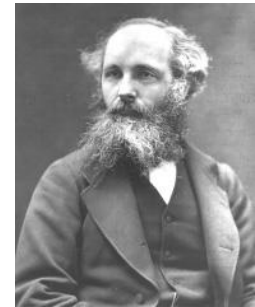




Magnetostatics

Maxwell's Equations



Electrostatics

$$\nabla \cdot \mathbf{D} = \rho_v, \quad (2.2a) \quad \underline{\text{Gauss' Law}}$$

$$\nabla \times \mathbf{E} = 0. \quad (2.2b) \quad \underline{??}$$

Magnetostatics



$$\nabla \cdot \mathbf{B} = 0, \quad (2.3a) \quad \text{Gauss' Law of magnetism (No magnetic charges)}$$

$$\nabla \times \mathbf{H} = \mathbf{J}. \quad (2.3b) \quad \underline{\text{Ampere's Law}}$$

Problems Involving Symmetries

Electrostatics

Gauss's Law

$$\nabla \cdot \vec{D} = \rho_v$$

$$\oint_S \vec{D} \cdot d\vec{s} = Q$$

Total charge enclosed
By Gaussian surface S

Magnetostatics

Ampere's Law

$$\nabla \times \vec{H} = \vec{J}$$

$$\oint_C \vec{H} \cdot d\vec{\ell} = I$$

Total current flowing
through surface S

Maxwell's Equation: $\nabla \times \vec{H} = \vec{J}$

Integrate both sides over an open surface S

$$\int_S (\nabla \times \vec{H}) \cdot d\vec{s} = \int_S \vec{J} \cdot d\vec{s}$$

Stokes's
theorem



Current I is equal to
the surface integral
of $\vec{J} \cdot d\vec{s}$



$$\oint_C \vec{H} \cdot d\vec{\ell} = I$$

Definition

Ampere's law states that the **line integral** of H (Magnetic Field Intensity) about any closed contour C is exactly equal to the **net current enclosed by that path**.

Mathematically:

$$\oint_C \vec{H} \cdot d\vec{l} = I$$

$$\vec{B} = \mu_0 \vec{H}$$

OR

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I$$

Net Current
Enclosed
by closed path
 C

Analyze Ampere's Law:

$$\oint_C \vec{H} \cdot d\vec{l} = I$$

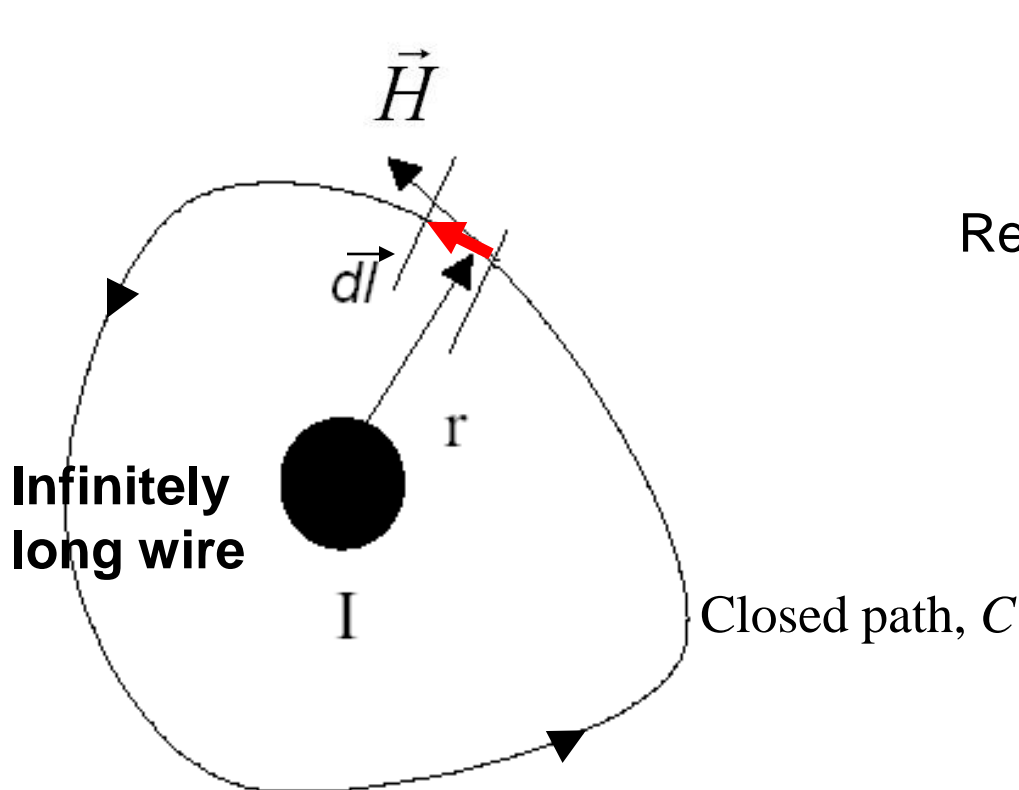
C : Closed Contour
(or Amperian Contour)
bounding the surface S

I : Total current flowing
through surface S

The sign convention for the direction of C is taken so that I and H satisfy Right-Hand Rule.

Thumb \rightarrow direction of current, I
Four fingers \rightarrow direction of the contour C

Ampere's law can be easily applied to determine \vec{H} in a magnetic circuit, when the *current distribution is symmetrical*. The proof is given as shown:



$$\vec{H} = H\hat{\phi}$$

Recall:

$$\vec{B} = \hat{\phi} \frac{\mu_o I}{2\pi r}$$

$$\vec{B} = \mu_o \vec{H}$$

$$\rightarrow \vec{H} = \hat{\phi} \frac{I}{2\pi r}$$

Assume that a closed path is drawn enclosing an infinite long conductor with current I . The magnetic field \mathbf{H} is :

$$\vec{H} = \hat{\phi} \frac{I}{2\pi r}$$

The integration along the closed contour C leads to:

$$d\vec{l} = \hat{r}dr + \hat{\phi}rd\phi + \hat{z}dz \quad \text{[Cylindrical coordinate system]}$$

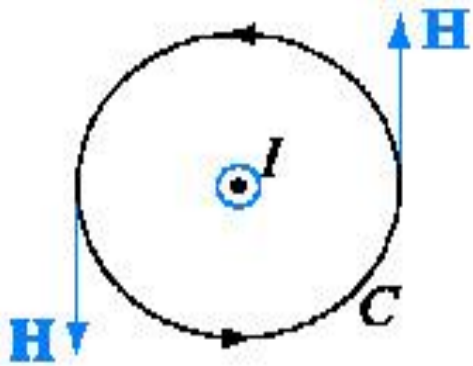
$$\oint_C \vec{H} \cdot d\vec{l} = \int_0^{2\pi} \hat{\phi} \frac{I}{2\pi r} \cdot (\hat{r}dr + \hat{\phi}rd\phi + \hat{z}dz) = \int_0^{2\pi} \frac{I}{2\pi} d\phi = I$$

In general,

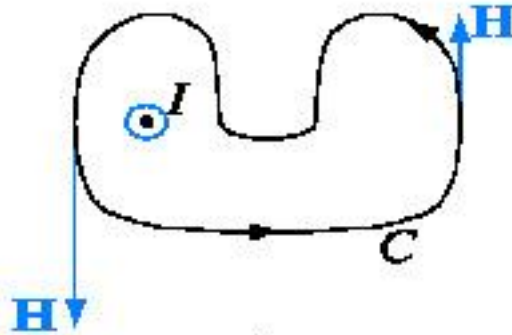
$$\oint_C \vec{H} \cdot d\vec{l} = \sum_i I_i$$

OR

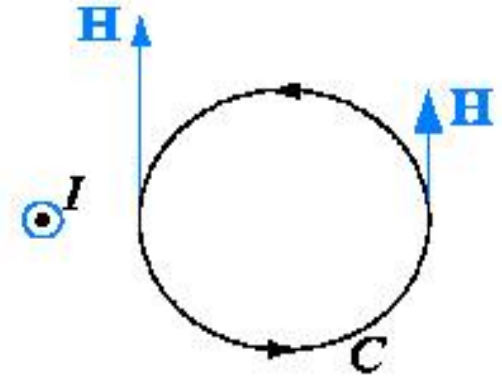
$$\oint_C \vec{B} \cdot d\vec{l} = \mu_o \sum_i I_i$$



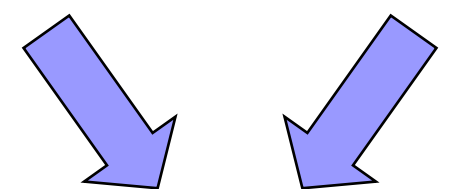
(a)




(b)

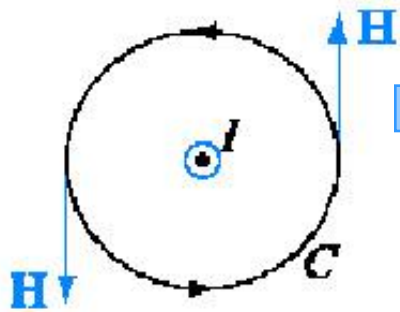


(c)


$$\oint_C \vec{H} \cdot d\vec{l} = I$$

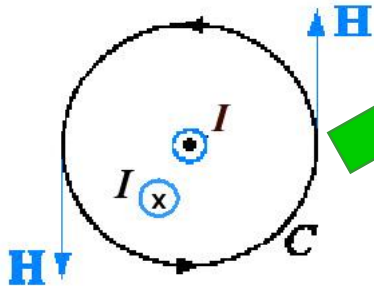
Contour C does not enclose the current I


$$\oint_C \vec{H} \cdot d\vec{l} = 0$$



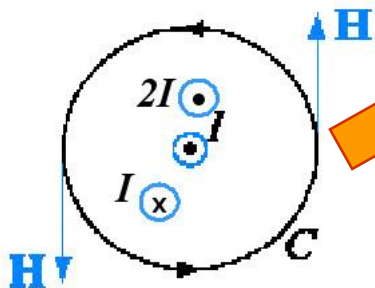
$$\oint_C \vec{H} \cdot d\vec{\ell} = I$$

Net current enclosed by the Amperian contour
 $= (I) + (-I) = 0$



$$\Rightarrow \oint_C \vec{H} \cdot d\vec{\ell} = 0$$

Net current enclosed by the Amperian contour
 $= (I) + (-I) + (2I) = 2I$



$$\Rightarrow \oint_C \vec{H} \cdot d\vec{\ell} = 2I$$

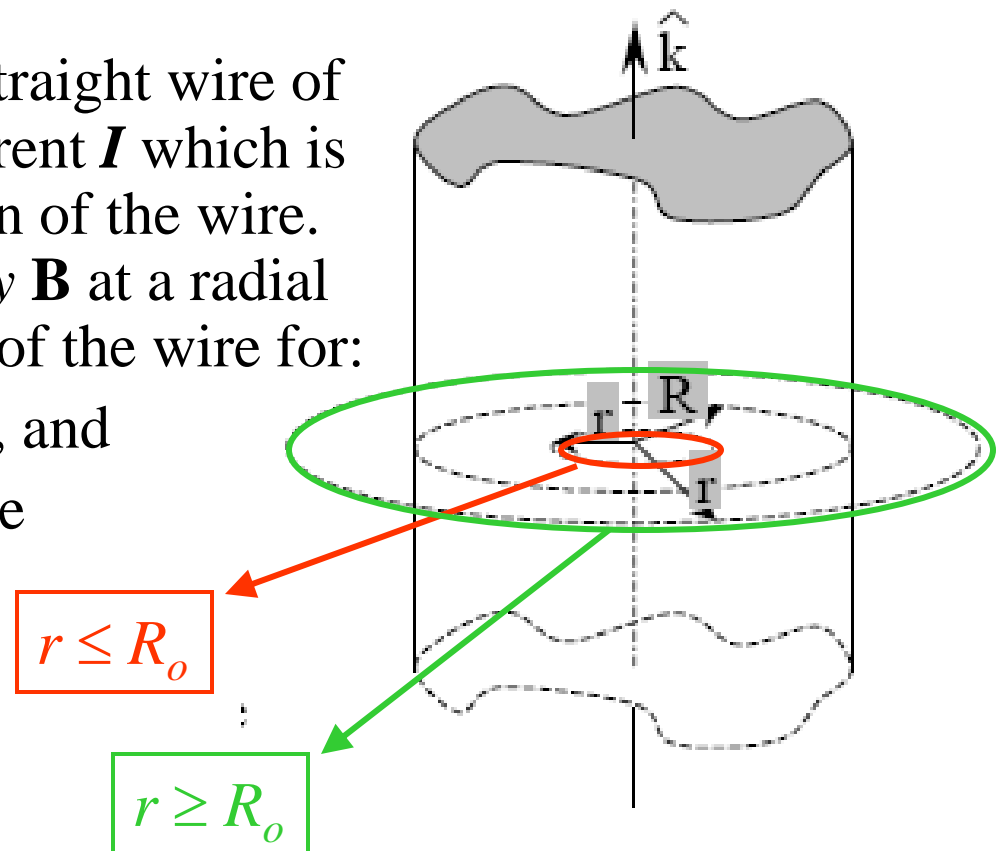
Application of Ampere's Law

- 1) Define Closed Contour C (Amperian Contour)
[use right hand rule]
- 2) Define net current I : current through the surface enclosed by Amperian Contour C .
- 3) Establish dl (magnitude and direction)
- 4) Establish H direction (Magnitude is H)
[use right hand rule]
- 5) Apply Ampere's Law

