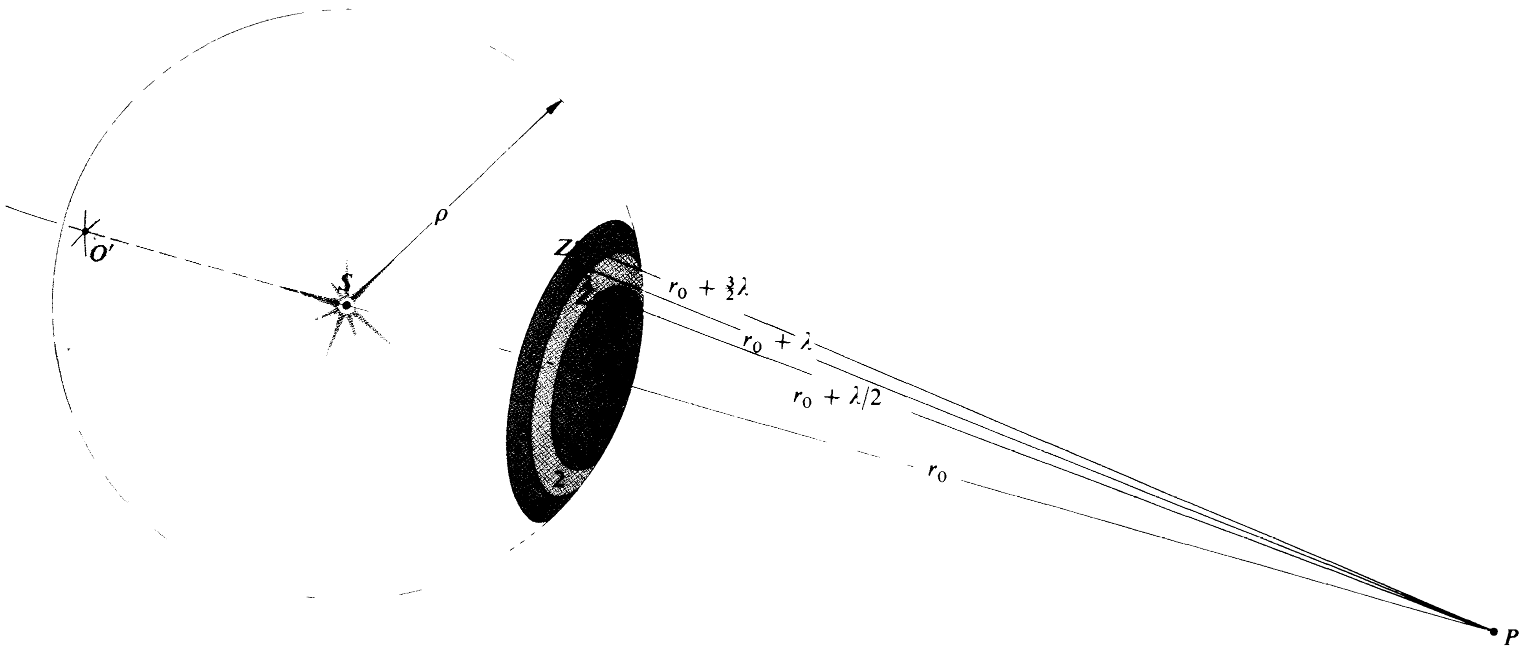
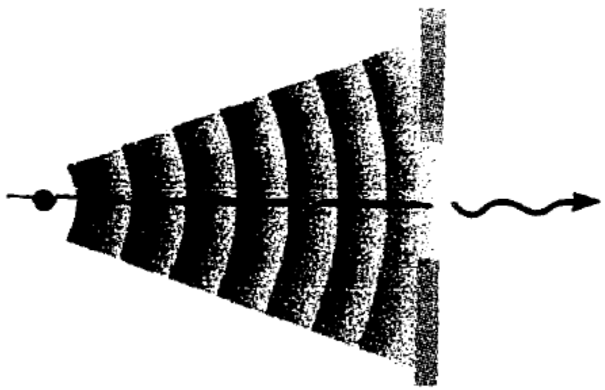
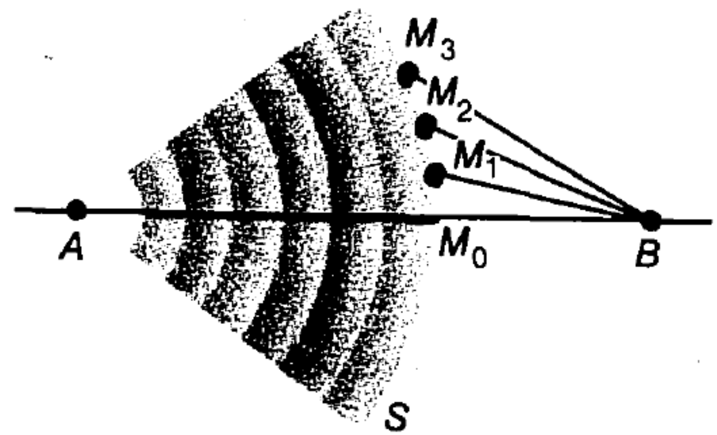
Fresnel Zone and Zone Plate



