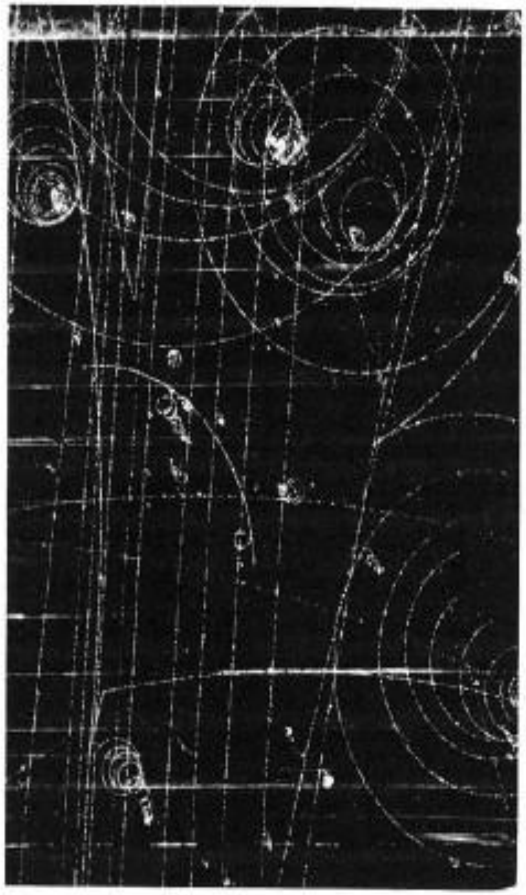
mesoni 1^-	S	Y	I_3	Q
K^{*+}	$+1$	$+1$	$+1/2$	$+1$
K^{*o}	$+1$	$+1$	$-1/2$	0
ρ^+	0	0	$+1$	$+1$
ρ^0	0	0	0	0
ρ^-	0	0	-1	-1
ω	0	0	0	0
$\bar{K}^{*o}$	-1	-1	$+1/2$	0
K^{*-}	-1	-1	$-1/2$	-1
ϕ	0	0	0	0

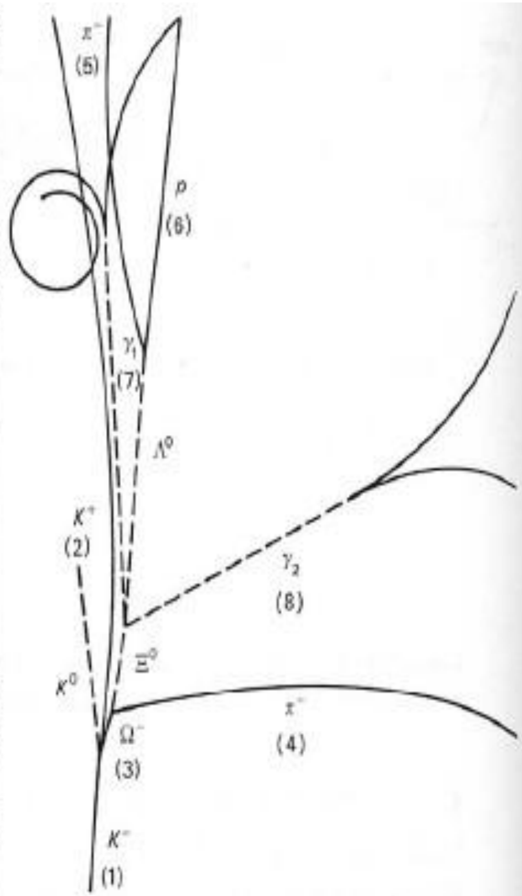
Barionic resonances $(3/2)^+$

barioni $\frac{3}{2}^+$	S	Y	I_3	Q
Δ^{++}	0	+1	+3/2	+2
Δ^{+}	0	+1	+1/2	+1
Δ^{0}	0	+1	−1/2	0
Δ^{-}	0	+1	−3/2	−1
Σ^{*+}	−1	0	+1	+1
Σ^{*o}	−1	0	0	0
Σ^{*-}	−1	0	−1	−1
Ξ^{*o}	−2	−1	+1/2	0
Ξ^{*-}	−2	−1	−1/2	−1
Ω^{-}	−3	−2	0	−1

The discovery of the Ω^-

The Ω^- was predicted by Gell-Mann by using his new particle classification (the eightfold way)





$K^- + p \rightarrow \Omega^- + K^+ + K^0$

$\quad \hookrightarrow \Xi^0 + \pi^-$ ($\Delta S = 1$ weak decay)

$\quad\quad \hookrightarrow \pi^0 + \Lambda$ ($\Delta S = 1$ weak decay)