Application 1: Magnetic Field of Long Wire

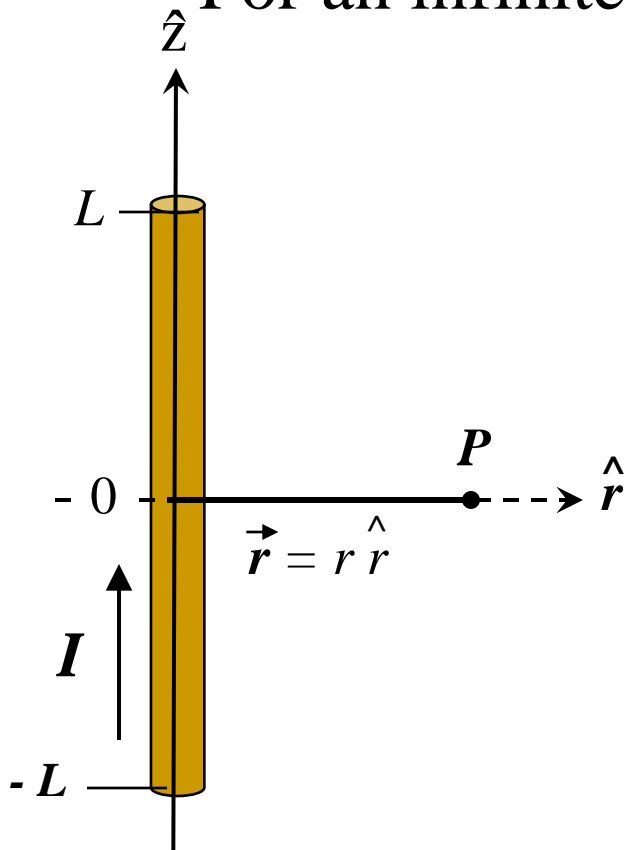
A long (practically infinite) straight wire of radius R_o carries a steady current I which is uniform over the cross-section of the wire. Find the *magnetic flux density* \mathbf{B} at a radial distance r from the long axis of the wire for:

- (a) $r \leq R_o$, i.e. inside the wire, and
- (b) $r \geq R_o$, i.e. outside the wire



$$\vec{B} = \hat{\phi} \frac{\mu_0 I L}{2\pi} \frac{1}{r(L^2 + r^2)^{1/2}}$$

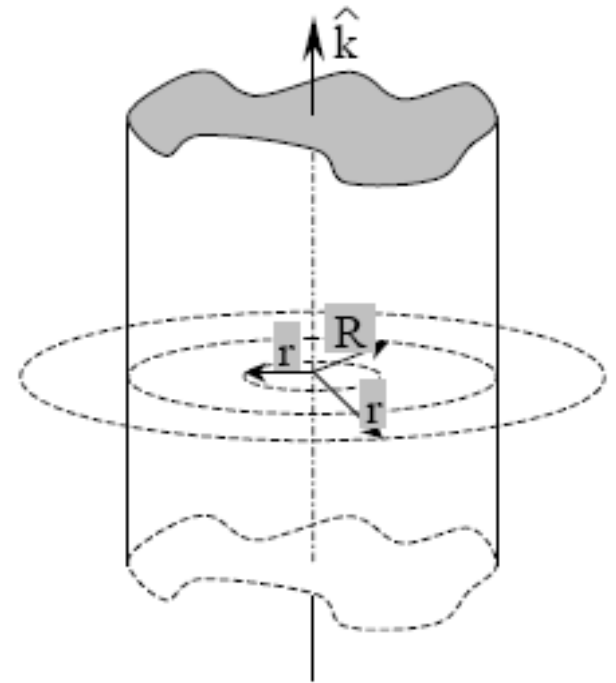
For an infinitely long wire such that $L \gg r$



$$\vec{B} = \hat{\phi} \frac{\mu_0 I}{2\pi r}$$

Application 1: Solution

- The wire is a long cylinder, therefore use cylindrical coordinates with the wire's axis points along \hat{z}
- I is chosen to be along the \hat{z} axis direction.
- From symmetry, the magnetic flux lines form concentric circles around the axis of wire.

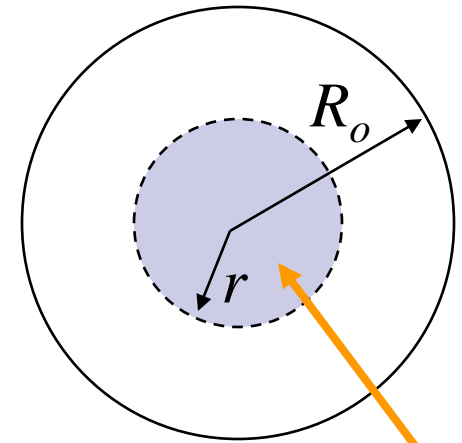


Part a) $r \leq R_o$, i.e. inside the wire:

The current density through the cross-section of the wire with radius R_o is:

$$J = \frac{I}{\pi R_o^2}$$

The cross sectional area for radius $r \leq R_o$ is: $A = \pi r^2$



\therefore Current through the cross section is

$$I_1 = JA = \frac{I}{\pi R_o^2} (\pi r^2) = I \frac{r^2}{R_o^2}$$

Part a) $r \leq R_o$, i.e. inside the wire:

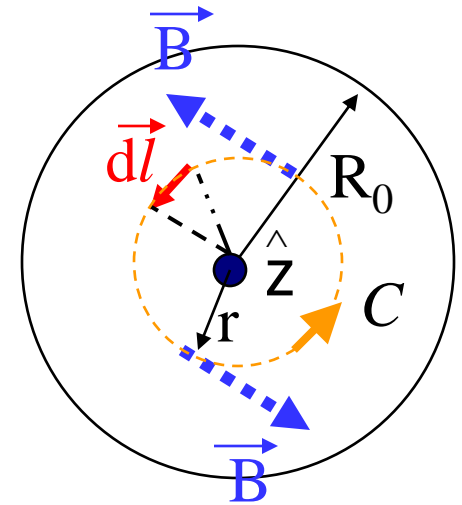
Applying Ampere's law: $\oint_C \vec{H} \cdot d\vec{\ell} = I_{enc}$

OR $\oint_C \vec{B} \cdot d\vec{\ell} = \mu_o I_{enc}$

$$I_{enc} = \frac{r^2}{R_o^2} I \quad \Rightarrow \quad B \oint_C d\ell = \mu_o \left(\frac{r^2}{R_o^2} I \right)$$

$$B(2\pi r) = \mu_o I \frac{r^2}{R_o^2}$$

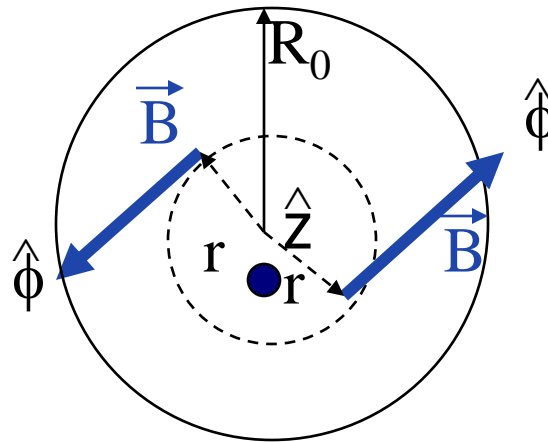
$$\therefore B = \frac{\mu_o I}{2\pi R_o^2} r^2$$



Part a) $r \leq R_0$, i.e. inside the wire:

Where \vec{B} is the magnetic flux density inside the wire. It points in the $\hat{\phi}$ direction, i.e. tangential to the circle with radius r . Hence:

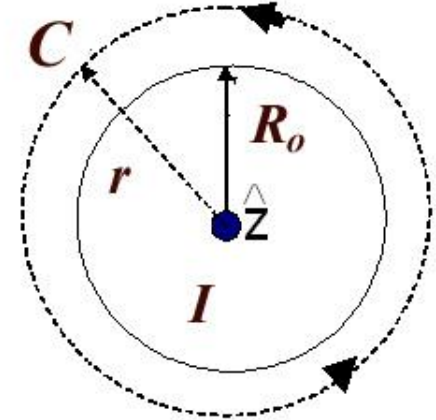
$$\vec{B} = \hat{\phi} \frac{\mu_0 I r^2}{2\pi R_0^2}$$



Part b) $r \geq R_0$, outside the wire

The current enclosed by the contour is I .

$$\left. \begin{aligned} \vec{B} &= B \hat{\phi} \\ \oint_C d\vec{l} &= 2\pi r \hat{\phi} \end{aligned} \right\} \Rightarrow \oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} \Rightarrow B(2\pi r) = \mu_0 I$$



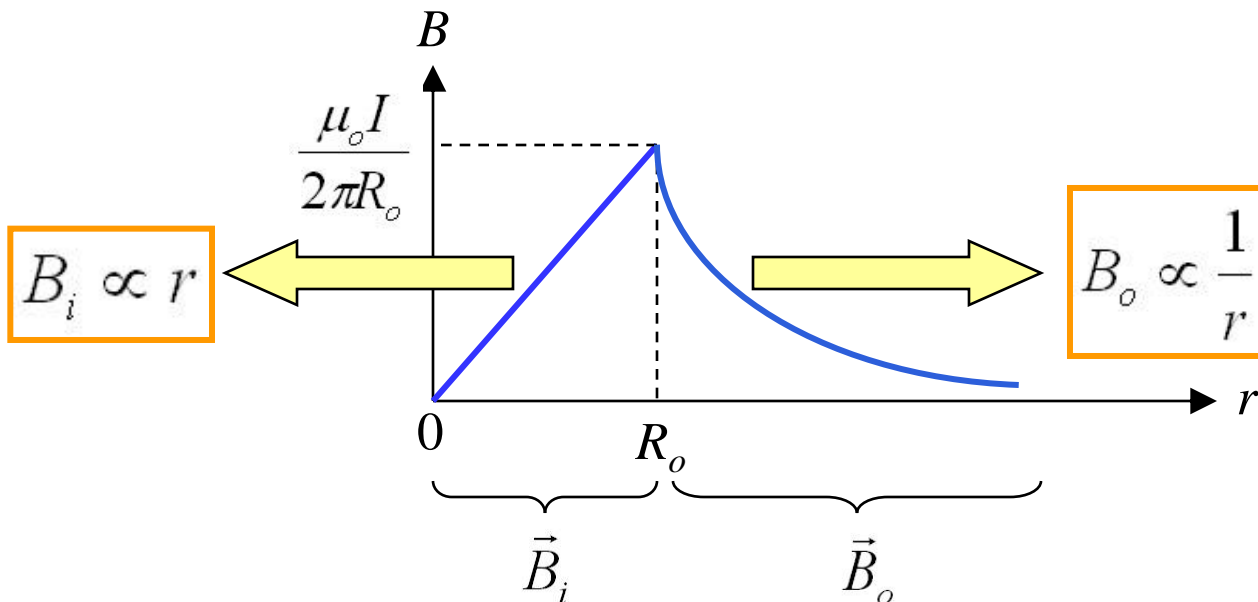
\vec{B} is the magnetic flux density outside the wire. It is pointed to the $\hat{\phi}$ direction