Fresnel Zone and Zone Plate



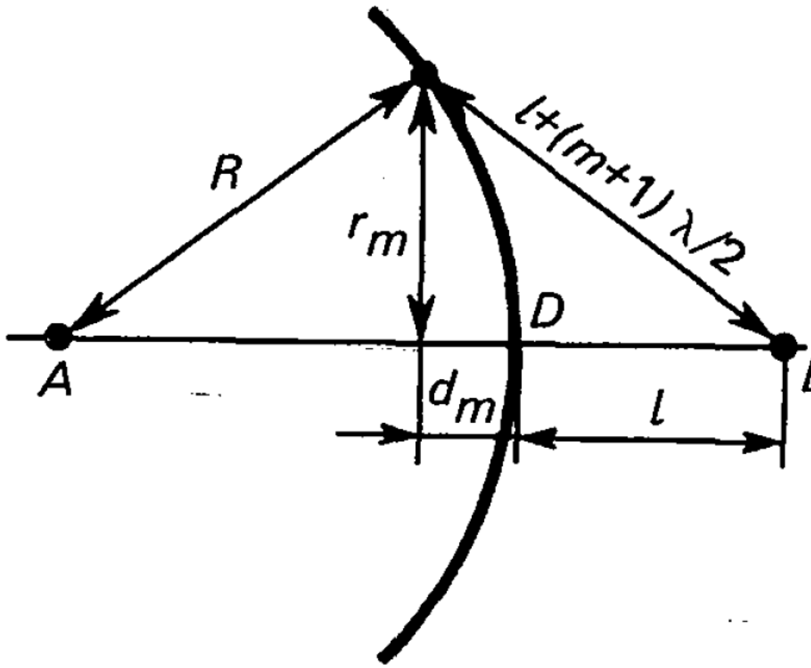


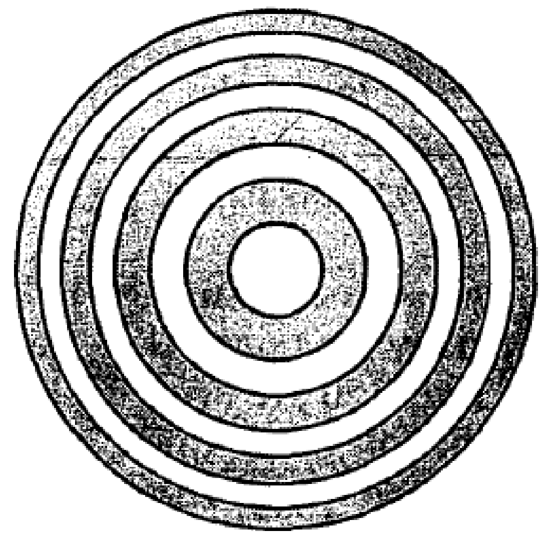
$$M_1B - M_0B = \frac{\lambda}{2}$$

$$M_2B - M_1B = \frac{\lambda}{2}$$

....

$$M_nB - Mn\ _{-\ 1}B = \frac{\lambda}{2}$$





$$r_m{}^2 = R^2 - (R - dm)^2 = [l + (m+1)\lambda/2]^2 - (l + dm)^2$$

$$d_m = \frac{l(m+1)\lambda}{(R+l)2} \qquad \mathrm{r_m} = \frac{Rl(m+1)\lambda}{(R+l)}$$