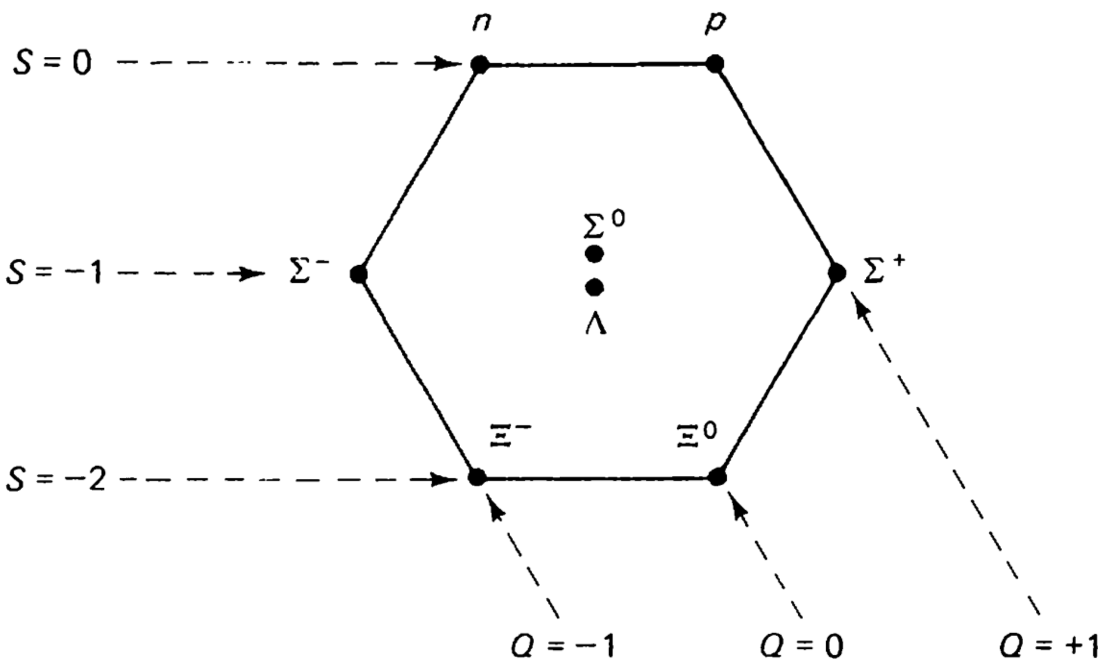
$\quad\quad\quad \hookrightarrow \pi^- + p$ ($\Delta S = 1$ weak decay)

$\quad\quad \hookrightarrow \gamma + \gamma$ (e.m. decay)

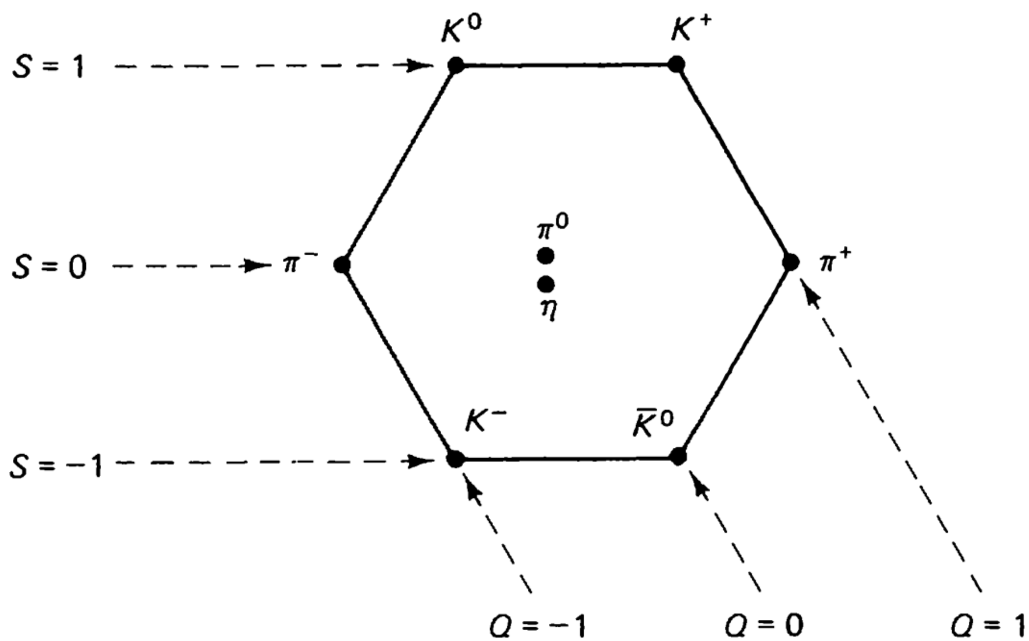
$\quad\quad\quad \downarrow \quad \downarrow$

$\quad\quad\quad e^+e^- \quad e^+e^-$.