$$\vec{B} = \frac{\mu_0 I}{2\pi r} \hat{\phi}$$

Summary: Plot of B vs. r

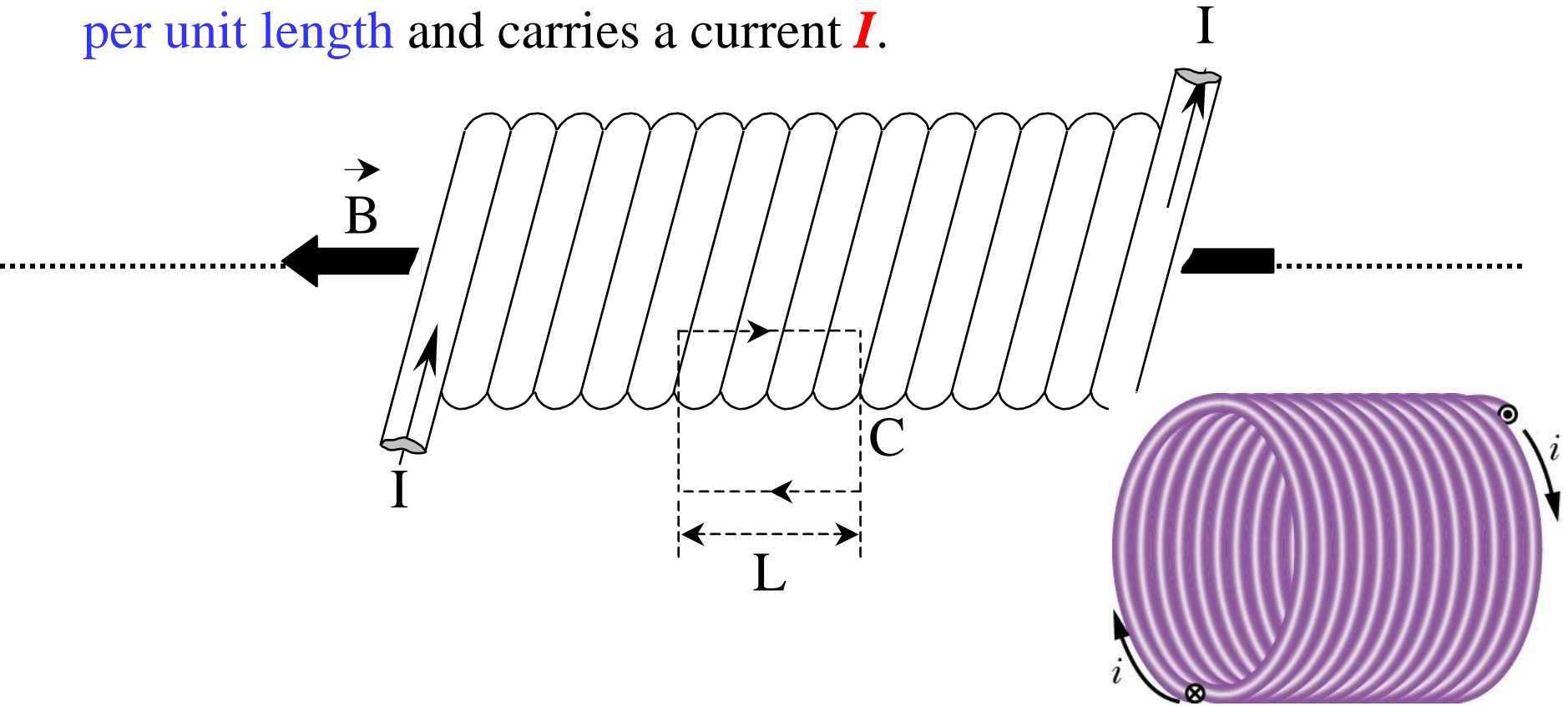
Part a) $r \leq R_o$, i.e. inside the wire: $\vec{B}_i = \frac{\mu_o I r}{2\pi R_o^2} \hat{\phi}$ ($r \leq R_o$)

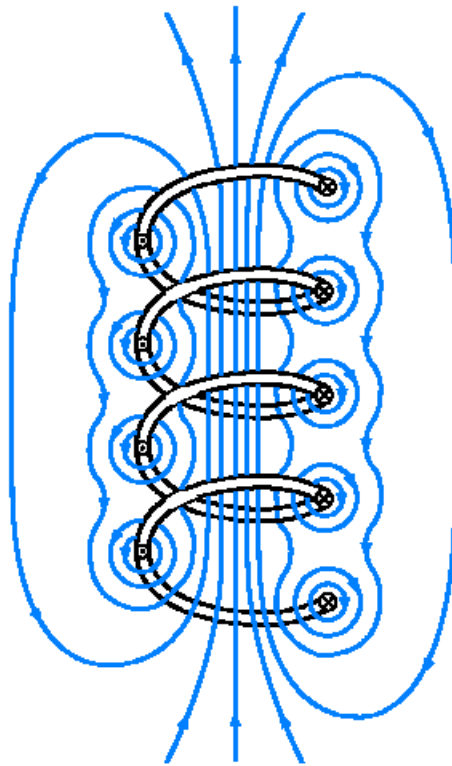
Part b) $r \geq R_o$, outside the wire: $\vec{B}_o = \frac{\mu_o I}{2\pi r} \hat{\phi}$ ($r \geq R_o$)



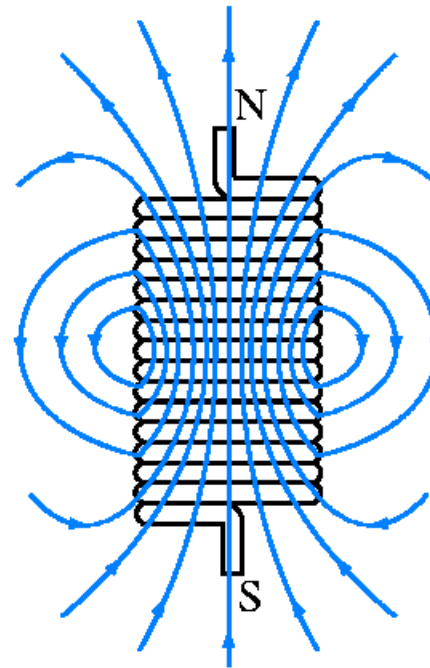
Application 2: Magnetic Field of Long Solenoid

Determine the **magnetic flux** inside an infinitely long solenoid with air core. The solenoid has **N number of closely wound turns per unit length** and carries a current **I** .





(a) Loosely wound solenoid

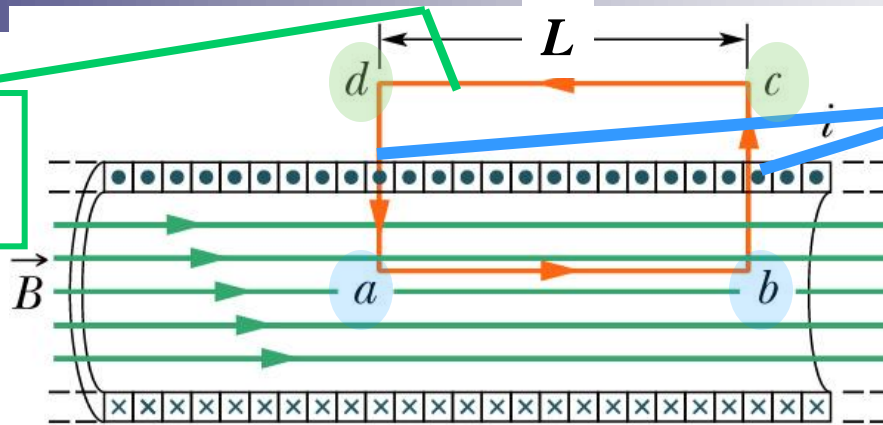


(b) Tightly wound solenoid

Application 2: Solution

- The magnetic field outside the solenoid is zero except in the vicinity of the ends which are at an infinite distance away.
- The magnetic field inside the solenoid is parallel to its long axis.
- In order to find the magnitude of the magnetic flux density, a **closed contour C** shown in the diagram is considered.
- The number of turns that the contour encloses is **NL** . The current enclosed by the contour is **NLI** . Since the magnetic flux density outside the solenoid is zero, therefore:

Outside the solenoid,
 $\rightarrow B = 0$



$$\vec{B} \perp d\vec{l}$$

$$\vec{B} \cdot d\vec{l}$$

$$= Bdl \cos 90^\circ$$

$$= 0$$

Ampere's Law: $\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

B is uniform within the solenoid and zero outside it, using the rectangular Amperian loop abcd:

$$\oint_C \vec{B} \cdot d\vec{l} = \int_a^b \vec{B} \cdot d\vec{l} + \int_b^c \vec{B} \cdot d\vec{l} + \int_c^d \vec{B} \cdot d\vec{l} + \int_d^a \vec{B} \cdot d\vec{l}$$

$$\oint_C \vec{B} \cdot d\vec{l} = BL + 0 + 0 + 0 = BL$$

$$\rightarrow BL = \mu_0 NLI$$

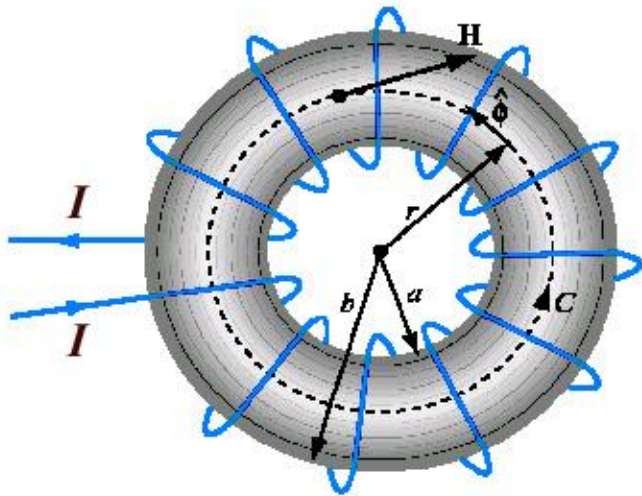
I_{enc}

N : number of turn per unit length

$$\therefore B = \mu_0 NI$$

Application 3: Magnetic Field of a Toroidal Coil (or Toroid)

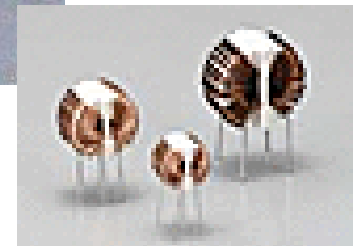
A toroid which direction dimensions as shown in the figure has N turns and carries current I . Determine magnetic flux intensity H inside and outside the toroid.