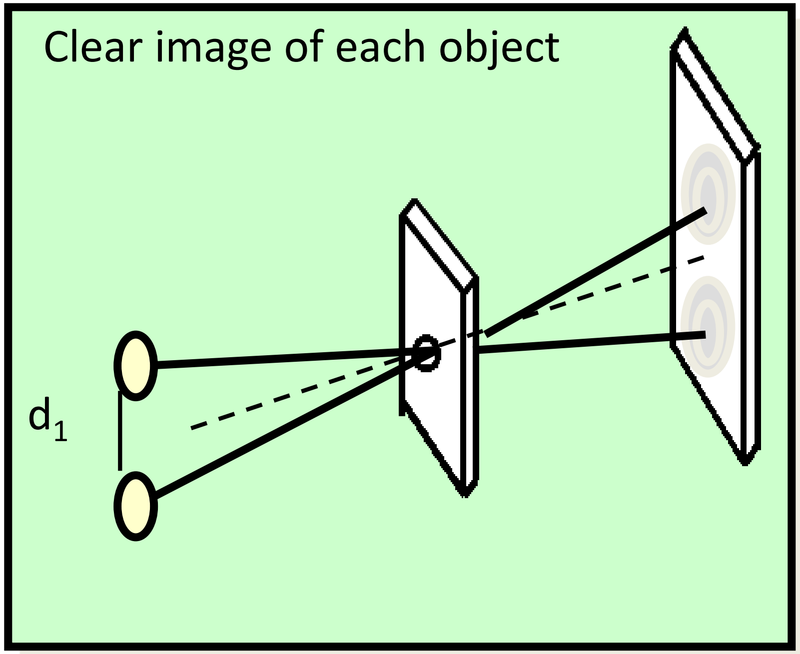
$$S_0 = \pi r_0{}^2 = \pi R l \lambda/(R+l)$$

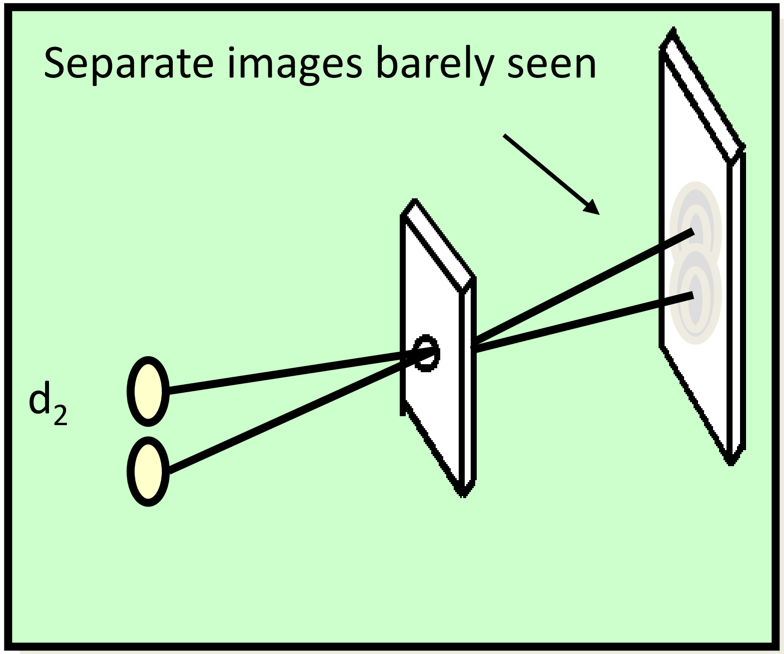
$$S_{0\,+\,1} = \pi R l 2\lambda/(R+l)$$

$$S_1 = S_{0\,+\,1} - S_0 = \pi R l/(R+l)$$

Resolution of Images

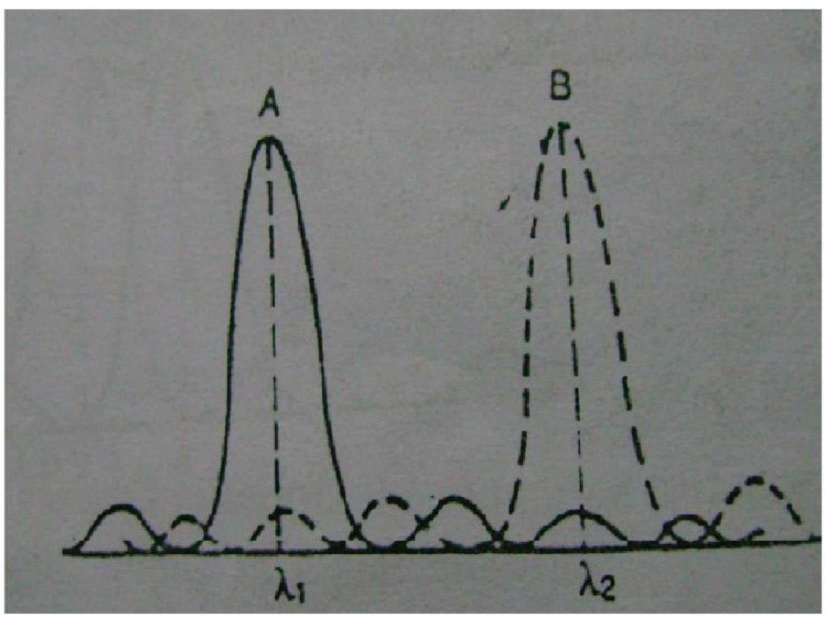
Consider light through a pinhole. As two objects get closer the interference fringes overlap, making it difficult to distinguish separate images.