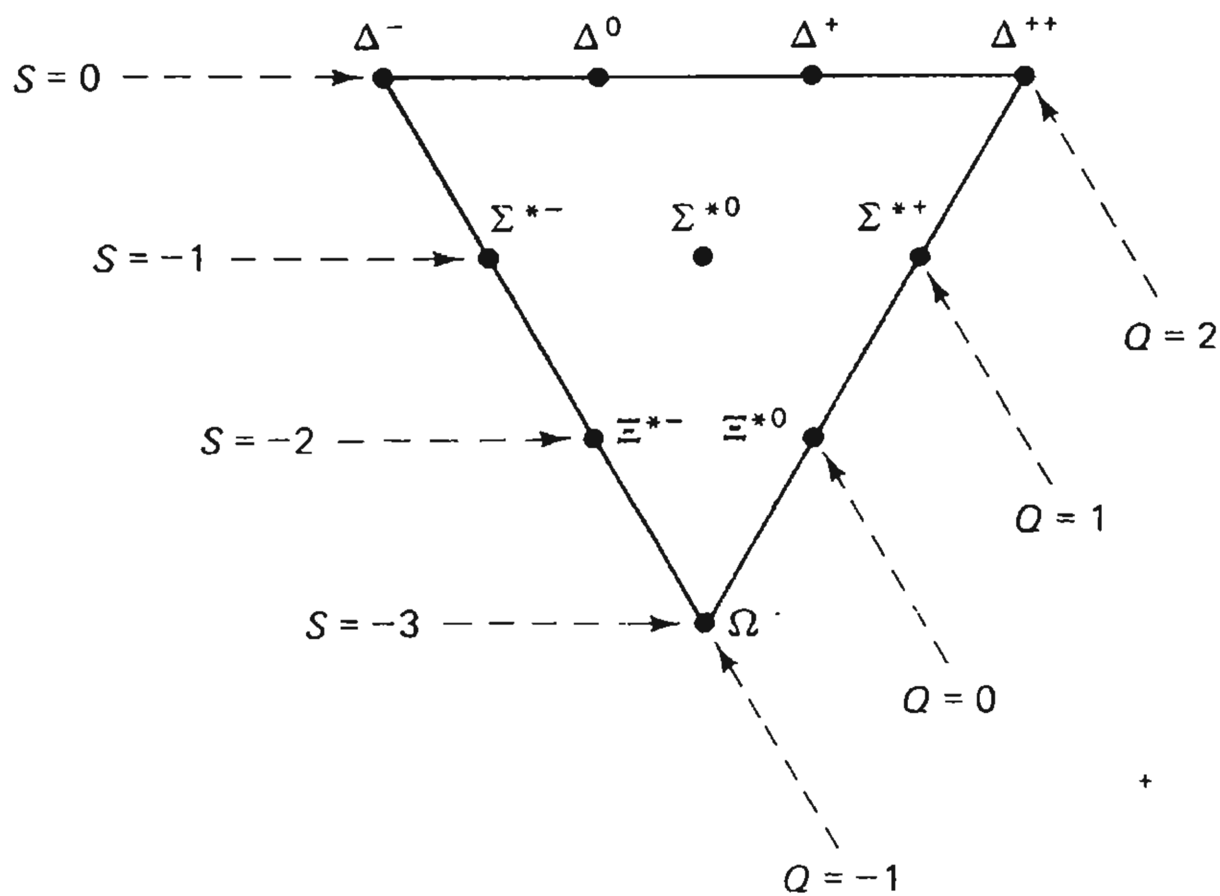
The Eightfold Way



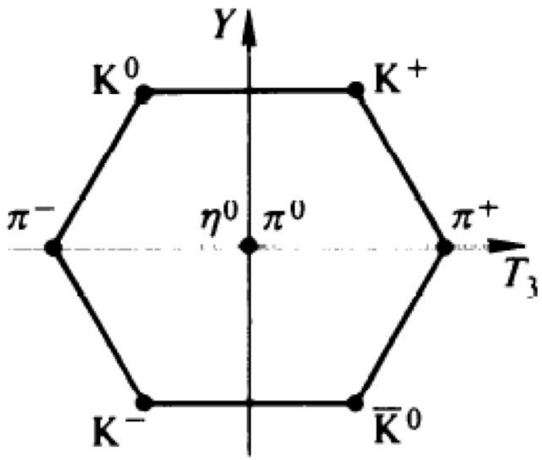
The baryon octet



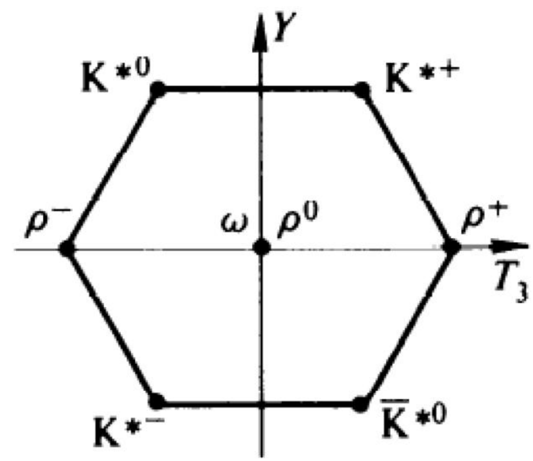
The meson octet



Δ⁻
Δ⁰
Δ⁺
Δ⁺⁺
S = 0
Σ*⁻
Σ*⁰
Σ*⁺
S = −1
Q = 2
Ξ*⁻
Ξ*⁰
S = −2
Q = 1
S = −3
Ω⁻
Q = 0
Q = −1



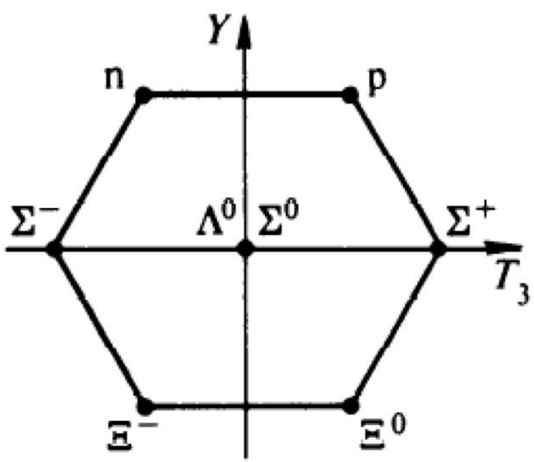
(a)



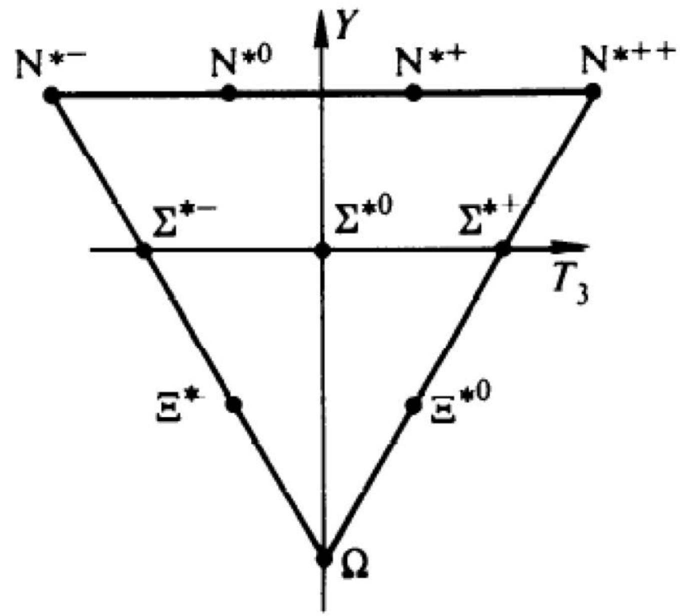
(b)

$$Q = T_3 + \frac{Y}{2}, \quad Y = (B + S)$$

$$Q = T_3 + \frac{(B+S)}{2}$$



(c)



(d)

Quark Properties

Quark Name	Symbol	Mass* (GeV/c^2)	Charge	Baryon Number	Strangeness S	Charm C	Bottomness B	Topness T