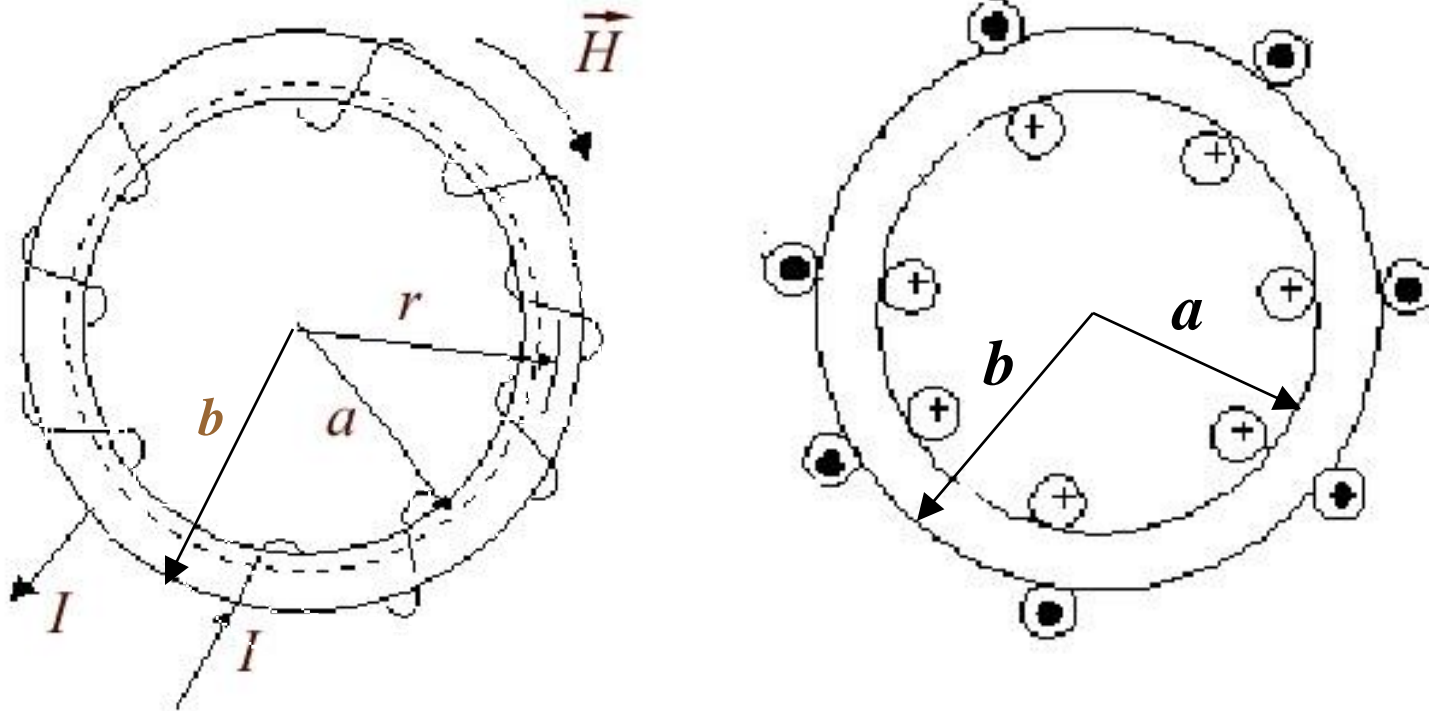
Only few turns are shown

What is toroid?

Doughnut-shaped structure with closed spaced turns of wire wrapped around it



A toroid which direction dimensions as shown in the figure has N turns and carries current I . Determine magnetic flux intensity H inside and outside the toroid.

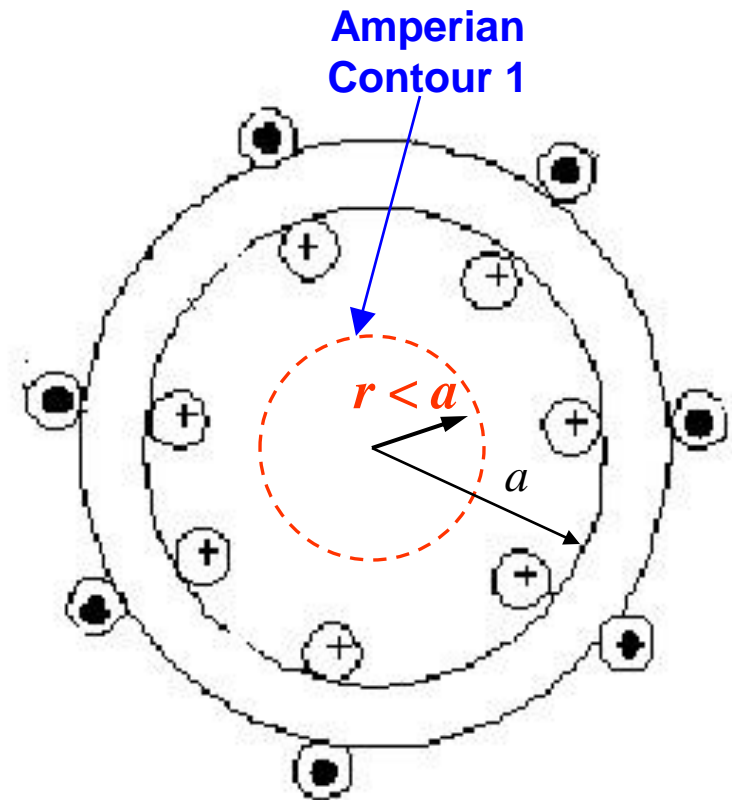


Inside the Toroid

Circular Amperian Contour 1 ($r < a$):

Ampere's Law:
$$\oint_C \vec{H} \cdot d\vec{l} = I$$

Current contained by contour 1 is zero, thus $\mathbf{H} = 0$



Outside the Toroid

Circular Amperian Contour 3 ($r > b$):

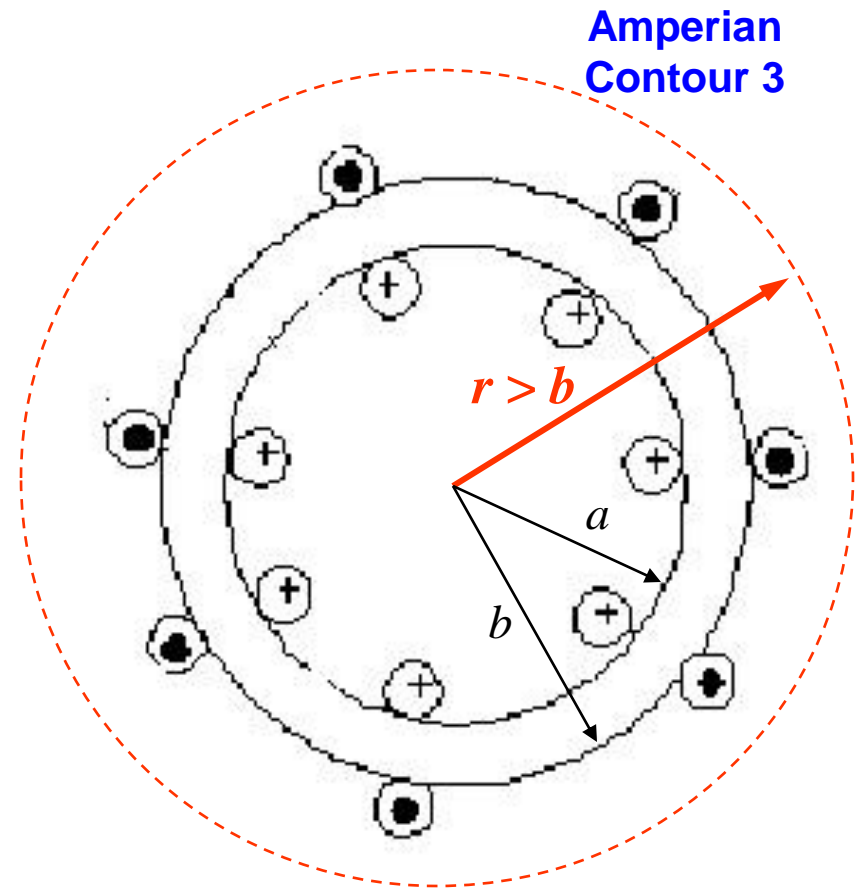
$$\text{Ampere's Law: } \oint_C \vec{H} \cdot d\vec{l} = I$$

Since the total current contained
by contour 3

$$= I + (-I)$$

$$= 0$$

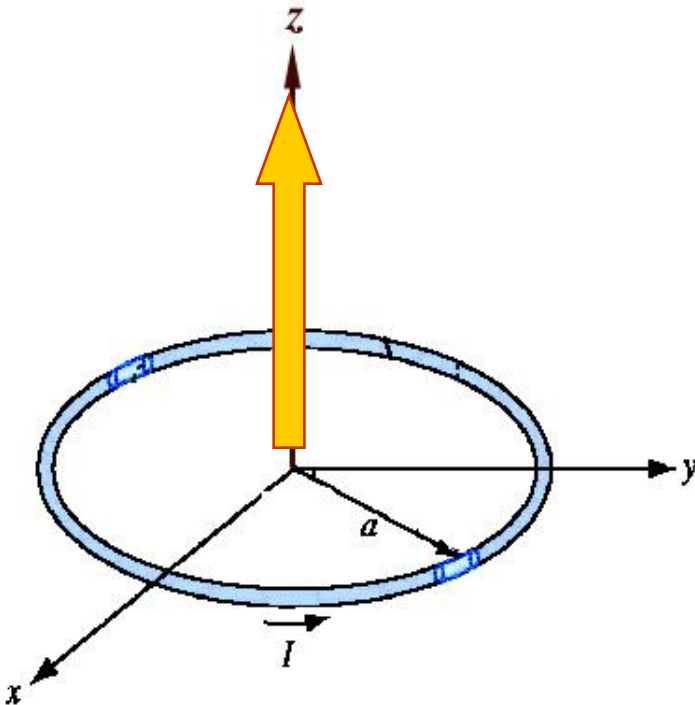
Thus, $H = 0$



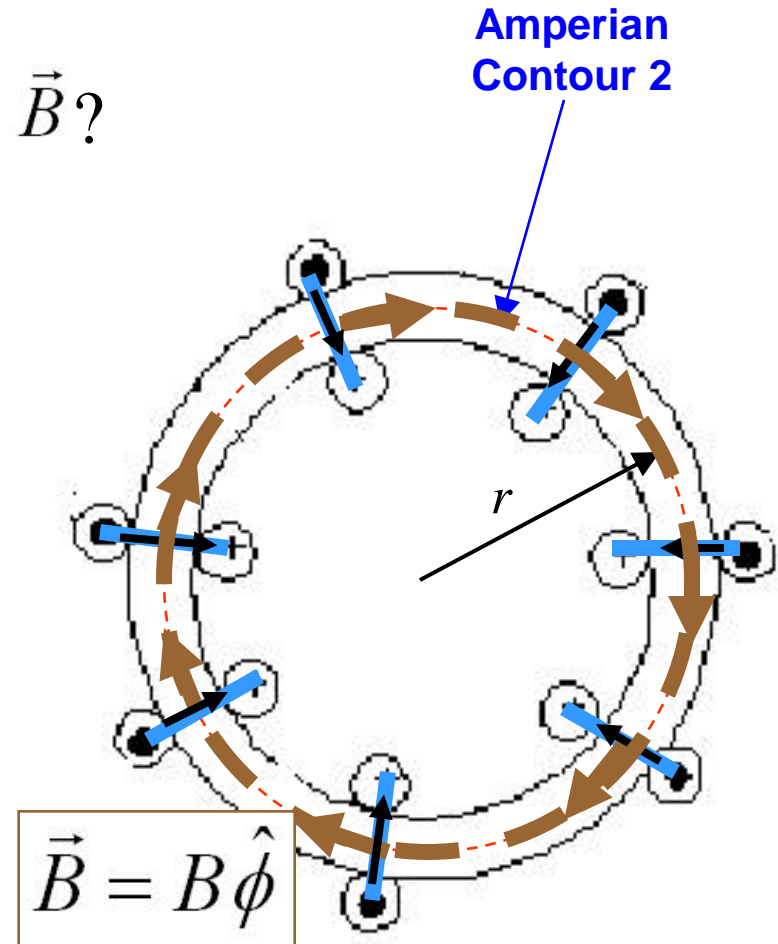
Inside the Toroid ($a < r < b$)

Recall:

The magnetic field at the center of the loop points along the axis of the loop



Direction of \vec{B} ?



The right-hand rule tells us that B must be in the negative ϕ -direction.