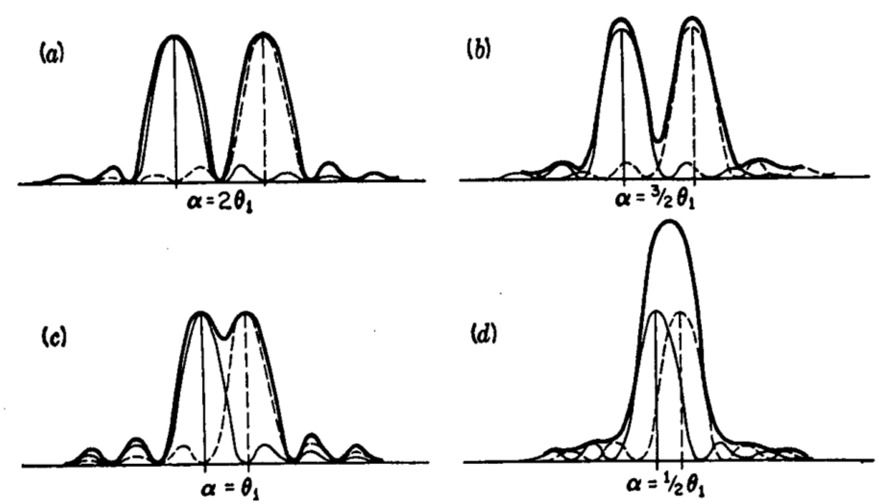




Basic Criterion of Resolution

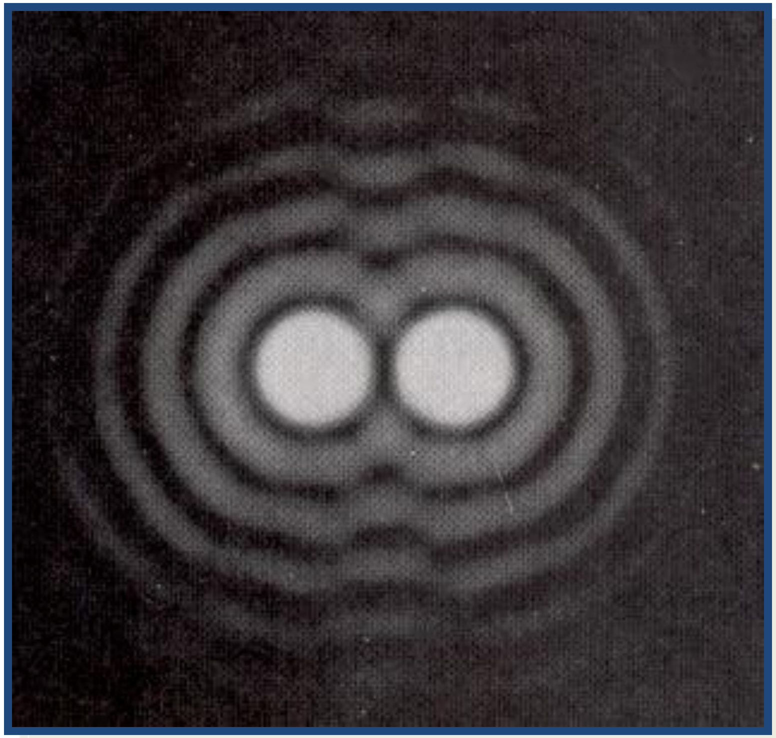
To express the resolving power of an optical instrument as a numerical value, Lord Rayleigh proposed an arbitrary criterion. According to him two nearby images are said to be resolved if the position of the central maximum of one coincides with the first secondary minimum of the other and vice-versa. The same criterion can be conveniently applied to calculate the resolving power of a telescope, microscope, grating, prism, etc.