Up	u	0.0015 to 0.004	$2e/3$	$\frac{1}{3}$	0	0	0	0
Down	d	0.004 to 0.008	$-e/3$	$\frac{1}{3}$	0	0	0	0
Strange	s	0.08 to 0.130	$-e/3$	$\frac{1}{3}$	−1	0	0	0
Charmed	c	1.15 to 1.35	$2e/3$	$\frac{1}{3}$	0	1	0	0
Bottom	b	4.1 to 4.3	$-e/3$	$\frac{1}{3}$	0	0	−1	0
Top	t	178	$2e/3$	$\frac{1}{3}$	0	0	0	1

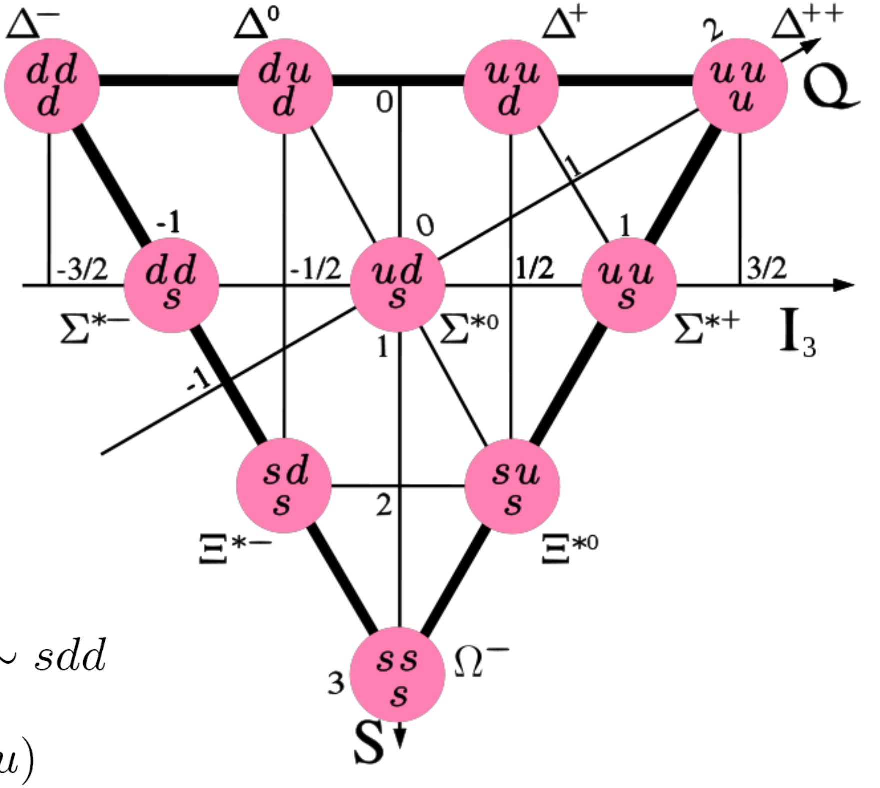
Antiquarks, $\bar{u}$, $\bar{d}$, $\bar{s}$, $\bar{c}$, $\bar{b}$, and $\bar{t}$, have opposite signs for charge, baryon number, S, C, B, and T.