Ampere's Law:

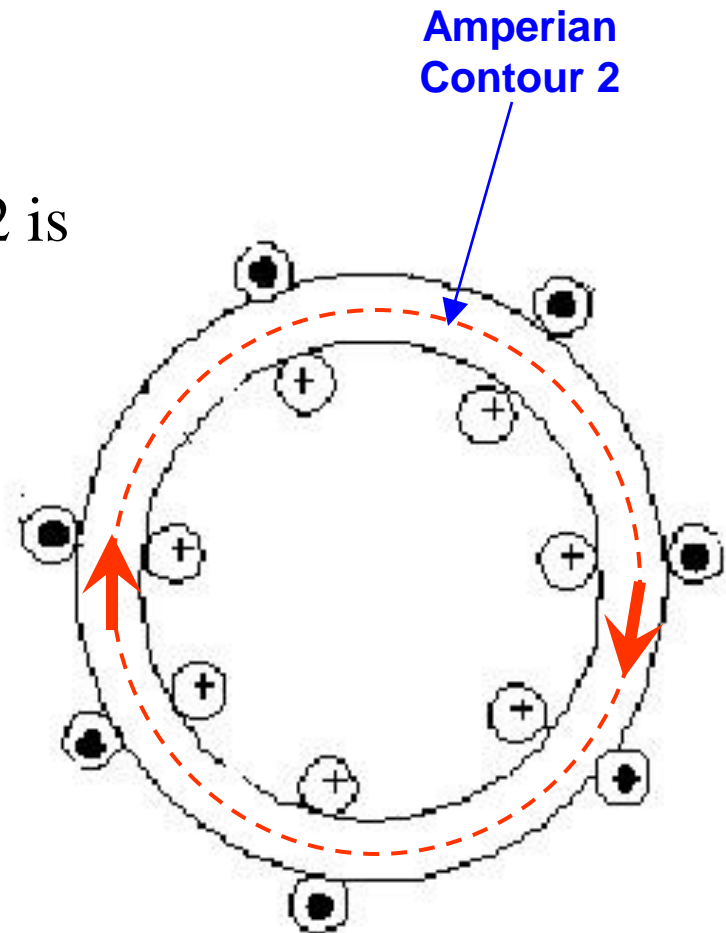
$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$$

Total current enclosed by the contour 2 is

$$I_{enc} = NI$$

N is the total number of turns in the winding

I_{enc} is positive if it crosses the surface of the contour in the direction of the four fingers of the right hand when thumb is pointing along the direction of the contour.



Ampere's Law:

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_o I_{enc}$$

$$I_{enc} = NI \quad \vec{B} = B\hat{\phi}$$

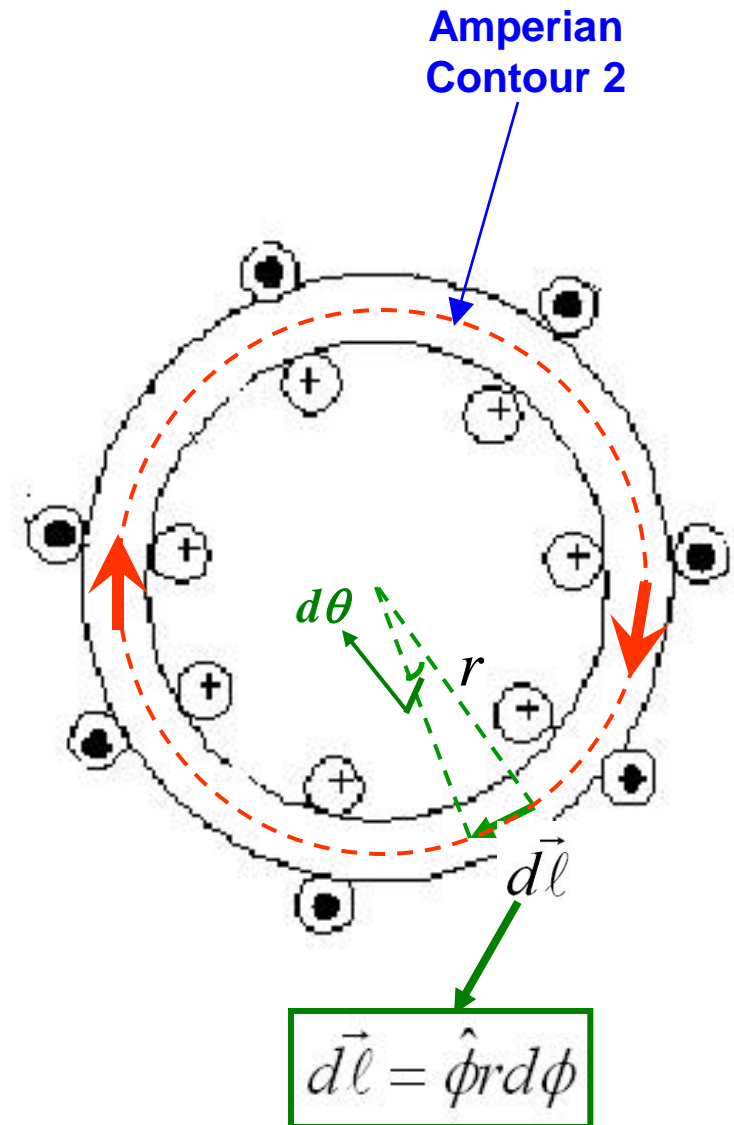
$$\Rightarrow \oint_C (B\hat{\phi}) \cdot (\hat{\phi}r d\phi) = \mu_o NI$$

$$Br \oint_C d\phi = \mu_o NI$$

$$Br(2\pi) = \mu_o NI$$

$$\Rightarrow B = \frac{\mu_o NI}{2\pi r}$$

$$\therefore \vec{B} = \hat{\phi} \frac{\mu_o NI}{2\pi r}$$



Ampere's Law: $\oint_C \vec{H} \cdot d\vec{\ell} = I_{enc}$ OR $\oint_C \vec{B} \cdot d\vec{\ell} = \mu_o I_{enc}$

Long Wire: Inside the wire: $\vec{B}_i = \frac{\mu_o I r}{2\pi R_o^2} \hat{\phi} \quad (r \leq R_o)$

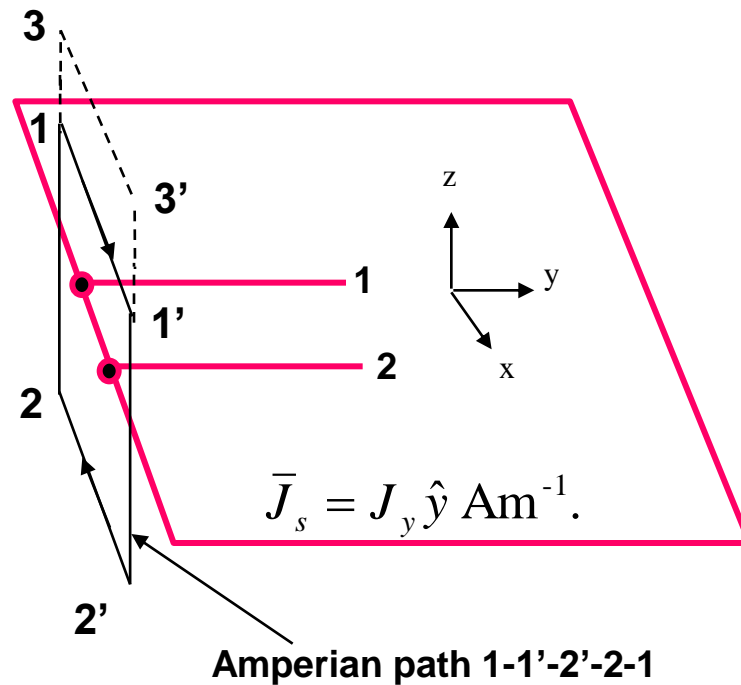
Outside the wire: $\vec{B}_o = \frac{\mu_o I}{2\pi r} \hat{\phi} \quad (r \geq R_o)$

Long Solenoid: $B = \mu_o NI$ N : number of turn per unit length

Toroid: $\vec{B} = \hat{\phi} \frac{\mu_o NI}{2\pi r}$ N : number of turns

Application 4: Infinite extent with a surface current density

Graphical display for finding \vec{H} and using Ampere's circuital law:



$$\oint_l \vec{H} \cdot d\vec{l} = \int_1^{1'} \vec{H} \cdot d\vec{l} + \int_{1'}^{2'} \vec{H} \cdot d\vec{l} + \int_{2'}^2 \vec{H} \cdot d\vec{l} + \int_2^1 \vec{H} \cdot d\vec{l} = I_{en} = J_y l$$

From the construction, we can see that \bar{H} above and below the surface current will be in the \hat{x} and $-\hat{x}$ directions, respectively.

$$\int_1^{1'} H_{x1} \hat{x} \cdot \hat{x} dx + \int_2^{2'} \bar{H}_{x2} (-\hat{x}) \cdot \hat{x} dx = J_y l$$

where

$$\int_{1'}^{2'} \text{ and } \int_2^1 = 0 \text{ since } \bar{H} \text{ is perpendicular to } d\bar{l}$$

Therefore:

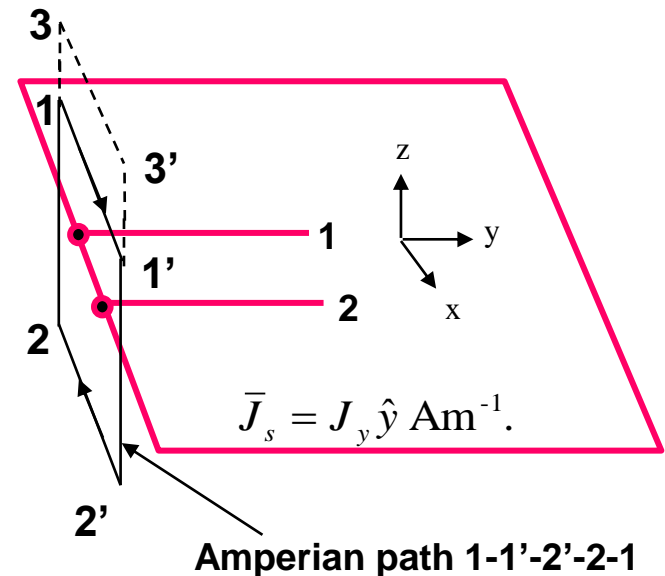
$$H_{x1} l - H_{x2} l = J_y l$$

Similarly if we takes on the path 3-3'-2'-2-3, the equation becomes:

$$H_{x3} l - H_{x2} l = J_y l$$

Hence:

$$H_{x1} = H_{x3} = H_x$$



And we deduce that $|\overline{H}|$ above and below the surface current are equal, its becomes:

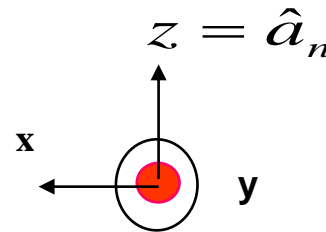
$$H_x l + H_x l = J_y l$$

$$H_x = \frac{1}{2} J_y \quad ; \quad z > 0 \quad \overline{H} = +\frac{1}{2} J_y \hat{x}$$

$$H_x = -\frac{1}{2} J_y \quad ; \quad z < 0 \quad \overline{H} = +\frac{1}{2} J_y (-\hat{x})$$

In vector form:

$$\overline{H} = \frac{1}{2} J \times \hat{a}_n$$



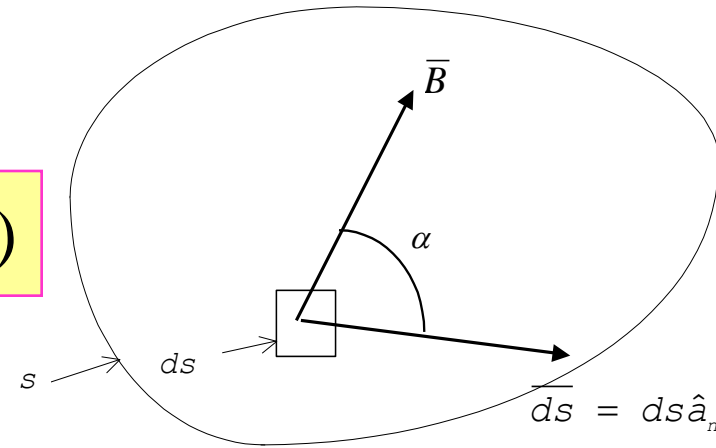
It can be shown for two parallel plate with separation h , carrying equal current density flowing in opposite direction the \overline{H} field is given by:

$$\begin{aligned} \overline{H} &= J \times \hat{a}_n \quad ; \quad (0 < z < h) \\ &= 0 \quad ; \quad (z > h \text{ and } z < 0) \end{aligned}$$

MAGNETIC FLUX DENSITY

Magnetic field : $\bar{B} = \mu_o \bar{H}$ Teslas (Wb/m^2)

where $\mu_o = 4\pi \times 10^{-7}$ H/m
permeability of free space



Magnetic flux : $\Psi_m = \int_s \bar{B} \bullet \bar{ds}$
that passes
through the surface S .