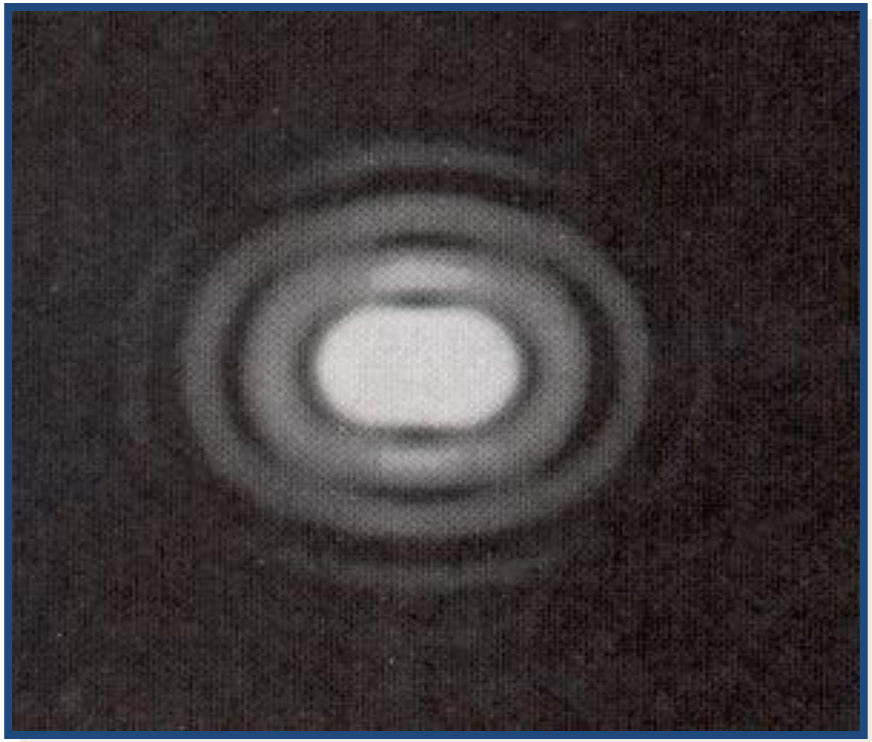


Resolution Limit

Images are just resolved when central maximum of one pattern coincides with first dark fringe of the other pattern.



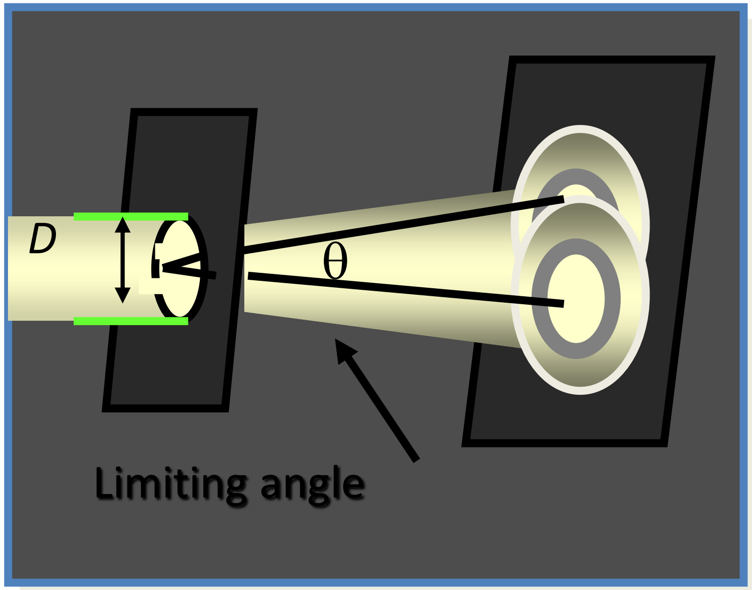
Separate images



Resolution Limit

Resolving Power of Instruments

The resolving power of an instrument is a measure of its ability to produce well-defined separate images.