The quark model (1964)

$$\pi^+ \sim \bar{d}u \quad \pi^0 \sim \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d). \quad \pi^- \sim \bar{u}d$$

$$K^+ \sim \bar{s}u \quad K^0 \sim \bar{s}d \ , \quad K^- \sim \bar{u}s. \quad \eta^0 \sim \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s)$$

	Q	T	T_3	Y	S	B
u	$2/3$	$1/2$	$+1/2$	$1/3$	0	$1/3$
d	$-1/3$	$1/2$	$-1/2$	$1/3$	0	$1/3$
s	$-1/3$	0	0	$-2/3$	-1	$1/3$



$$p \sim uud \ , \quad n \sim ddu$$

$$\Sigma^+ \sim suu \ , \quad \Sigma^0 \sim s\left(\frac{ud+du}{\sqrt{2}}\right) \ , \quad \Sigma^- \sim sdd$$

$$\Xi^0 \sim ssu \ , \quad \Xi^- \sim ssd \ , \quad \Lambda^0 \sim \frac{s(ud-du)}{\sqrt{2}}.$$

Quark Composition of Selected Hadrons

Particle	Quark Composition
Mesons	
π^+	$u\bar{d}$
π^-	$\bar{u}d$
K^+	$u\bar{s}$
K^0	$d\bar{s}$
D^+	$c\bar{d}$
D^0	$c\bar{u}$
Baryons	
p	uud
n	udd
Λ	uds
Σ^+	uus
Σ^0	uds
Ξ^0	uss
Ξ^-	dss
Ω^-	sss
Λ_C^+	udc

The Hadrons

Particle Name	Symbol	Anti-particle	Mass (MeV/c^2)	Mean Lifetime (s)	Main Decay Modes	Spin	Baryon Number B	Strangeness Number S	Charm Number C
Mesons									
Pion	π^-	π^+	140	2.6×10^{-8}	$\mu^+\nu_\mu$	0	0	0	0
	π^0	Self	135	8.4×10^{-17}	2γ	0	0	0	0
Kaon	K^+	K^-	494	1.2×10^{-8}	$\mu^+\nu_\mu$, $\pi^+\pi^0$	0	0	1	0
	K_S^0	$\overline{K}_S^0$	498	8.9×10^{-11}	$\pi^+\pi^-$, $2\pi^0$	0	0	1	0
	K_L^0	$\overline{K}_L^0$	498	5.2×10^{-8}	$\pi^\pm e^\mp \nu_e$, $3\pi^0$, $\pi^\pm\mu^\mp\nu_\mu$, $\pi^+\pi^-\pi^0$	0	0	1	0
Eta	η^0	Self	547	5×10^{-19}	2γ, $3\pi^0$, $\pi^+\pi^-\pi^0$	0	0	0	0
Charmed D's	D^+	D^-	1868	1.0×10^{-12}	e^+, $K^\pm$, K^0, $\overline{K}^0$ + anything	0	0	0	1
	D^0	$\overline{D}^0$	1864	4.1×10^{-13}	Same as D^+	0	0	0	1
	D_S^+	$\overline{D}_S^-$	1969	4.9×10^{-13}	Various	0	0	1	1
Bottom B's	B^+	B^-	5279	1.7×10^{-12}	Various	0	0	0	0
	B^0	$\overline{B}^0$	5279	1.5×10^{-12}	Various	0	0	0	0
J/Psi	J/ψ	Self	3097	10^{-20}	Various	0	0	0	0
Upsilon	$\Upsilon(1S)$	Self	9460	10^{-20}	Various	0	0	0	0
Baryons									
Proton	p	$\bar{p}$	938.3	Stable (?)		$\frac{1}{2}$	1	0	0
Neutron	n	$\bar{n}$	939.6	886	$pe^-\bar{\nu}_e$	$\frac{1}{2}$	1	0	0
Lambda	Λ	$\overline{\Lambda}$	1116	2.6×10^{-10}	$p\pi^-$, $n\pi^0$	$\frac{1}{2}$	1	−1	0
Sigmas	Σ^+	$\overline{\Sigma}^-$	1189	8.0×10^{-11}	$p\pi^0$, $n\pi^+$	$\frac{1}{2}$	1	−1	0
	Σ^0	$\overline{\Sigma}^0$	1193	7.4×10^{-20}	$\Lambda\gamma$	$\frac{1}{2}$	1	−1	0
	Σ^-	$\overline{\Sigma}^+$	1197	1.5×10^{-10}	$n\pi^-$	$\frac{1}{2}$	1	−1	0
Xi	Ξ^0	$\overline{\Xi}^0$	1315	2.9×10^{-10}	$\Lambda\pi^0$	$\frac{1}{2}$	1	−2	0
	Ξ^-	Ξ^+	1321	1.6×10^{-10}	$\Lambda\pi^-$	$\frac{1}{2}$	1	−2	0
Omega	Ω^-	Ω^+	1672	0.82×10^{-10}	ΛK^-, $\Xi^0\pi^-$	$\frac{1}{2}$	1	−3	0
Charmed lambda	Λ_C^+	$\overline{\Lambda}_C^-$	2285	2.0×10^{-13}	Various	$\frac{1}{2}$	1	0	1