In magnetics, magnet poles have not been isolated:

$$\Psi_m = \oint_s \bar{B} \bullet \bar{ds} = 0 \quad (Wb)$$

$$\begin{aligned} d\Psi_m &= |\mathbf{ds}| |\bar{B}| |\cos \alpha| \\ &= \bar{B} \bullet \bar{ds} \end{aligned}$$

Hence:

$$\Psi_m = \int_s \bar{B} \bullet \bar{ds}$$

$$\begin{aligned} \oint_s \bar{B} \bullet \bar{ds} &= \int_v (\nabla \bullet \bar{B}) dv = 0 \\ \nabla \bullet \bar{B} &= 0 \end{aligned}$$

For $\vec{H} = \hat{\phi}10^3 r$ (Am^{-1}), **find** the ψ_m that passes through a plane surface by, ($\phi = \pi/2$), ($2 \leq r \leq 4$), and ($0 \leq z \leq 2$).

Solution:

$$\begin{aligned}\Psi_m &= \int_s \vec{B} \cdot \vec{ds} = \int_0^2 \int_2^4 (\mu_o \hat{\phi} 10^3 r) \cdot \hat{\phi} (dr dz) \\ &= \mu_o 10^3 \int_0^2 \int_2^4 r dr dz = \mu_o 10^3 (12) \\ &= 150.8 \times 10^{-4} \text{ Wb}\end{aligned}$$

MAXWELL'S EQUATIONS

| POINT FORM | INTEGRAL FORM |
|-----------------------------------|--|
| $\nabla \cdot \bar{D} = \rho_v$ | $\int_v \nabla \cdot \bar{D} dv = \oint_s \bar{D} \cdot d\bar{s} = \int_v \rho_v dv = Q_{enc}$ |
| $\nabla \times \bar{E} = 0$ | $\int_s (\nabla \times \bar{E}) \cdot d\bar{s} = \oint_l \bar{E} \cdot d\bar{l} = 0$ |
| $\nabla \times \bar{H} = \bar{J}$ | $\int_s (\nabla \times \bar{H}) \cdot d\bar{s} = \oint_l \bar{H} \cdot d\bar{l} = \int_s \bar{J} \cdot d\bar{s} = I_{enc}$ |
| $\nabla \cdot \bar{B} = 0$ | $\int_v \nabla \cdot \bar{B} dv = \oint_s \bar{B} \cdot d\bar{s} = 0$ |

Electrostatic fields : $\bar{D} = \epsilon\bar{E}$

Magnetostatic fields: $\bar{B} = \mu\bar{H}$

VECTOR MAGNETIC POTENTIAL

To define vector magnetic potential, we start with:

$$\oint_S \bar{B} \cdot d\bar{s} = 0$$

=> magnet poles have not been isolated

Using divergence theorem:

$$\oint_S \bar{B} \cdot d\bar{s} = \int_V \nabla \cdot \bar{B} dv = 0$$

$$\Leftrightarrow \nabla \cdot \bar{B} = 0$$

From vector identity:

$$\nabla \cdot (\nabla \times \bar{A}) = 0$$

where \bar{A} is any vector.

Therefore from Maxwell and identity vector, we can defined if \bar{A} is a vector magnetic potential, hence:

$$\bar{B} = \nabla \times \bar{A}$$

FORCE ON A MOVING POINT CHARGE

Force in electric field:

$$\bar{F}_e = Q\bar{E}$$

Force in magnetic field:

$$\bar{F}_m = Q\bar{U} \times \bar{B}$$

Total force:

$$\bar{F} = \bar{F}_e + \bar{F}_m \quad \text{or} \quad \bar{F} = Q(\bar{E} + \bar{U} \times \bar{B})$$

Also known as Lorentz force equation.

Force on charge in the influence of fields:

| Charge Condition | \vec{E} Field | \vec{B} Field | Combination \vec{E} and \vec{B} |
|------------------|-----------------|---------------------------|--|
| Stationary | $Q\vec{E}$ | - | $Q\vec{E}$ |
| Moving | $Q\vec{E}$ | $Q\vec{U} \times \vec{B}$ | $Q(\vec{E} + \vec{U} \times \vec{B})$ |

FORCE ON A FILAMENTARY CURRENT

The force on a differential current element , $I \bar{dl}$ due to the uniform magnetic field, \bar{B} :

$$d\bar{F} = I d\bar{l} \times \bar{B}$$

$$\bar{F} = \oint I d\bar{l} \times \bar{B} = -I \oint \bar{B} \times d\bar{l}$$

$$\bar{F} = -I \bar{B} \times \oint d\bar{l} = 0$$

It is shown that the net force for any close current loop in the uniform magnetic field is zero.

A semi-circle conductor carrying current I , is located in plane xy as shown in Fig.. The conductor is under the influence of uniform magnetic field, $\vec{B} = \hat{y}B_0$. Find:

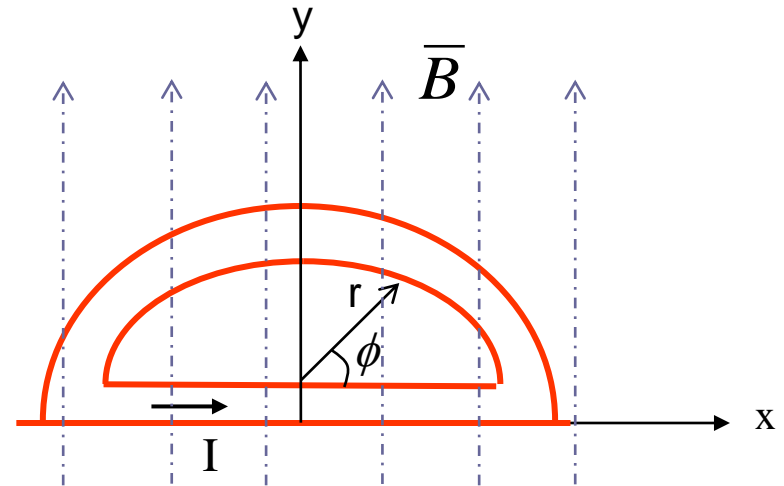
- (a) Force on a straight part of the conductor.
- (b) Force on a curve part of the conductor.

Solution:

- (a) The straight part length = $2r$.
Current flows in the x direction.

$$\vec{F} = \int I \vec{dl} \times \vec{B}$$

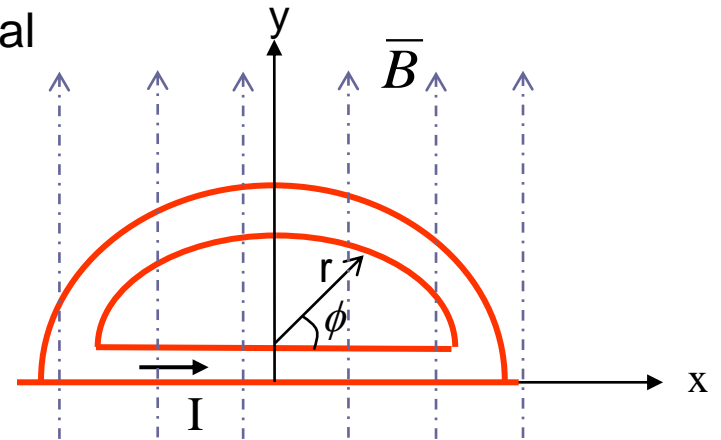
$$\vec{F}_1 = \hat{x}(2Ir) \times \hat{y}B_0 = \hat{z}2IrB_0 \text{ (N)}$$



(b) For curve part, $\overline{dl} \times \overline{B}$ will be in the (-ve) z direction and the magnitude is proportional to $\sin \phi$

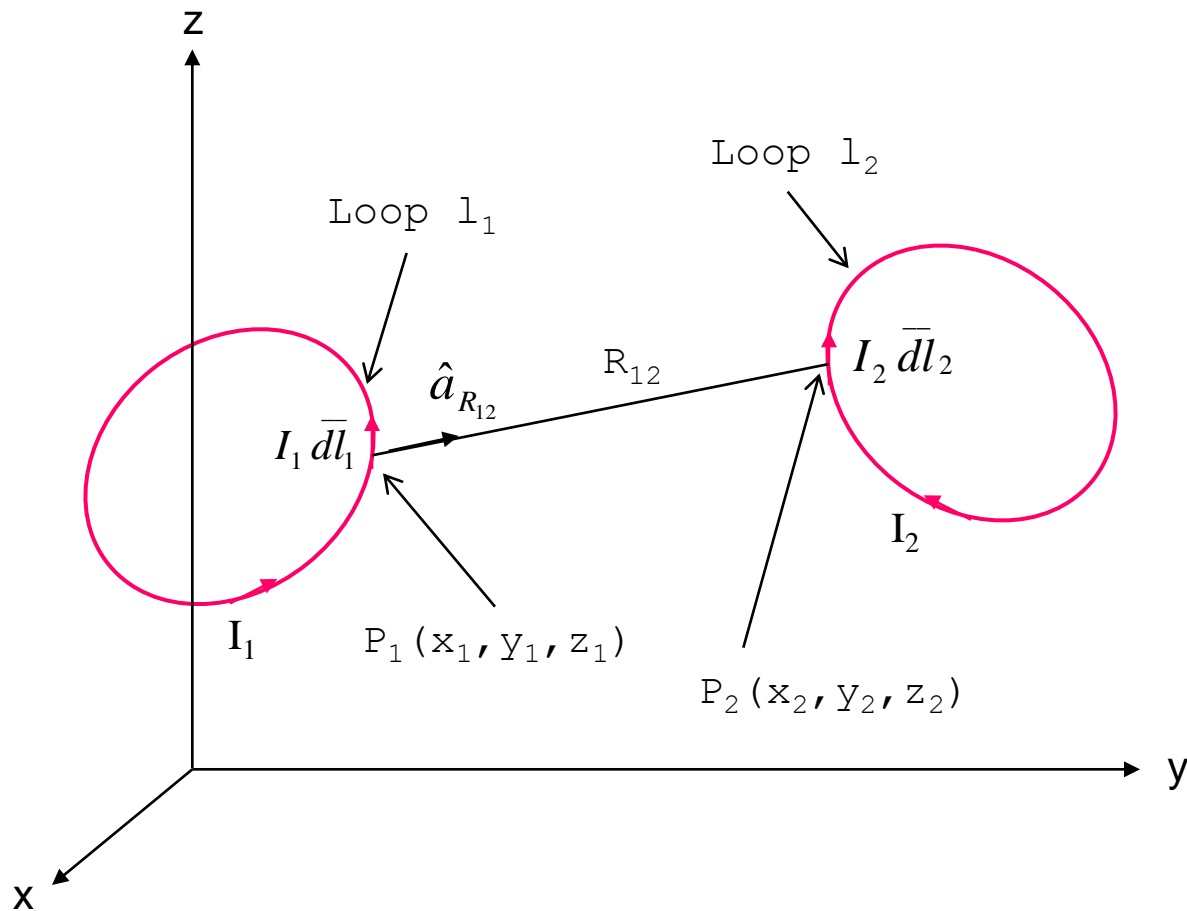
$$\overline{F}_2 = I \int_{\phi=0}^{\pi} \overline{dl} \times \overline{B}$$

$$= -\hat{z}I \int_{\phi=0}^{\pi} rB_0 \sin \phi d\phi = -\hat{z}2IrB_0 \text{ (N)}$$



Hence, it is observed that $\overline{F}_2 = -\overline{F}_1$ and it is shown that the net force on a close loop is zero.