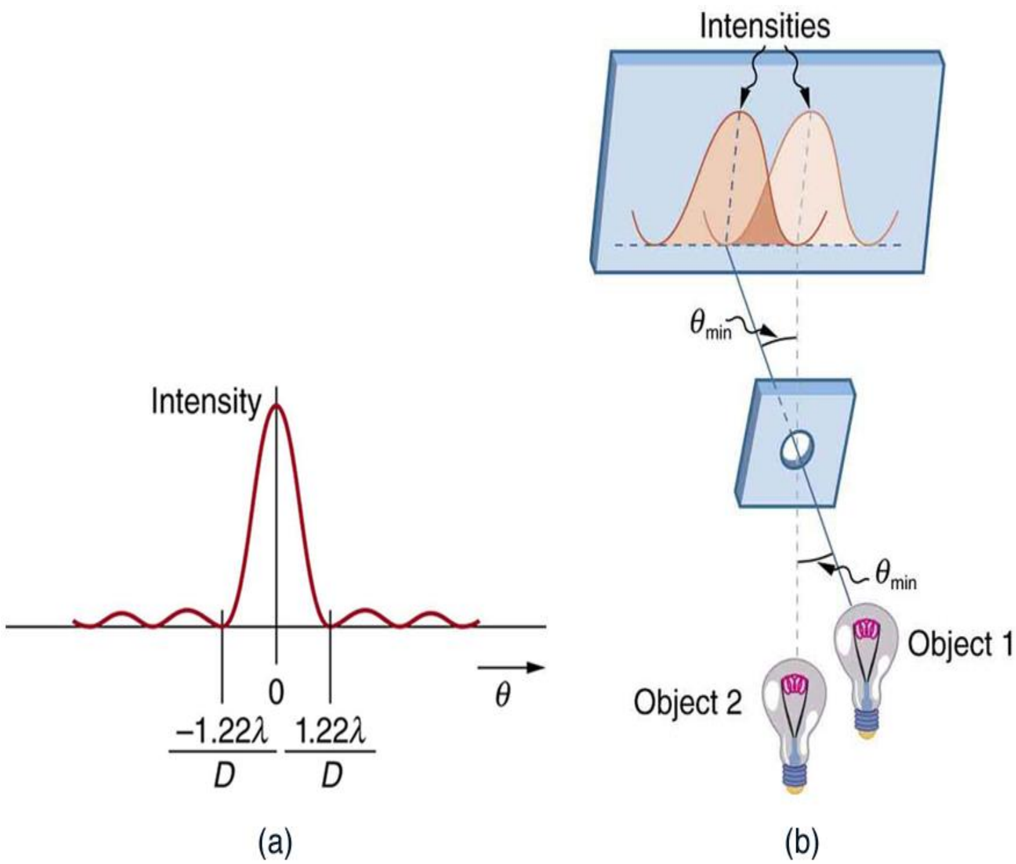


For small angles, sin θ ≅ θ, and the limiting angle of resolution for a circular opening is:

Limiting angle of resolution:

$$\theta_0 = 1.22\frac{\lambda}{D}$$

Rayleigh's Criterion



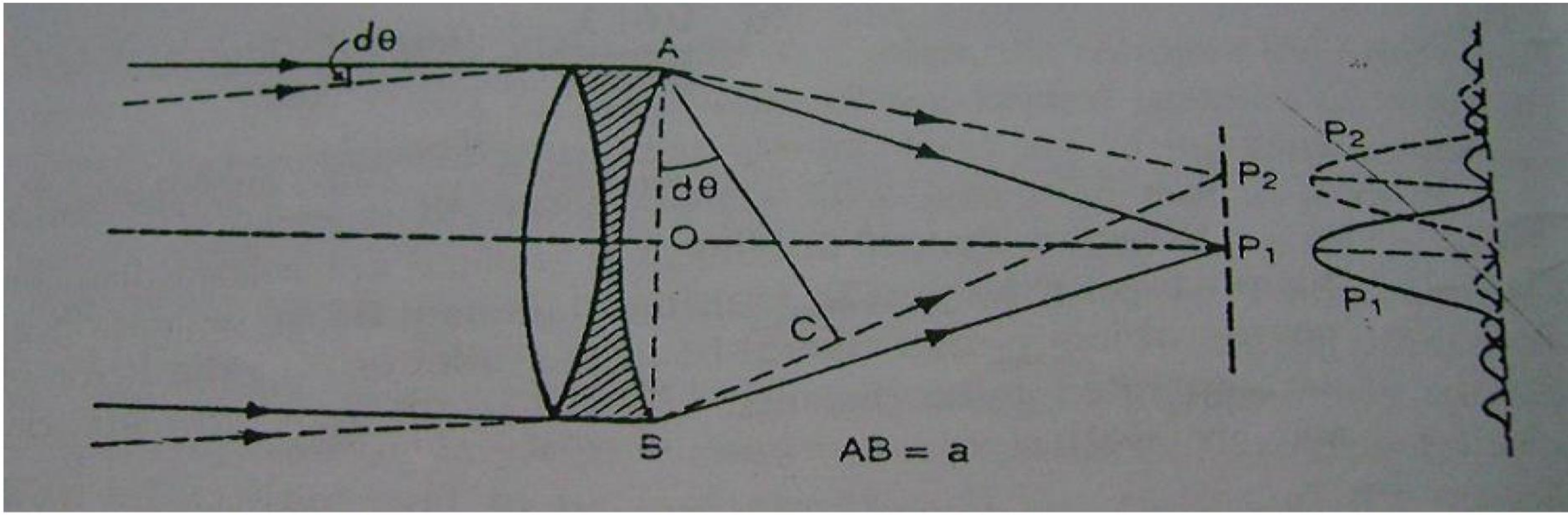
The **Rayleigh criterion** for the diffraction limit to resolution states that *two images are just resolvable when the center of the diffraction pattern of one is directly over the first minimum of the diffraction pattern of the other*.