Boson Properties: Gauge, Higgs, and Graviton

Boson	Mass	Spin	Electric Charge	Comments
Gauge:				
Photon	0	1	0	Stable
W^+, W^-	80.42 GeV/c^2	1	1, −1	Γ = 2.12 GeV, decays observed
Z	91.19 GeV/c^2	1	0	Γ = 2.50 GeV, decays observed
Gluon	0	1	0	Bound in hadrons, not free
Higgs H^0	>117 GeV/c^2	0	0	Not observed
Graviton	0	2	0	Stable, not observed

The quark model: beyond

symmetrybreaking
extra dimensions of particle physics
A joint Fermilab/SLAC publication

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VIEW CURREN

Belle experiment makes exotic discovery

January 11, 2012 | 4:29 am

The Belle Experiment at KEK laboratory in Japan has discovered two unexpected new types of hadrons. Hadrons are composite particles made up of quarks, the smallest known components of matter.

These new particles are thought to contain at least four quarks, making them exotic hadrons — hadrons that do not fit the quark model originally developed in 1961.

The Belle experiment discovered a new type of exotic hadron. Image: KEK

The B Factory experiment at KEK previously discovered exotic hadrons containing charm quarks. With this new finding, the Belle experiment has identified the first of this type of exotic hadrons discovered to contain bottom quarks, the second-heaviest type of quarks among the six known types of quarks. The particles, termed Zb, contain both one bottom quark and one anti-bottom quark.

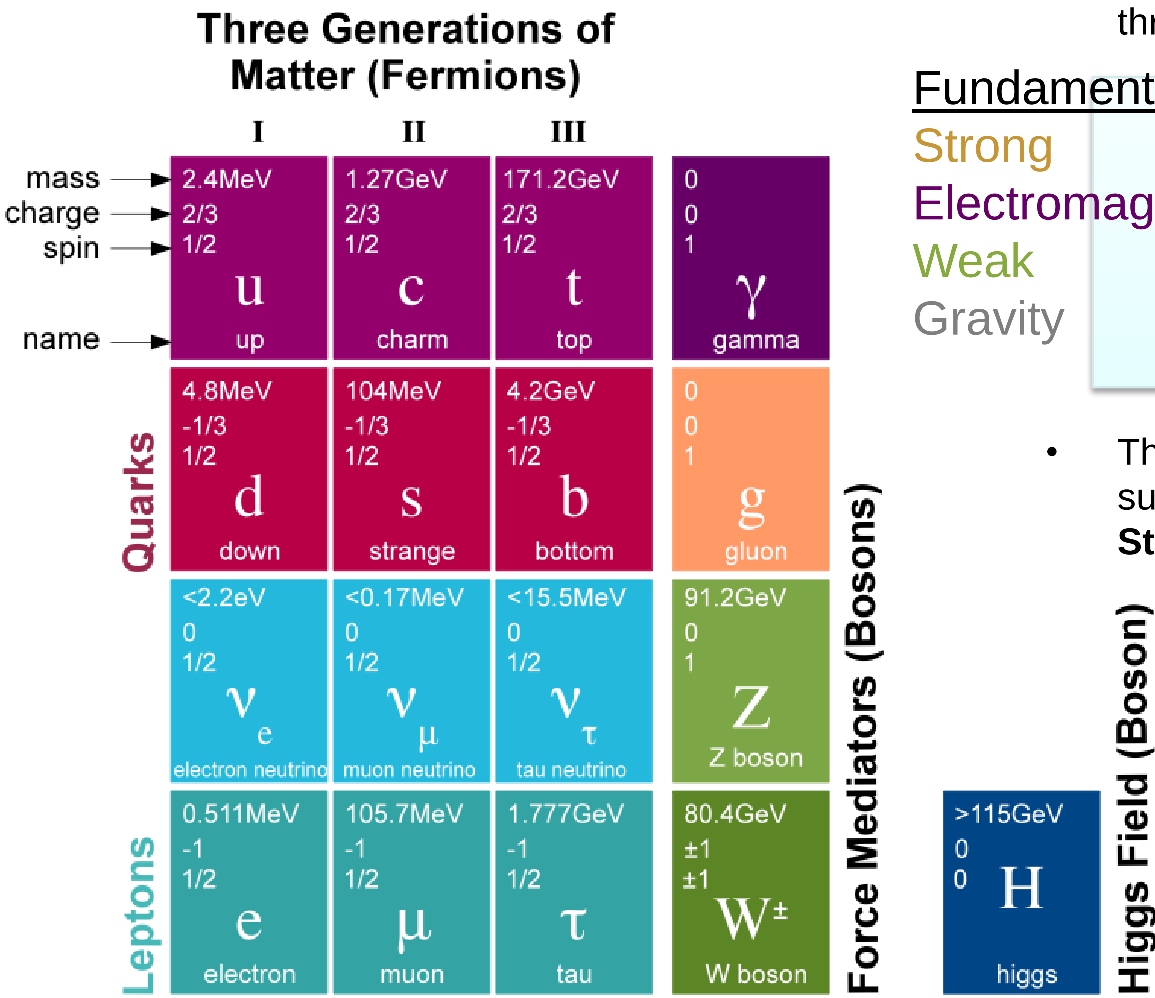
http://www.symmetrymagazine.org/breaking/2012/01/11/belle-experiment-makes-exotic-discovery/