FORCE BETWEEN TWO FILAMENTARY CURRENT

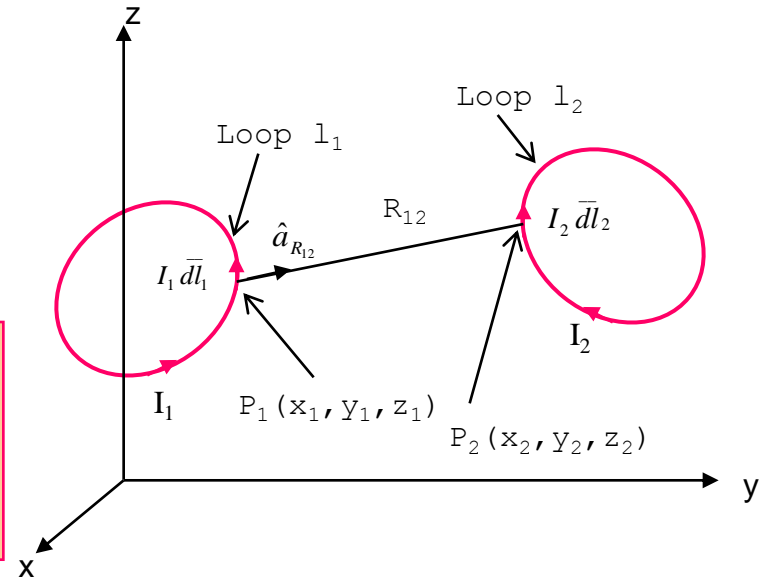


We have :

$$d\vec{F} = I d\vec{l} \times \vec{B} \quad (\text{N})$$

The magnetic field at point P_2 due to the filamentary current $I_1 d\vec{l}_1$:

$$d\vec{H}_2 = \frac{I_1 d\vec{l}_1 \times \hat{a}_{R_{12}}}{4\pi R_{12}^2} \quad (\text{A/m})$$



$$d(d\vec{F}_2) = I_2 d\vec{l}_2 \times \frac{\mu_o I_1 d\vec{l}_1 \times \hat{a}_{R_{12}}}{4\pi R_{12}^2}$$

$$(d\vec{F}_2) = I_2 d\vec{l}_2 \times \oint_{l_1} \frac{\mu_o I_1 d\vec{l}_1 \times \hat{a}_{R_{12}}}{4\pi R_{12}^2} = I_2 d\vec{l}_2 \times \vec{B}_2$$

where $d\vec{F}_2$ is the force due to $I_2 d\vec{l}_2$ and due to the magnetic field of loop I_1

Integrate:

$$\bar{F}_2 = \oint_{l_2} I_2 d\bar{l}_2 \times \oint_{l_1} \left[\frac{\mu_o I_1 d\bar{l}_1 \times \hat{a}_{R_{12}}}{4\pi R_{12}^2} \right]$$

$$\bar{F}_2 = \frac{\mu_o I_1 I_2}{4\pi} \oint_{l_2} \left[\oint_{l_1} \frac{(\hat{a}_{R_{12}} \times d\bar{l}_1)}{R_{12}^2} \right] \times d\bar{l}_2$$

For surface current :

$$\bar{F}_2 = \int_s \bar{J}_{s2} \times \bar{B}_2 ds$$

For volume current :

$$\bar{F}_2 = \int_v \bar{J}_2 \times \bar{B}_2 dv$$

A square conductor current loop is located in $z = 0$ plane with the edge given by coordinate $(1,0,0)$, $(1,2,0)$, $(3,0,0)$ and $(3,2,0)$ carrying a current of 2 mA in anti clockwise direction. A filamentary current carrying conductor of infinite length along the y axis carrying a current of 15 A in the $-y$ direction. Find the force on the square loop.

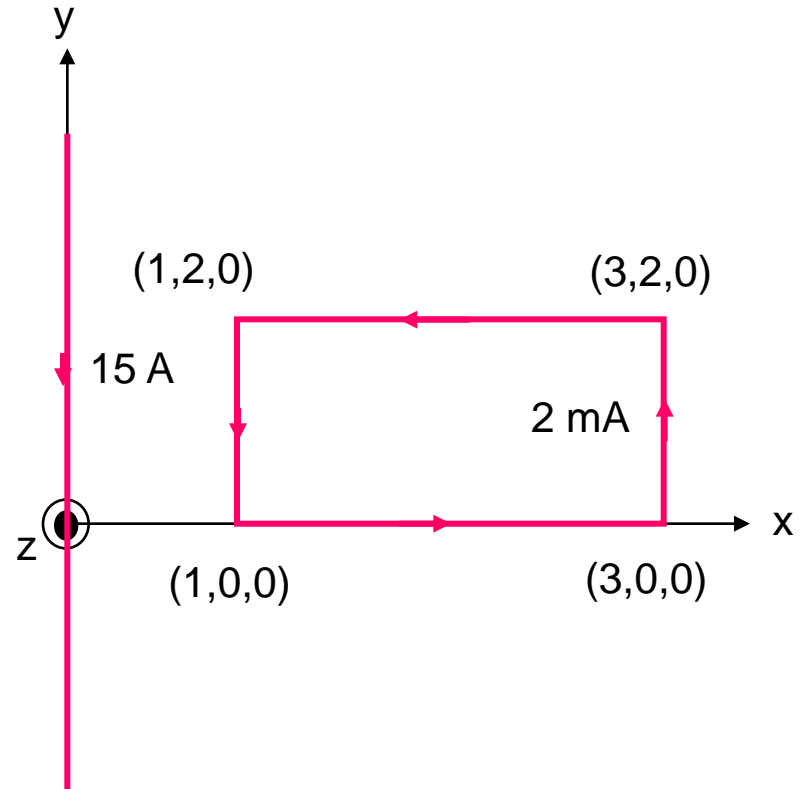
Solution:

Field created in the square loop due to filamentary current :

$$\bar{H} = \frac{I}{2\pi x} \hat{z} = \frac{15}{2\pi x} \hat{z} \text{ A/m}$$

$$\begin{aligned} \hat{\phi} &= \hat{a}_l \times \hat{a}_R \\ &= -\hat{y} \times \hat{x} = \hat{z} \end{aligned}$$

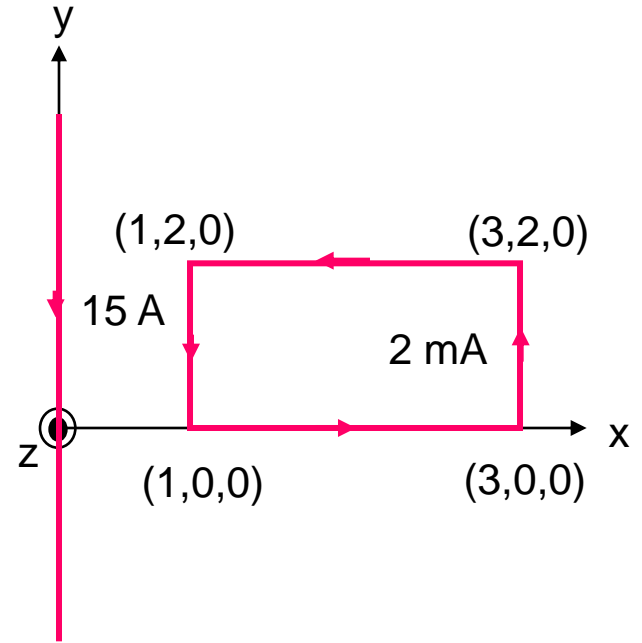
$$\therefore \bar{B} = \mu_0 \bar{H} = 4\pi \times 10^{-7} \bar{H} = \frac{3 \times 10^{-6}}{x} \hat{z} \text{ T}$$



Hence:

$$\bar{F} = \oint I \bar{dl} \times \bar{B} = -I \oint \bar{B} \times \bar{dl}$$

$$\bar{F} = -2 \times 10^{-3} \times 3 \times 10^{-6} \left[\int_{x=1}^3 \frac{\hat{z}}{x} \times dx \hat{x} + \int_{y=0}^2 \frac{\hat{z}}{3} \times dy \hat{y} + \int_{x=3}^1 \frac{\hat{z}}{x} \times dx \hat{x} + \int_{y=2}^0 \frac{\hat{z}}{1} \times dy \hat{y} \right]$$



$$\begin{aligned} \bar{F} &= -6 \times 10^{-9} \left[\ln x \Big|_1^3 \hat{y} + \frac{1}{3} y \Big|_0^2 (-\hat{x}) + \ln x \Big|_3^1 \hat{y} + y \Big|_2^0 (-\hat{x}) \right] \\ &= -6 \times 10^{-9} \left[(\ln 3) \hat{y} - \frac{2}{3} \hat{x} + \left(\ln \frac{1}{3} \right) \hat{y} + 2 \hat{x} \right] \\ &= -8 \hat{x} \text{ nN} \end{aligned}$$

MAGNETIC MATERIAL

The prominent characteristic of magnetic material is magnetic polarization – the alignment of its magnetic dipoles when a magnetic field is applied.

Through the alignment, the magnetic fields of the dipoles will combine with the applied magnetic field.

The resultant magnetic field will be increased.

MAGNETIC POLARIZATION (MAGNETIZATION)

Magnetic dipoles were the results of three sources of magnetic moments that produced magnetic dipole moments : (i) the orbiting electron about the nucleus (ii) the electron spin and (iii) the nucleus spin.

The effect of magnetic dipole moment will produce bound current or magnetization current.

Magnetic dipole moment in microscopic view is given by :

$$\overline{dm} = I \overline{ds} \quad \text{Am}^2$$

where \overline{dm} is magnetic dipole moment in discrete and I is the bound current.

In macroscopic view, magnetic dipole moment per unit volume can be written as:

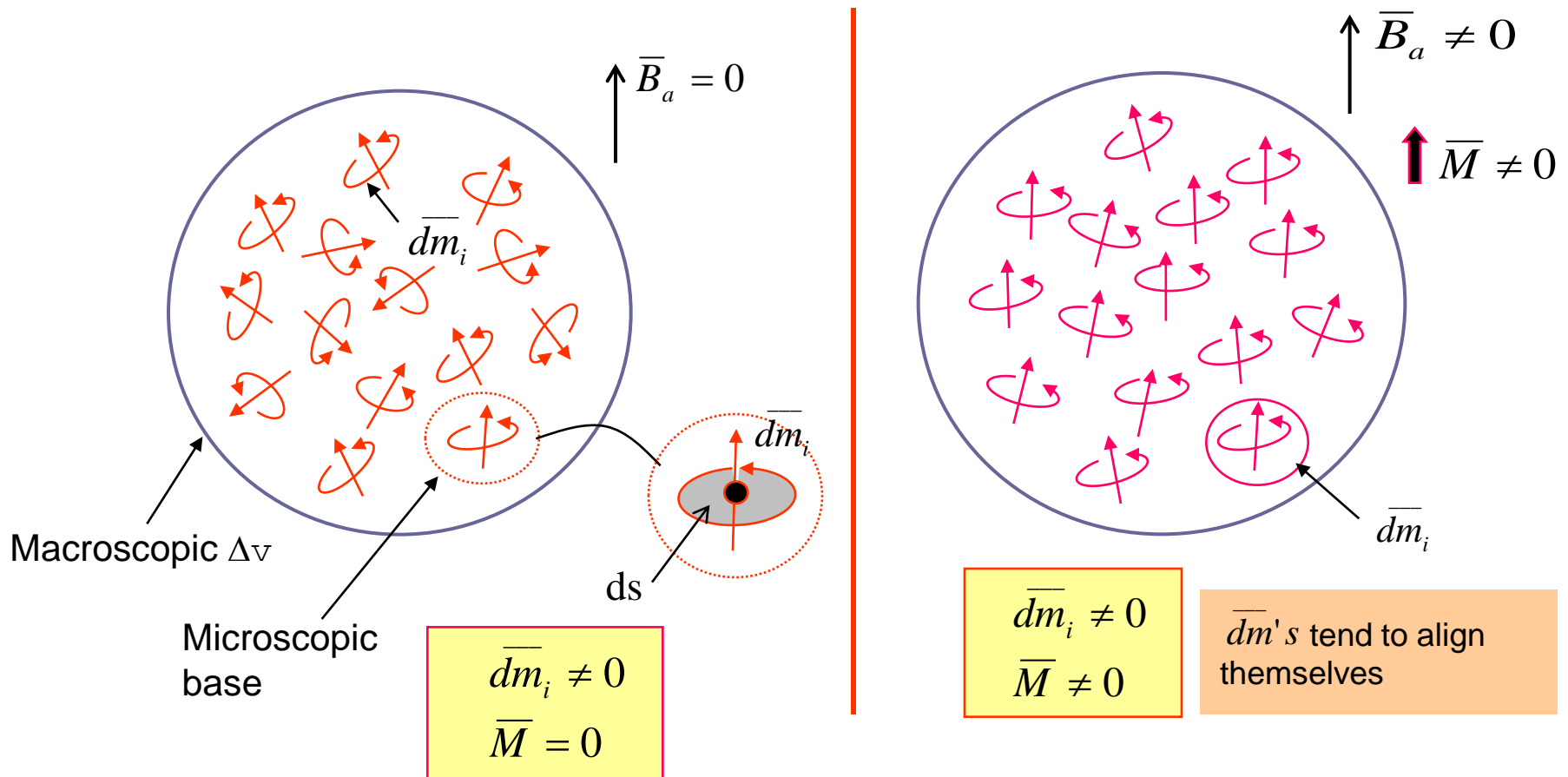
$$\overline{M} \cong \lim_{\Delta v \rightarrow 0} \left[\frac{1}{\Delta v} \sum_{i=1}^{n\Delta v} \overline{dm}_i \right] \quad \text{A/m}$$

where \overline{M} is a magnetization and n is the volume dipole density when $\Delta v \rightarrow 0$.