The first minimum is at an angle of $\theta = \frac{1.22\lambda}{D}$, so that two point objects are just resolvable if they are separated by the angle

$$\theta = \frac{1.22\lambda}{D},$$

where λ is the wavelength of light (or other electromagnetic radiation) and D is the diameter of the aperture, lens, mirror, etc., with which the two objects are observed. In this expression, θ has units of radians.

Resolving Power of Telescope



Let 'd' be the diameter of the objective of the telescope considering the incident ray of light from two neighboring points of a distant object. The image of each point object is a Fraunhofer diffraction pattern. Let P1 and P2 be the position of the central maximum of two images. These two images are resolved if the position of central maximum of second image coincides with the first maximum of the first image and vice-versa. The path difference between the second wave traveling in the directions AP1 and BP1 is low and hence they reinforce with one another at P1. The secondary waves traveling in the directions AP2 and BP2 will meet at P2 on the screen. Let the angle P2AP1 be dθ. The path difference between the secondary waves traveling in the directions BP2 and AP2 is equal to BC.

$$The\ path\ difference\ in\ BP2\ and\ AP2\ is\ equal\ to\ BC.$$

Therefore,

$$BC = AN \sin d\theta = AB\ d\theta = a d\theta$$

If this path difference [$a\ d\theta = \lambda$], the position of P2 corresponds to the first minimum of the first image. But P2 is also the position of the central maximum of the second image. Thus Rayleigh's condition of resolution is satisfied if

$$a\ d\theta = \lambda \quad or \quad d\theta = \lambda/a$$

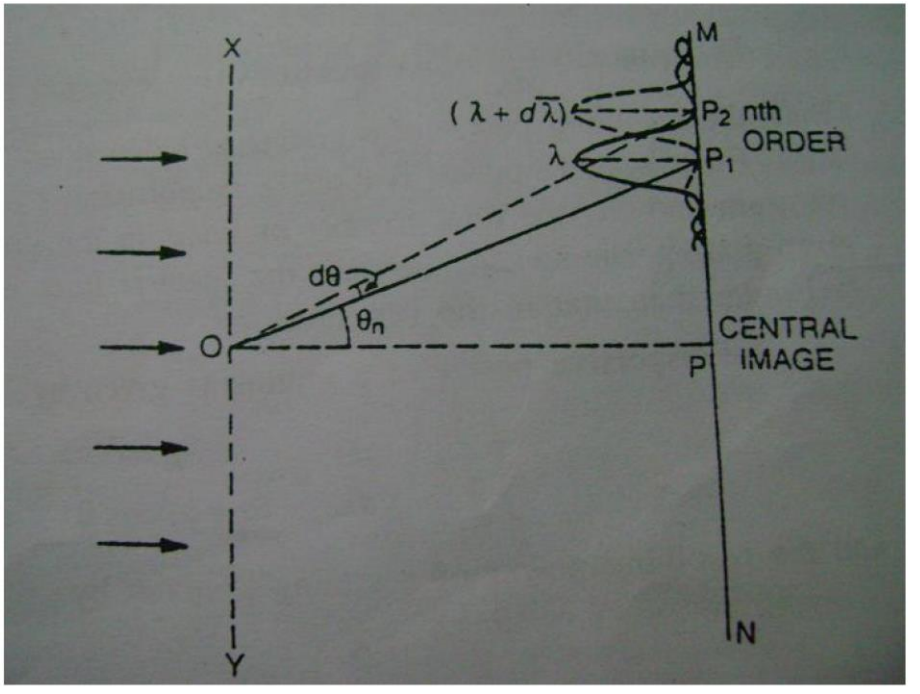
The whole aperture AB can be considered to be made up of two halves AO and OB. The path difference between the secondary waves from the corresponding points in the two halves will be $\lambda/2$. The equation $d\theta = \frac{\lambda}{a}$ holds good for rectangular aperture. For circular aperture this equation can be written as