The Standard Model Now

Quarks (spin ½)		
Flavor	Charge	Bare Mass (MeV/c^2)
u	+2/3	1.5 – 4.5
d	-1/3	5 – 8.5
s	-1/3	80 – 155
c	+2/3	1,000 – 1,400
b	-1/3	4,000 – 4,500
t	+2/3	17,430±5,100

Leptons (spin ½)		
Lepton	Charge	Mass (MeV/c^2)
e	-1	0.511
ν_e	0	< 0.000003
μ	-1	105.7
ν_μ	0	< 0.19
τ	-1	1,776.99
ν_τ	0	< 18.2

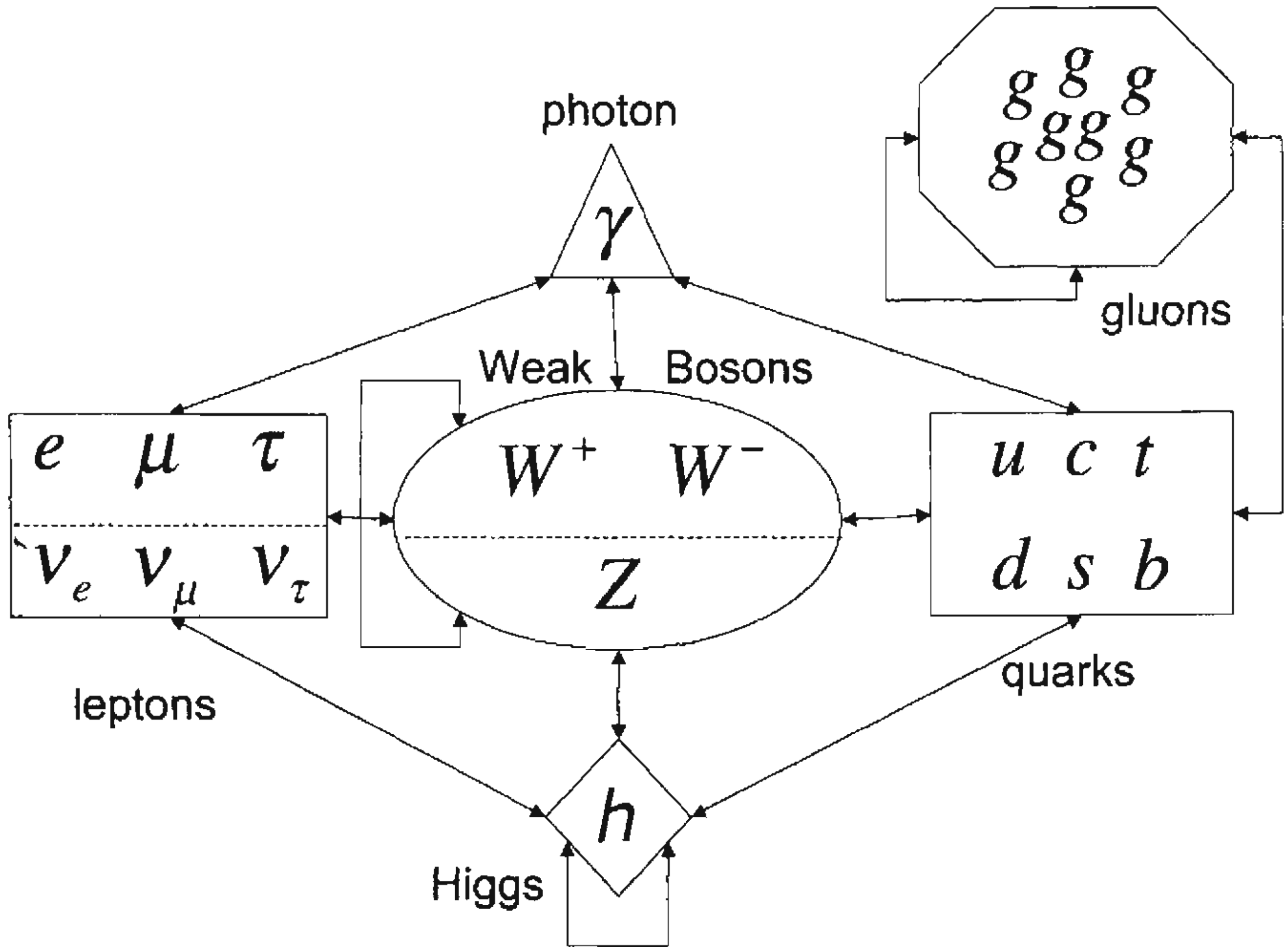
Force Mediators (spin 1)				
Mediator	Charge	Mass (MeV/c^2)	Lifetime (s)	Force
gluon	0	0	∞	strong
photon (γ)	0	0	∞	electromagnetic
$W^\pm$	±1	80,423 ± 39	~10^{-25}	weak (charged)
Z^0	0	91,188 ± 2	~10^{-25}	weak (neutral)