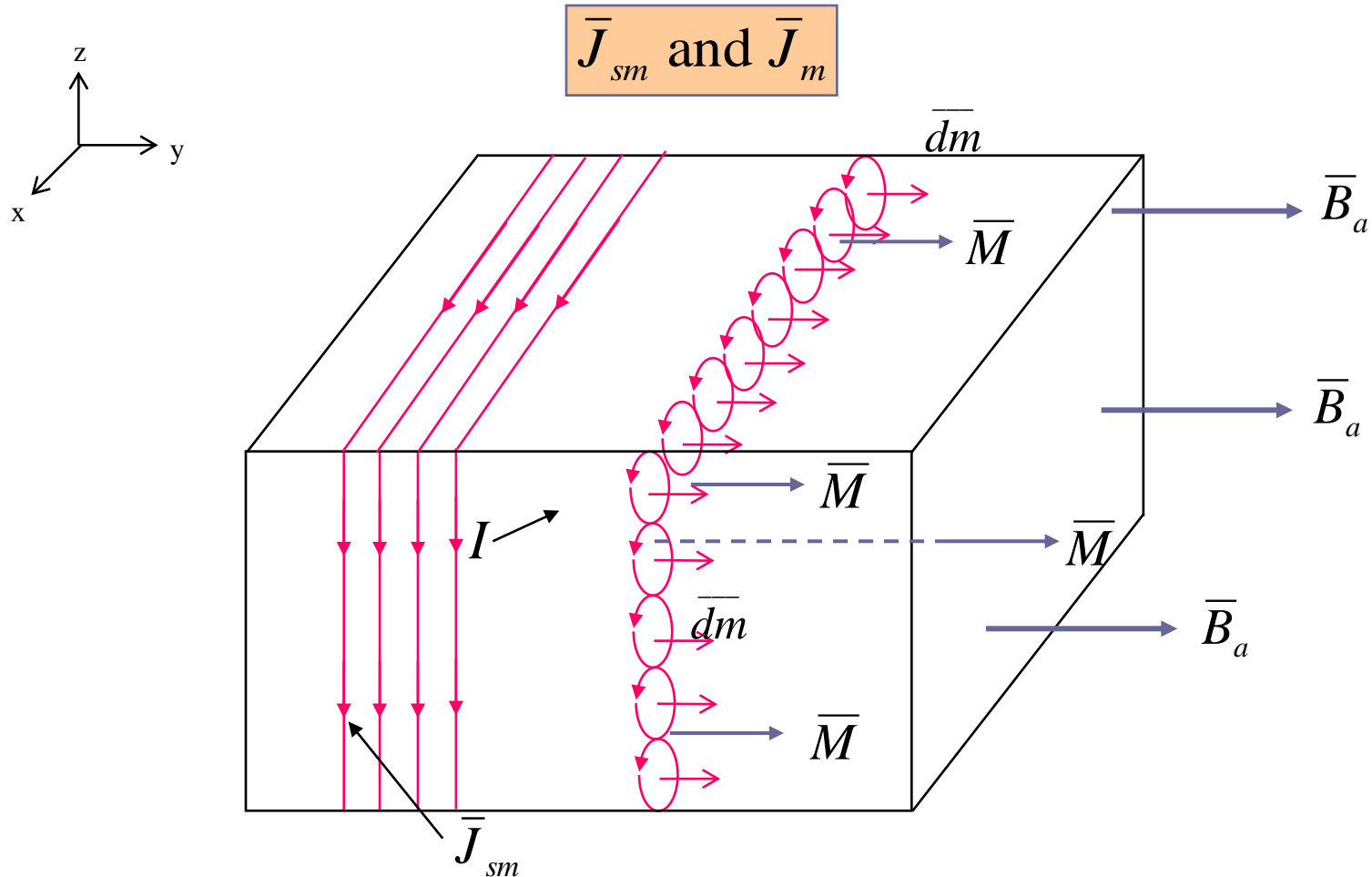
If the dipole moments become totally aligned :

$$\bar{M} = n \bar{dm} = nI \bar{ds} \quad \text{Am}^{-1}$$

Magnetic dipole moments in a magnetic material



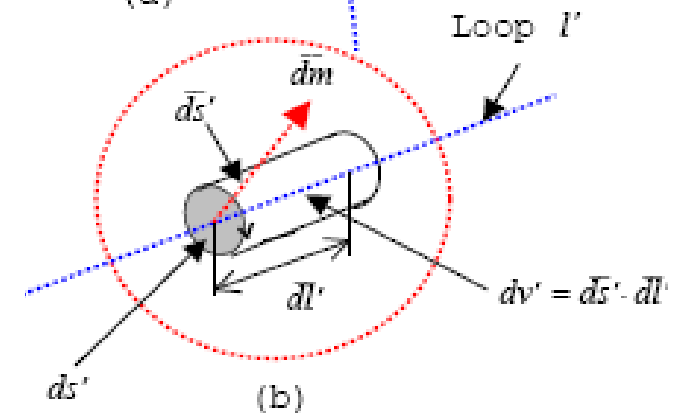
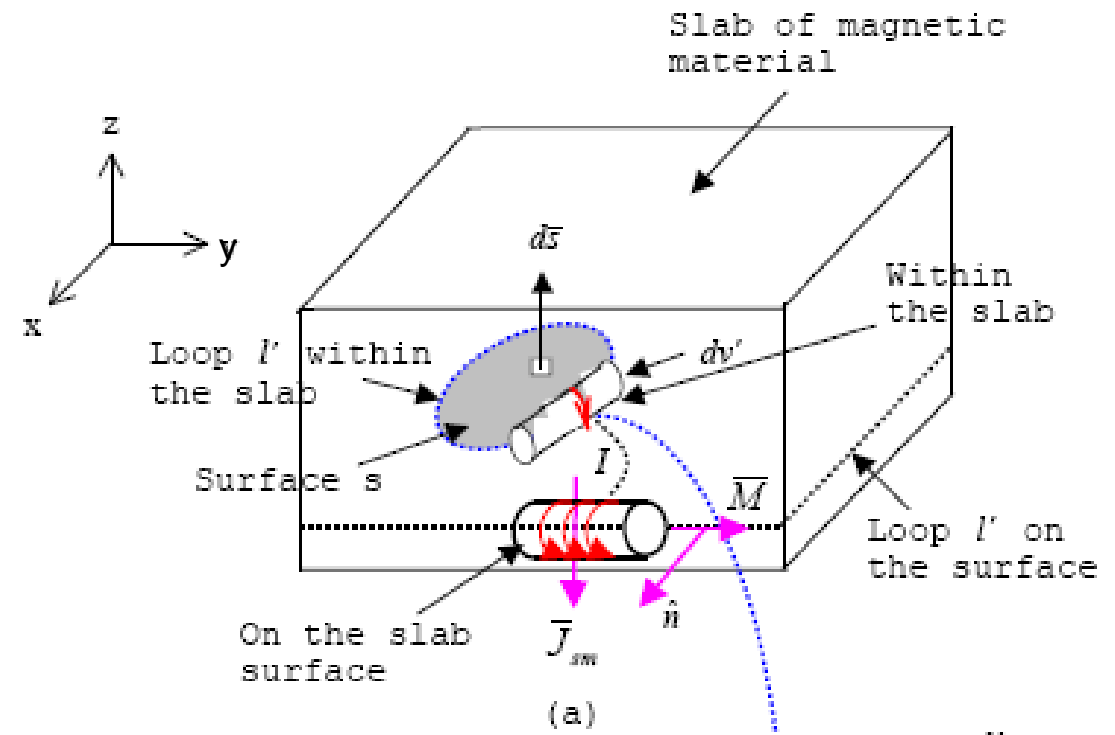
BOUND MAGNETIZATION CURRENT DENSITIES



Alignment of \bar{dm} 's within a magnetic material under uniform \bar{B}_a conditions to form a non zero \bar{J}_{sm} on the slab surfaces, and a $\bar{J}_m = 0$ within the material.

TO FIND

\bar{J}_{sm} and \bar{J}_m



Graphical display for finding expressions for \bar{J}_{sm} Am⁻¹ and \bar{J}_m Am⁻²: (a) slab of magnetic material with closed loop l' within the material and on the slab surface (b) expanded view of dv' about the loop l' .

Bound magnetization current :

$$dI_m = I n d\mathbf{v}'$$

$$dI_m = I(n \overline{d\mathbf{s}'} \cdot \overline{d\mathbf{l}'}) = (nI \overline{d\mathbf{s}'}) \cdot (\overline{d\mathbf{l}'})$$

We have:

$$\overline{\mathbf{M}} = n \overline{d\mathbf{m}} = nI \overline{d\mathbf{s}} \quad \text{Am}^{-1}$$

Hence:

$$dI_m = \overline{\mathbf{M}} \cdot \overline{d\mathbf{l}'}$$

Using Stoke's Theorem:

$$I_m = \int_s \overline{\mathbf{J}}_m \cdot \overline{d\mathbf{s}} = \oint_l \overline{\mathbf{M}} \cdot \overline{d\mathbf{l}'} = \int_s \nabla \times \overline{\mathbf{M}} \cdot \overline{d\mathbf{s}}$$

$$\overline{\mathbf{J}}_m = \nabla \times \overline{\mathbf{M}} \quad (\text{Am}^{-2})$$

is the bound magnetization current density within the magnetic material.

EFFECT OF MAGNETIZATION ON MAGNETIC FIELDS

Due to magnetization in a material, we have seen the formation of **bound magnetization** and **surface bound magnetization currents** density.

Maxwell's equation:

$$\begin{aligned}\nabla \times \bar{H} &= \bar{J} && \text{(free charge)} \\ \nabla \times \frac{\bar{B}}{\mu_o} &= \bar{J} && ; \quad \bar{B} = \mu_o \bar{H}\end{aligned}$$

$$\Rightarrow \nabla \times \frac{\bar{B}}{\mu_o} = (\bar{J} + \bar{J}_m)$$

$$\nabla \times \bar{M} = \bar{J}_m$$

$$\therefore \nabla \times \frac{\bar{B}}{\mu_o} = (\bar{J} + \nabla \times \bar{M})$$

$$\nabla \times \left(\frac{\bar{B}}{\mu_o} - \bar{M} \right) = \bar{J}$$

← due to free charges and bound magnetization currents

Define:

$$\bar{H} \cong \left(\frac{\bar{B}}{\mu_o} - \bar{M} \right)$$

$$\therefore \nabla \times \bar{H} = \bar{J}$$

Hence:

$$\bar{B} = \mu_o (\bar{H} + \bar{M})$$

Magnetization in isotropic material:

$$\bar{M} = \chi_m \bar{H}$$

χ_m = magnetic susceptibility

Hence:

$$\bar{B} = \mu_o \bar{H} (1 + \chi_m)$$

$$\mu_r \cong (1 + \chi_m)$$

$$\therefore \bar{B} = \mu \bar{H}$$

$\mu = \mu_o \mu_r = \text{permeability}$