$$d\theta = \frac{1.22\lambda}{a}$$

The reciprocal of $d\theta$ measures the resolving power of the telescope.
Therefore,

$$\frac{1}{d\theta} = \frac{a}{1.22\lambda}$$

Resolving Power of Grating



The resolving power of a grating is defined as the ratio of the wavelength of any spectral line to the difference in wavelength between this line and a neighboring line such that two lines appear to be just resolved.

In above figure, XY is a grating surface and MN is the field of view of the telescope.
P1 isthe n^{th} primary maximum of a spectral line of wavelength λ at an angle of diffraction θ_n .
P2 is the n^{th} primary maximum of a second spectral line of wavelength $\lambda + d\lambda$ at a diffracting angle $\theta_n + d\theta$.

P1 and P2 are the spectral lines in the n^th order. The direction of n^th primary maximum for a wavelength λ is given by

$$(a+b)sin\theta_n = n\lambda \qquad \text{--------------------} \quad 1$$

The direction of n^th primary maximum for a wavelength $(\lambda + d\lambda)$ is given by

$$(a+b)\sin(\theta_n + d\theta) = n(\lambda + d\lambda) \qquad \text{------------------------} \quad 2$$

The two lines will appear just resolved if the angle of diffraction $(\theta_n + d\theta)$ also corresponds to the direction of the first secondary minimum after the n^th primary maximum at P1.

This is possible if the extra path difference introduced is $\frac{\lambda}{N}$ where N is the total number of lines of the grating surface. Therefore,

$$(a+b)\sin(\theta_n + d\theta) = n\lambda + \lambda/N \qquad \text{---------------------------} \quad 3$$

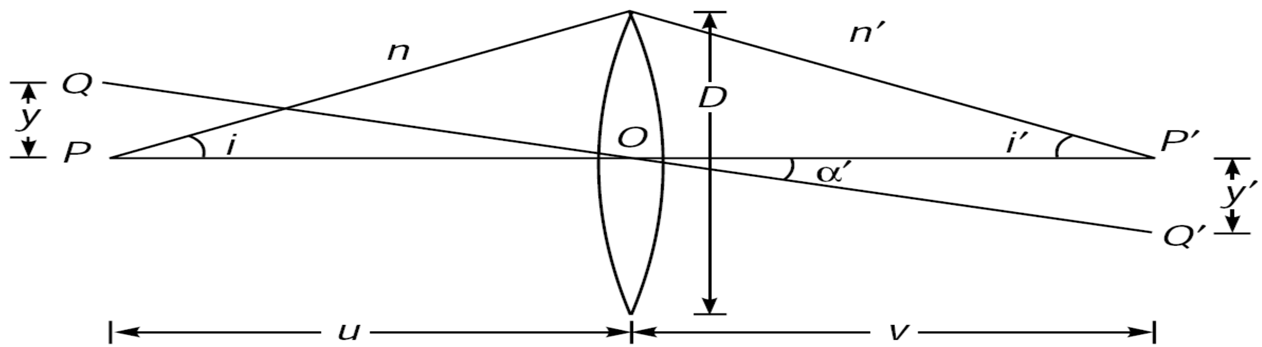
Equating the right hand sides of (2) and (3), we get

$$n(\lambda + d\lambda) = n\lambda + \lambda/N$$
$$\frac{\lambda}{d\lambda} = nN$$

The quantity $\frac{\lambda}{d\lambda} = nN$, measures the resolving power of a grating.

Resolving Power of Microscope

$$\sin \alpha' \approx \frac{1.22\lambda}{D} = \frac{1.22\lambda_0}{n'D}$$



$$\sin \alpha' \approx \frac{y'}{OP'} = \frac{y' \tan i'}{D/2} \approx \frac{y' \sin i'}{D/2}$$

$$y' \approx \frac{0.61\lambda_0}{n' \sin i'}$$

$$y \approx \frac{0.61\lambda_0}{n \sin i}$$