- Current model of matter: matter consists of point particles (< 10^{-18}m) interacting through four forces.

Fundamental forces	Strength
Strong	1
Electromagnetic	10^{-2}
Weak	10^{-5}
Gravity	10^{-39}

- This picture (excluding gravity) summarizes the **Standard Model** of particle physics.


photon
γ
g g g
g g g g g
g
gluons
Weak Bosons
W⁺ W⁻
Z
e μ τ
νe νμ ντ
u c t
d s b
leptons
quarks
h
Higgs

Gellmann and Okubo mass formula

Generally, It was observed that the masses of the different members of a SU(3) symmetry is not exact and seems to be broken.

- Gellmann and Okubo mass formula was the consequence of SU(3) symmetry. This formula correctly predicts the relations of masses of the members of SU(3) multiplets in term of hypercharge Y and isospin I.

- It was one of the early indicator of the existence of Quarks.

- The mass formula was first formulated by Murray Gellmann in 1961 and was independently proposed by Susumu Okubo in 1962.

$$M(Y,I) = M_0 + aY + b[I(I+1) - \frac{1}{4}Y^2]$$

$$Nucleon: Y = 1; I = \frac{1}{2} \Rightarrow M(Y,I) = M_0 + aY + b[I(I+1) - \frac{1}{4}Y^2]$$

$Baryon \frac{1}{2}^+ \ Octet$

$$n, p : Y = 1; \ I = \frac{1}{2} \Rightarrow M_N = M_0 + a + \frac{1}{2}b$$

$$\Sigma : Y = 0; I = 1 \Rightarrow M_\Sigma = M_0 + 2b$$

$$\Lambda : Y = 0; \ I = 0 \Rightarrow M_\Lambda = M_0$$

$$\Xi : \ Y = -1; \ I = \frac{1}{2} \Rightarrow M_\Xi = M_0 - a + \frac{1}{2}b$$

$$M_N + M_\Xi = 2M_0 + b$$

$$\frac{1}{3}(3M_\Lambda + M_\Sigma) = 2M_0 + b$$

$$\frac{1}{2}(3M_\Lambda + M_\Sigma) = M_N + M_\Xi$$

$$Experimentally, \ \frac{1}{2}(3M_\Lambda + M_\Sigma) = 2.23 GeV \ and \ M_N + M_\Xi = 2.25 GeV$$

$$Baryon\ \frac{3}{2}^{+}\ Octet$$

$$M(Y,I) = M_0 + aY + b[I(I+1) - \frac{1}{4}Y^2]$$

$$I = 1 + \frac{Y}{2}$$

$$M(Y,I) = M_0 + 2b + [a + \frac{3}{2}b]Y$$

$$\Delta : Y = 1, I = 3/2$$

$$\Sigma : Y = 0, I = 1$$

$$\Xi : Y = -1, I = 1/2$$

$$\Omega : Y = -2, I = 0$$

$$M_{\Sigma} - M_{\Lambda} = -(a + \frac{3}{2}b)$$

$$M_{\Xi} - M_{\Sigma} = -(a + \frac{3}{2}b)$$

$$M_{\Omega} - M_{\Xi} = -(a + \frac{3}{2}b)$$

$$M_{\Sigma} - M_{\Delta} = M_{\Xi} - M_{\Sigma} = M_{\Omega} - M_{\Xi}$$

The above formula is called equal spacing rule and also predicted equal mass spacing in the decuplet which is in agreement with the observed mass.

This Formula was used by Gellmann to predict the existence, nature and mass of the Ω particle.

Meson Octet

$$M^2(Y,I) = M_0{}^2 + aY + b[I(I+1) - \frac{1}{4}Y^2]$$

Gellmann and Okubo mass formula for the mesons differ slightly from that of baryons because of chiral symmetry breaking as

By putting the value for Y and I for charge multiplets of meson octet, we get the following relation

$$(M_{K^-}{}^2 + M_K{}^2) = \frac{3M_\eta{}^2 + M_\pi{}^2}{4}$$

$$4M_K{}^2 = 0.98\,GeV^2 \;\; and \;\; 3M_\eta{}^2 + M_\pi{}^2 = 0.92\,GeV^2$$